Chapter 4

1 through 8 - Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss:

\[ \begin{align*}
\sum F_x &= 0: \quad B_x = 0 \\
\sum F_y &= 0: \quad B_y = 0 \\
\sum M_C &= 0: \quad -B_x(3.2\,\text{m}) - (48\,\text{kN})(7.2\,\text{m}) = 0
\end{align*} \]

\[ B_x = -108\,\text{kN} \quad B_y = 108\,\text{kN} \]

Free body: Joint B:

\[ F_{AB} = \frac{108\,\text{kN}}{3} = 36\,\text{kN} \]

\[ F_{BC} = 108\,\text{kN} \]

\[ F_{AB} = 180.0\,\text{kN} \quad \text{T} \downarrow \]

\[ F_{BC} = 144.0\,\text{kN} \quad \text{T} \downarrow \]

Free body: Joint C:

\[ F_{BC} = 144\,\text{kN} \]

\[ F_{AC} = \frac{60\,\text{kN}}{5} = 12\,\text{kN} \]

\[ F_{BC} = 144\,\text{kN} \quad \text{(checks)} \]

\[ F_{AC} = 156.0\,\text{kN} \quad \text{C} \uparrow \]
SOLUTION

Reactions:

\[ \Sigma M_A = 0: \quad C = 1260 \text{ lb} \uparrow \]
\[ \Sigma F_x = 0: \quad \Delta x = 0 \]
\[ \Sigma F_y = 0: \quad \Delta y = 960 \text{ lb} \downarrow \]

Joint B:

\[ \frac{F_{AB}}{12} = \frac{F_{BC}}{13} = \frac{300 \text{ lb}}{5} \]

\[ F_{AB} = 720 \text{ lb} \quad T \downarrow \]
\[ F_{BC} = 780 \text{ lb} \quad C \downarrow \]

Joint A:

\[ + \Sigma F_y = 0: \quad -960 \text{ lb} - \frac{4}{5} F_{AC} = 0 \]

\[ F_{AC} = 1200 \text{ lb} \]

\[ - \Sigma F_x = 0: \quad 720 \text{ lb} - (1200 \text{ lb}) \frac{3}{5} = 0 \] (checks)
**SOLUTION**

\[ AB = \sqrt{3^2 + 1.25^2} = 3.25 \text{ m} \]
\[ BC = \sqrt{3^2 + 4^2} = 5 \text{ m} \]

**Reactions:**

\[ + \sum \Sigma M_A = 0: \quad (84 \text{ kN})(3 \text{ m}) - C(5.25 \text{ m}) = 0 \]
\[ C = 48 \text{ kN} \]

\[ \sum \Sigma F_x = 0: \quad A_x - C = 0 \]
\[ A_x = 48 \text{ kN} \]

\[ \sum \Sigma F_y = 0: \quad A_y = 84 \text{ kN} = 0 \]
\[ A_y = 84 \text{ kN} \]

**Joint A:**

\[ \sum \Sigma F_x = 0: \quad 48 \text{ kN} - \frac{12}{13} F_{AB} = 0 \]
\[ F_{AB} = 52 \text{ kN} \]

\[ \sum \Sigma F_y = 0: \quad 84 \text{ kN} - \frac{5}{13} (52 \text{ kN}) - F_{AC} = 0 \]
\[ F_{AC} = 64 \text{ kN} \]

**Joint C:**

\[ F_{BC} = \frac{48 \text{ kN}}{3} \]

\[ F_{BC} = 80 \text{ kN} \]
SOLUTION

Free body: Truss:

From the symmetry of the truss and loading, we find

\[ C = D = 600 \text{ lb} \]

Free body: Joint B:

\[ \frac{F_{AB}}{\sqrt{5}} = \frac{F_{BC}}{2} = \frac{300 \text{ lb}}{1} \]

\[ F_{AB} = 671 \text{ lb} \quad T \quad F_{BC} = 600 \text{ lb} \quad C \uparrow \]

Free body: Joint C:

\[ \Sigma F_x = 0: \quad \frac{3}{5} F_{AC} + 600 \text{ lb} = 0 \]

\[ F_{AC} = -1000 \text{ lb} \quad F_{AC} = 1000 \text{ lb} \quad C \uparrow \]

\[ \Sigma F_y = 0: \quad \frac{4}{5} (-1000 \text{ lb}) + 600 \text{ lb} + F_{CD} = 0 \]

\[ F_{CD} = 200 \text{ lb} \quad T \uparrow \]

From symmetry:

\[ F_{AD} = F_{AC} = 1000 \text{ lb} \quad C, \quad F_{AE} = F_{AB} = 671 \text{ lb} \quad T, \quad F_{DE} = F_{BC} = 600 \text{ lb} \quad C \uparrow \]
SOLUTION

Reactions:

\[ \Sigma M_D = 0: \quad F_y (1) - (4 + 2.4)(12) - (1)(24) = 0 \]
\[ F_y = 4.2 \text{ kips} \]

\[ \Sigma F_x = 0: \quad F_x = 0 \]
\[ \Sigma F_y = 0: \quad D - (1 + 4 + 1 + 2.4) + 4.2 = 0 \]
\[ D = 4.2 \text{ kips} \]

Joint A:

\[ \Sigma F_x = 0: \quad F_{AB} = 0 \]
\[ \Sigma F_y = 0: \quad -1 - F_{AD} = 0 \]
\[ F_{AD} = -1 \text{ kip} \]
\[ F_{AB} = 1.00 \text{ kip} \]

Joint D:

\[ \Sigma F_x = 0: \quad -1 + 4.2 + \frac{8}{17} F_{BD} = 0 \]
\[ F_{BD} = -6.8 \text{ kips} \]
\[ F_{BD} = 6.80 \text{ kips} \]

Joint E:

\[ \Sigma F_y = 0: \quad F_{BE} = 2.4 \text{ kips} \]
\[ F_{BE} = +2.4 \text{ kips} \]

Truss and loading symmetrical about \( L \).
SOLUTION

Reactions:

\[ \Sigma F_x = 0: \quad D_x = 0 \]
\[ \Sigma M_x = 0: \quad D_y (21 \text{ ft}) - (693 \text{ lb})(5 \text{ ft}) = 0 \quad D_y = 165 \text{ lb} \]
\[ \Sigma F_y = 0: \quad 165 \text{ lb} - 693 \text{ lb} + E = 0 \quad E = 528 \text{ lb} \]

Joint \( D \):

\[ \Sigma F_x = 0: \quad \frac{5}{13} F_{AD} + \frac{4}{5} F_{DC} = 0 \]
\[ \Sigma F_y = 0: \quad \frac{12}{13} F_{AD} + \frac{3}{5} F_{DC} + 165 \text{ lb} = 0 \]

Solving Eqs. (1) and (2) simultaneously,

\[ F_{AD} = -260 \text{ lb} \]
\[ F_{DC} = +125 \text{ lb} \]

\[ F_{AD} = 260 \text{ lb} \quad C \uparrow \]
\[ F_{DC} = 125 \text{ lb} \quad T \downarrow \]
Joint E:

\[ \Sigma F_x = 0: \quad \frac{5}{13} F_{BE} + \frac{4}{5} F_{CE} = 0 \]  \quad (3)

\[ \Sigma F_y = 0: \quad \frac{12}{13} F_{BE} + \frac{3}{5} F_{CE} + 528 \text{ lb} = 0 \]  \quad (4)

Solving Eqs. (3) and (4) simultaneously,

\[ F_{BE} = 832 \text{ lb} \]
\[ F_{CE} = 400 \text{ lb} \]

Joint C:

Force polygon is a parallelogram (see Fig. 6.11, p. 209).

\[ F_{AC} = 400 \text{ lb} \]
\[ F_{BC} = 125.0 \text{ lb} \]

Joint A:

\[ \Sigma F_x = 0: \quad \frac{5}{13} (260 \text{ lb}) + \frac{4}{5} (400 \text{ lb}) + F_{AB} = 0 \]

\[ F_{AB} = -420 \text{ lb} \]

\[ \Sigma F_y = 0: \quad \frac{12}{13} (260 \text{ lb}) - \frac{3}{5} (400 \text{ lb}) = 0 \]

\[ 0 = 0 \quad \text{(Checks)} \]
SOLUTION

Free body: Entire truss:
\[ \pm \Sigma F_x = 0: \quad C_x + 2(5 \text{ kN}) = 0 \]
\[ C_x = -10 \text{ kN} \quad \bar{C}_x = 10 \text{ kN} \]
\[ \pm \Sigma M_C = 0: \quad D(2 \text{ m}) - (5 \text{ kN})(8 \text{ m}) - (5 \text{ kN})(4 \text{ m}) = 0 \]
\[ D = +30 \text{ kN} \quad \bar{D} = 30 \text{ kN} \]
\[ \pm \Sigma F_y = 0: \quad C_y + 30 \text{ kN} = 0 \quad C_y = -30 \text{ kN} \quad \bar{C}_y = 30 \text{ kN} \]

Free body: Joint A:
\[ F_{AB} = \frac{5 \text{ kN}}{\sqrt{17}} = 5 \text{ kN} \]
\[ F_{AD} = 20.6 \text{ kN} \quad T \]

Free body: Joint B:
\[ F_{BD} = -5\sqrt{5} \text{ kN} \]
\[ F_{BD} = 11.18 \text{ kN} \quad C \]
\[ F_{BC} = +30 \text{ kN} \quad F_{BC} = 30.0 \text{ kN} \quad T \]

Free body: Joint C:
\[ F_{CD} = +10 \text{ kN} \quad F_{CD} = 10.00 \text{ kN} \quad T \]
\[ F_{CD} = 10 \text{ kN} \quad F_{CD} = +10 \text{ kN} \]
\[ \pm \Sigma F_y = 0: \quad 30 \text{ kN} - 30 \text{ kN} = 0 \quad \text{(checks)} \]
SOLUTION

Reactions:

\[ \Sigma M_C = 0: \quad A_y = 16 \text{kN} \]
\[ \Sigma F_y = 0: \quad A_y = 9 \text{kN} \]
\[ \Sigma F_x = 0: \quad C = 16 \text{kN} \]

Joint E:

\[ \frac{F_{BE}}{5} = \frac{F_{DE}}{4} = \frac{3 \text{kN}}{3} \]
\[ F_{BE} = 5.00 \text{kN} \quad T \]
\[ F_{DE} = 4.00 \text{kN} \quad C \]

Joint B:

\[ \begin{align*}
\Sigma F_x &= 0: \quad \frac{4}{5} (5 \text{kN}) - F_{AB} = 0 \\
F_{AB} &= +4 \text{kN} \\
F_{AB} &= 4.00 \text{kN} \quad T \\
\Sigma F_y &= 0: -6 \text{kN} - \frac{3}{5} (5 \text{kN}) - F_{BD} = 0 \\
F_{BD} &= -9 \text{kN} \\
F_{BD} &= 9.00 \text{kN} \quad C \\
\end{align*} \]

Joint D:

\[ \begin{align*}
\Sigma F_x &= 0: -9 \text{kN} + \frac{3}{5} F_{AD} = 0 \\
F_{AD} &= +15 \text{kN} \\
F_{AD} &= 15.00 \text{kN} \quad T \\
\Sigma F_y &= 0: -4 \text{kN} - \frac{4}{5} (15 \text{kN}) - F_{CD} = 0 \\
F_{CD} &= -16 \text{kN} \\
F_{CD} &= 16.00 \text{kN} \quad C \\
\end{align*} \]
9- Determine the force in each member of the Gambrel roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:

\[ \Sigma F_y = 0: \quad H_y = 0 \]

Because of the symmetry of the truss and loading,

\[ A = H_y = \frac{1}{2} \text{ total load} \]

\[ A = H_y = 1200 \text{ lb} \]

Free body: Joint A:

\[ \frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3} \]

\[ F_{AB} = 1500 \text{ lb} \quad C \uparrow \]

\[ F_{AC} = 1200 \text{ lb} \quad T \uparrow \]

Free body: Joint C:

\[ F_{BC} = 0 \]

Free body: Joint B:

\[ + \Sigma F_x = 0: \quad \frac{24}{25} F_{BD} + \frac{4}{5} F_{BE} + \frac{4}{5} (1500 \text{ lb}) = 0 \]

or

\[ 24 F_{BD} + 20 F_{BE} = -30,000 \text{ lb} \quad (1) \]

or

\[ + \Sigma F_y = 0: \quad \frac{7}{25} F_{BD} - \frac{3}{5} F_{BE} + \frac{3}{5} (1500) - 600 = 0 \]

or

\[ 7 F_{BD} - 15 F_{BE} = -7,500 \text{ lb} \quad (2) \]
Multiply Eq. (1) by 3, Eq. (2) by 4, and add:
\[ 100F_{BD} = -120,000 \text{ lb} \]
\[ F_{BD} = 1200 \text{ lb} \]

Multiply Eq. (1) by 7, Eq. (2) by −24, and add:
\[ 500F_{BE} = -30,000 \text{ lb} \]
\[ F_{BE} = 60.0 \text{ lb} \]

**Free body: Joint D:**
\[ \sum F_x = 0: \quad \frac{24}{25} (1200 \text{ lb}) + \frac{24}{25} F_{DF} = 0 \]
\[ F_{DF} = -1200 \text{ lb} \]
\[ F_{DF} = 1200 \text{ lb} \]

\[ \sum F_y = 0: \quad \frac{7}{25} (1200 \text{ lb}) - \frac{7}{25} (-1200 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0 \]
\[ F_{DE} = 72.0 \text{ lb} \]
\[ F_{DE} = 72.0 \text{ lb} \]

Because of the symmetry of the truss and loading, we deduce that
\[ F_{EF} = F_{BE} \]
\[ F_{EG} = F_{CE} \]
\[ F_{FG} = F_{BC} \]
\[ F_{FH} = F_{AB} \]
\[ F_{GH} = F_{AC} \]
10- Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:

\[ \Sigma F_x = 0: \quad H_x = 0 \]

Because of the symmetry of the truss and loading,

\[ A = H_y = \frac{1}{2} \text{total load} \]

\[ A = H_y = 1200 \text{ lb} \uparrow \]

Free body: Joint A:

\[ \frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3} \]

\[ F_{AB} = 1500 \text{ lb} \quad C \uparrow \]

\[ F_{AC} = 1200 \text{ lb} \quad T \uparrow \]

Free body: Joint C:

BC is a zero-force member.

\[ F_{BC} = 0 \]

\[ F_{CE} = 1200 \text{ lb} \quad T \uparrow \]
Free body: Joint R:

\[ \pm \sum F_x = 0: \quad \frac{4}{5} F_{BD} + \frac{4}{5} F_{BC} + \frac{4}{5} (1500 \text{ lb}) = 0 \]

or

\[ F_{BD} + F_{BE} = -1500 \text{ lb} \quad (1) \]

or

\[ F_{BD} - F_{BE} = -500 \text{ lb} \quad (2) \]

Add Eqs. (1) and (2):

\[ 2F_{BD} = -2000 \text{ lb} \quad F_{BD} = 1000 \text{ lb} \quad C \]

Subtract Eq. (2) from Eq. (1):

\[ 2F_{BE} = -1000 \text{ lb} \quad F_{BE} = 500 \text{ lb} \quad C \]

Free Body: Joint D:

\[ \pm \sum F_x = 0: \quad \frac{4}{5} (1000 \text{ lb}) + \frac{4}{5} F_{DF} = 0 \]

\[ F_{DF} = -1000 \text{ lb} \quad F_{DF} = 1000 \text{ lb} \quad C \]

\[ \sum F_y = 0: \quad \frac{3}{5} (1000 \text{ lb}) - \frac{3}{5} (-1000 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0 \]

\[ F_{DE} = +600 \text{ lb} \quad F_{DE} = 600 \text{ lb} \quad T \]

Because of the symmetry of the truss and loading, we deduce that

\[ F_{EF} = F_{BE} \]

\[ F_{EG} = F_{CE} \]

\[ F_{FG} = F_{BC} \]

\[ F_{FH} = F_{AB} \]

\[ F_{GH} = F_{AC} \]

\[ F_{EF} = 500 \text{ lb} \quad C \]

\[ F_{EG} = 1200 \text{ lb} \quad T \]

\[ F_{FG} = 0 \quad T \]

\[ F_{FH} = 1500 \text{ lb} \quad C \]

\[ F_{GH} = 1200 \text{ lb} \quad T \]
11- Determine the force in each member of the Pratt roof truss shown. State whether each member is in tension or compression.

**SOLUTION**

Free body: Truss:

Due to symmetry of truss and load, \( A_y = H = \frac{1}{2} \) total load = 21 kN

Free body: Joint A:

\[
\begin{align*}
F_{AB} &= \frac{35}{37} F_{AC} = 15.3 \text{ kN} \\
F_{AB} &= 47.175 \text{ kN} \quad F_{AC} = 44.625 \text{ kN}
\end{align*}
\]

Free body: Joint B:

From force polygon:

\[
F_{BD} = 47.175 \text{ kN}, \quad F_{BC} = 10.5 \text{ kN}
\]

\[
F_{BC} = 10.50 \text{ kN} \quad F_{BD} = 47.2 \text{ kN}
\]
12- Determine the force in each member of the Fink roof truss shown. State whether each member is in tension or compression.
SOLUTION

Free body: Truss:

\[ \sum F_x = 0: \quad A_x = 0 \]

Because of the symmetry of the truss and loading,

\[ A_y = G = \frac{1}{2} \text{ total load} \]

\[ A_y = G = 6.00 \text{ kN} \]

Free body: Joint A:

\[ \frac{F_{AB}}{2.462} = \frac{F_{AC}}{2.25} = \frac{4.50 \text{ kN}}{1} \]

\[ F_{AB} = 11.08 \text{ kN} \quad C \uparrow \]

\[ F_{AC} = 10.125 \text{ kN} \quad T \uparrow \]

Free body: Joint B:

\[ \pm \sum F_y = 0: \quad \frac{3}{5} F_{BC} + \frac{2.25}{2.462} F_{BD} + \frac{2.25}{2.462} (11.08 \text{ kN}) = 0 \quad (1) \]

\[ + \sum F_y = 0: \quad -\frac{4}{5} F_{BC} + \frac{F_{BD}}{2.462} + 11.08 \text{ kN} - 3 \text{ kN} = 0 \quad (2) \]

Multiply Eq. (2) by \(-2.25\) and add to Eq. (1):

\[ \frac{12}{5} F_{BC} + 6.75 \text{ kN} = 0 \quad F_{BC} = -2.8125 \quad F_{BC} = 2.81 \text{ kN} \quad C \uparrow \]

Multiply Eq. (1) by 4, Eq. (2) by 3, and add:

\[ \frac{12}{2.462} F_{BD} + \frac{12}{2.462} (11.08 \text{ kN}) - 9 \text{ kN} = 0 \]

\[ F_{BD} = -9.2335 \text{ kN} \quad F_{BD} = 9.23 \text{ kN} \quad C \uparrow \]

Free body: Joint C:

\[ + \sum F_y = 0: \quad \frac{4}{5} F_{CD} - \frac{4}{5} (2.8125 \text{ kN}) = 0 \]

\[ F_{CD} = 2.8125 \text{ kN}, \quad F_{CD} = 2.81 \text{ kN} \quad T \uparrow \]

\[ \pm \sum F_x = 0: \quad F_{CE} - 10.125 \text{ kN} + \frac{3}{5} (2.8125 \text{ kN}) + \frac{3}{5} (2.8125 \text{ kN}) = 0 \]

\[ F_{CE} = -6.7500 \text{ kN} \quad F_{CE} = 6.75 \text{ kN} \quad T \uparrow \]

Because of the symmetry of the truss and loading, we deduce that

\[ F_{DE} = F_{CD} \quad F_{CD} = 2.81 \text{ kN} \quad T \uparrow \]

\[ F_{DF} = F_{BD} \quad F_{BD} = 9.23 \text{ kN} \quad C \uparrow \]

\[ F_{EF} = F_{BC} \quad F_{BC} = 2.81 \text{ kN} \quad C \uparrow \]

\[ F_{EG} = F_{AC} \quad F_{AC} = 10.13 \text{ kN} \quad T \uparrow \]

\[ F_{FG} = F_{AB} \quad F_{AB} = 11.08 \text{ kN} \quad C \uparrow \]
13- The truss shown is one of several supporting an advertising panel. Determine the force in each member of the truss for a wind load equivalent to the two forces shown. State whether each member is in tension or compression.

\[ +\sum M_A = 0: \quad (800 \text{ N})(7.5 \text{ m}) + (800 \text{ N})(3.75 \text{ m}) - A(2 \text{ m}) = 0 \]
\[ A = +2250 \text{ N} \quad A = 2250 \text{ N} \]

\[ +\sum F_y = 0: \quad 2250 \text{ N} + F_y = 0 \]
\[ F_y = -2250 \text{ N} \]

\[ +\sum F_x = 0: \quad -800 \text{ N} - 800 \text{ N} + F_x = 0 \]
\[ F_x = +1600 \text{ N} \]

**Joint D:**

\[ F_{BD} = \frac{800 \text{ N}}{8} = 100 \text{ N} \]

**Joint A:**

\[ F_{AB} = \frac{2250 \text{ N}}{15} = 150 \text{ N} \]

**Joint F:**

\[ +\sum F_x = 0: \quad 1600 \text{ N} - F_{CF} = 0 \]
\[ F_{CF} = +1600 \text{ N} \]

\[ +\sum F_y = 0: \quad F_{EF} - 2250 \text{ N} = 0 \]
\[ F_{EF} = +2250 \text{ N} \]
14- Determine the force in each of the members located to the left of FG for the scissors roof truss shown. State whether each member is in tension or compression.
SOLUTION

Free Body: Truss:

\[ \sum F_x = 0; \quad \Lambda_x = 0 \]
\[ \sum M_L = 0: \quad (1 \text{kN})(12 \text{ m}) + (2 \text{kN})(10 \text{ m}) + (2 \text{kN})(8 \text{ m}) + (1 \text{kN})(6 \text{ m}) - A_y(12 \text{ m}) = 0 \]
\[ A_y = 4.50 \text{ kN} \uparrow \]

We note that BC is a zero-force member:
\[ F_{BC} = 0 \]

Also,
\[ F_{CE} = F_{AC} \quad (1) \]

Free body: Joint A:
\[ \sum F_x = 0: \quad \frac{1}{\sqrt{2}} F_{AB} + \frac{2}{\sqrt{5}} F_{AC} = 0 \quad (2) \]
\[ \sum F_y = 0: \quad \frac{1}{\sqrt{2}} F_{AB} + \frac{1}{\sqrt{5}} F_{AC} + 3.50 \text{ kN} = 0 \quad (3) \]

Multiply Eq. (3) by \(-2\) and add Eq. (2):
\[ \frac{-1}{\sqrt{2}} F_{AB} - 7 \text{ kN} = 0 \]
\[ F_{AB} = 9.90 \text{ kN} \]

C
Subtract Eq. (3) from Eq. (2):

\[
\frac{1}{\sqrt{5}} F_{AC} - 3.50 \text{ kN} = 0 \quad F_{AC} = 7.826 \text{ kN} \quad F_{AC} = 7.83 \text{ kN} \quad T \uparrow
\]

From Eq. (1):

\[
F_{CE} = F_{AC} = 7.826 \text{ kN} \quad F_{CE} = 7.83 \text{ kN} \quad T \uparrow
\]

Free body: Joint B:

\[
\begin{align*}
2 \text{ kN} & + \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}} F_{BD} + \frac{1}{\sqrt{2}} (9.90 \text{ kN}) - 2 \text{ kN} = 0 \\
& \Rightarrow F_{BD} = 7.071 \text{ kN} \quad F_{BD} = 7.07 \text{ kN} \quad C \uparrow
\end{align*}
\]

\[
\begin{align*}
\Sigma F_x &= 0: \quad F_{BE} + \frac{1}{\sqrt{2}} (9.90 - 7.071) \text{ kN} = 0 \\
& \Rightarrow F_{BE} = 2.000 \text{ kN} \quad F_{BE} = 2.00 \text{ kN} \quad C \uparrow
\end{align*}
\]

Free body: Joint E:

\[
\begin{align*}
\Sigma F_y &= 0: \frac{2}{\sqrt{5}} (F_{EG} - 7.826 \text{ kN}) + 2.00 \text{ kN} = 0 \\
& \Rightarrow F_{EG} = 5.590 \text{ kN} \quad F_{EG} = 5.59 \text{ kN} \quad T \uparrow
\end{align*}
\]

\[
\begin{align*}
\Sigma F_x &= 0: \quad F_{DE} - \frac{1}{\sqrt{5}} (7.826 - 5.590) \text{ kN} = 0 \\
& \Rightarrow F_{DE} = 1.000 \text{ kN} \quad F_{DE} = 1.00 \text{ kN} \quad T \uparrow
\end{align*}
\]

Free body: Joint D:

\[
\begin{align*}
\Sigma F_x &= 0: \frac{2}{\sqrt{5}} (F_{DF} + F_{DG}) + \frac{1}{\sqrt{2}} (7.071 \text{ kN}) \\
& \quad \text{or} \\
F_{DF} + F_{DG} &= -5.590 \text{ kN} \quad \text{(4)}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_y &= 0: \frac{1}{\sqrt{5}} (F_{DF} - F_{DG}) + \frac{1}{\sqrt{2}} (7.071 \text{ kN}) = 2 \text{ kN} - 1 \text{ kN} = 0 \\
& \quad \text{or} \\
F_{DE} - F_{DG} &= -4.472 \quad \text{(5)}
\end{align*}
\]

Add Eqs. (4) and (5):

\[
2F_{DF} = -10.062 \text{ kN} \quad F_{DF} = 5.03 \text{ kN} \quad C \uparrow
\]

Subtract Eq. (5) from Eq. (4):

\[
2F_{DG} = -1.1180 \text{ kN} \quad F_{DG} = 0.559 \text{ kN} \quad C \uparrow
\]
15- The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above HJ. State whether each member is in tension or compression.
SOLUTION

Free body: Joint A:
\[
\begin{align*}
F_{AB} &= F_{AC} = 1.2 \text{ kN} \\
2.29 &= 2.29 = 1.2
\end{align*}
\]
\[
F_{AB} = 2.29 \text{ kN} \quad T\updownarrow
\]
\[
F_{AC} = 2.29 \text{ kN} \quad C\updownarrow
\]

Free body: Joint F:
\[
\begin{align*}
F_{DF} &= F_{EF} = 1.2 \text{ kN} \\
2.29 &= 2.29 = 1.2
\end{align*}
\]
\[
F_{DF} = 2.29 \text{ kN} \quad T\updownarrow
\]
\[
F_{EF} = 2.29 \text{ kN} \quad C\updownarrow
\]

Free body: Joint D:
\[
\begin{align*}
F_{BD} &= F_{DE} = 2.29 \text{ kN} \\
2.21 &= 0.6 = 2.29
\end{align*}
\]
\[
F_{BD} = 2.21 \text{ kN} \quad T\updownarrow
\]
\[
F_{DE} = 0.600 \text{ kN} \quad C\updownarrow
\]

Free body: Joint B:
\[
\begin{align*}
\pm \Sigma F_x &= 0: \quad \frac{4}{5} F_{BE} + 2.21 \text{ kN} - \frac{2.21}{2.29} (2.29 \text{ kN}) = 0 \\
F_{BE} &= 0 \quad \updownarrow
\end{align*}
\]
\[
\pm \Sigma F_y &= 0: \quad -F_{BC} - \frac{3}{5} (0) - \frac{0.6}{2.29} (2.29 \text{ kN}) = 0 \\
F_{BC} &= -0.600 \text{ kN} \quad F_{BC} = 0.600 \text{ kN} \quad C\updownarrow
\]

Free body: Joint C:
\[
\begin{align*}
\pm \Sigma F_x &= 0: \quad F_{CE} + \frac{2.21}{2.29} (2.29 \text{ kN}) = 0 \\
F_{CE} &= -2.21 \text{ kN} \quad F_{CE} = 2.21 \text{ kN} \quad C\updownarrow
\end{align*}
\]
\[
\pm \Sigma F_y &= 0: \quad -F_{CH} - 0.600 \text{ kN} - \frac{0.6}{2.29} (2.29 \text{ kN}) = 0 \\
F_{CH} &= -1.200 \text{ kN} \quad F_{CH} = 1.200 \text{ kN} \quad C\updownarrow
\]

Free body: Joint E:
\[
\begin{align*}
\pm \Sigma F_x &= 0: \quad 2.21 \text{ kN} - \frac{2.21}{2.29} (2.29 \text{ kN}) - \frac{4}{5} F_{EH} = 0 \\
F_{EH} &= 0 \quad \updownarrow
\end{align*}
\]
\[
\pm \Sigma F_y &= 0: \quad -F_{EJ} - 0.600 \text{ kN} - \frac{0.6}{2.29} (2.29 \text{ kN}) - 0 = 0 \\
F_{EJ} &= -1.200 \text{ kN} \quad F_{EJ} = 1.200 \text{ kN} \quad C\updownarrow
\]
1- Determine the force in members CD and DF of the truss shown.

SOLUTION

Reactions:

\[ \sum M_B = 0: \ (12 \text{ kN})(4.8 \text{ m}) + (12 \text{ kN})(2.4 \text{ m}) - B(9.6 \text{ m}) = 0 \]

\[ B = 9.00 \text{ kN} \]

\[ \sum F_y = 0: \ 9.00 \text{ kN} - 12.00 \text{ kN} - 12.00 \text{ kN} + J = 0 \]

\[ J = 15.00 \text{ kN} \]

Member CD:

\[ \sum F_y = 0: \ 9.00 \text{ kN} + F_{CD} = 0 \]

\[ F_{CD} = 9.00 \text{ kN} \]

Member DF:

\[ \sum M_C = 0: \ F_{DF}(1.8 \text{ m}) - (9.00 \text{ kN})(2.4 \text{ m}) = 0 \]

\[ F_{DF} = 12.00 \text{ kN} \]
2- Determine the force in members FG and FH of the truss shown.

**SOLUTION**

**Reactions:**

\[ + \sum M_J = 0: \quad (12 \text{kN})(4.8 \text{ m}) + (12 \text{kN})(2.4 \text{ m}) - B(9.6 \text{ m}) = 0 \]

\[ B = 9.00 \text{kN} \]

\[ + \sum F_y = 0: \quad 9.00 \text{kN} - 12.00 \text{kN} - 12.00 \text{kN} + J = 0 \]

\[ J = 15.00 \text{kN} \]

**Member FG:**

\[ + \sum F_y = 0: \quad - \frac{3}{5} F_{FG} - 12.00 \text{kN} + 15.00 \text{kN} = 0 \]

\[ F_{FG} = 5.00 \text{kN} \quad \uparrow \]

**Member FH:**

\[ + \sum M_G = 0: \quad (15.00 \text{kN})(2.4 \text{ m}) - F_{FH}(1.8 \text{ m}) = 0 \]

\[ F_{FH} = 20.0 \text{kN} \quad \uparrow \]
3- Determine the force in members DF, EF, and EG of the truss shown.

4- Determine the force in members GI, GJ, and HI of the truss shown.
5- Determine the force in members $AD$, $CD$, and $CE$ of the truss shown.
6- A stadium roof truss is loaded as shown. Determine the force in members $AB$, $AG$, and $FG$. 

\[ \sum M_A = 0: \quad 36(1.2) - 26.4(2.25) - F_{AD}(1.2) = 0 \]

\[ F_{AD} = -13.5 \text{ kN} \]

\[ F_{CD} = 0 \]

\[ F_{CE} = 56.1 \text{ kN} \]
SOLUTION

We pass a section through members AB, AG, and FG, and use the free body shown.

\[ + \gamma M_G = 0: \left( \frac{40}{41} F_{AB} \right) (6.3 \text{ ft}) - (1.8 \text{ kips})(14 \text{ ft}) \]
\[ - (0.9 \text{ kips})(28 \text{ ft}) = 0 \]
\[ F_{AB} = 8.20 \text{ kips} \]

\[ F_{AB} = 8.20 \text{ kips} \quad T \uparrow \]

\[ + \gamma M_D = 0: - \left( \frac{3}{5} F_{AG} \right) (28 \text{ ft}) + (1.8 \text{ kips})(28 \text{ ft}) \]
\[ + (1.8 \text{ kips})(14 \text{ ft}) = 0 \]
\[ F_{AG} = 4.50 \text{ kips} \]

\[ F_{AG} = 4.50 \text{ kips} \quad T \uparrow \]

\[ + \gamma M_A = 0: - F_{FG} (9 \text{ ft}) - (1.8 \text{ kips})(12 \text{ ft}) - (1.8 \text{ kips})(26 \text{ ft}) \]
\[ - (0.9 \text{ kips})(40 \text{ ft}) = 0 \]
\[ F_{FG} = 11.60 \text{ kips} \]

\[ F_{FG} = 11.60 \text{ kips} \quad C \uparrow \]

7- Determine the force in members CD and DF of the truss shown.
SOLUTION

\[
\tan \alpha = \frac{5}{12} \quad \alpha = 22.62^\circ \\
\sin \alpha = \frac{5}{13} \quad \cos \alpha = \frac{12}{13}
\]

**Member CD:**

\[+ \sum M_I = 0: \quad F_{CD}(9 \text{ m}) + (10 \text{ kN})(9 \text{ m}) + (10 \text{ kN})(6 \text{ m}) + (10 \text{ kN})(3 \text{ m}) = 0\]

\[F_{CD} = -20 \text{ kN} \quad F_{CD} = 20.0 \text{ kN} \quad \text{C} \quad \uparrow\]

**Member DF:**

\[+ \sum M_C = 0: \quad (F_{DF} \cos \alpha)(3.75 \text{ m}) + (10 \text{ kN})(3 \text{ m}) + (10 \text{ kN})(6 \text{ m}) + (10 \text{ kN})(9 \text{ m}) = 0\]

\[F_{DF} \cos \alpha = -48 \text{ kN}\]

\[F_{DF} \left(\frac{12}{13}\right) = -48 \text{ kN} \quad F_{DF} = -52.0 \text{ kN} \quad F_{DF} = 52.0 \text{ kN} \quad \text{C} \quad \uparrow\]
8- The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members $FG$, $EG$, and $EH$.

**SOLUTION**

Reactions at supports. Because of the symmetry of the loading,

$$A_x = 0, \quad A_y = O = \frac{1}{2} \text{total load}$$

$$A = 0 = 4.48 \text{ kN}$$

We pass a section through members $FG$, $EG$, and $EH$, and use the free body shown.

\[
\begin{align*}
\text{Slope } FG &= \text{Slope } FI = \frac{1.75 \text{ m}}{6 \text{ m}} \\
\text{Slope } EG &= \frac{5.50 \text{ m}}{2.4 \text{ m}}
\end{align*}
\]

\[
\begin{align*}
+ \sum M_k &= 0: \quad (0.6 \text{ kN})(7.44 \text{ m}) + (1.24 \text{ kN})(3.84 \text{ m}) \\
&\quad - (4.48 \text{ kN})(7.44 \text{ m}) \\
&\quad - \left( \frac{6}{6.25} F_{FG} \right)(4.80 \text{ m}) = 0 \\
F_{FG} &= -5.231 \text{ kN}
\end{align*}
\]

\[
\begin{align*}
+ \sum M_G &= 0: \quad F_{EH}(5.50 \text{ m}) + (0.6 \text{ kN})(9.84 \text{ m}) \\
&\quad + (1.24 \text{ kN})(6.24 \text{ m}) + (1.04 \text{ kN})(2.4 \text{ m}) \\
&\quad - (4.48 \text{ kN})(9.84 \text{ m}) = 0 \\
F_{EH} &= 5.08 \text{ kN}
\end{align*}
\]

\[
\begin{align*}
+ \sum F_j &= 0: \quad \frac{5.50}{6.001} F_{EG} + \frac{1.75}{6.25} (-5.231 \text{ kN}) + 4.48 \text{ kN} - 0.6 \text{ kN} - 1.24 \text{ kN} - 1.04 \text{ kN} = 0 \\
F_{EG} &= -0.1476 \text{ kN}
\end{align*}
\]

\[
\begin{align*}
F_{FG} &= 5.23 \text{ kN} \quad C \downarrow \\
F_{EH} &= 5.08 \text{ kN} \quad T \downarrow \\
F_{EG} &= 0.1476 \text{ kN} \quad C \downarrow
\end{align*}
\]
9- A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members DF, EF, and EG.

**SOLUTION**

Free body: Truss:

\[ \Sigma F_y = 0: \quad N_y = 0 \]
\[ + \Sigma M_N = 0: \quad (200 \text{ lb})(8a) + (400 \text{ lb})(7a + 6a + 5a) + (350 \text{ lb})(4a) + (300 \text{ lb})(3a + 2a + a) - A(8a) = 0 \]
\[ A = 1500 \text{ lb} \]
\[ + \Sigma F_y = 0: \quad 1500 \text{ lb} - 200 \text{ lb} - 3(400 \text{ lb}) - 350 \text{ lb} - 3(300 \text{ lb}) - 150 \text{ lb} + N_y = 0 \]
\[ N_y = 1300 \text{ lb} \]

We pass a section through DF, EF, and EG, and use the free body shown. (We apply \( F_{DF} \) at \( F \).)

\[ + \Sigma M_E = 0: \quad (200 \text{ lb})(18 \text{ ft}) + (400 \text{ lb})(12 \text{ ft}) + (400 \text{ lb})(6 \text{ ft}) - (1500 \text{ lb})(18 \text{ ft}) - \left( \frac{18}{\sqrt{18^2 + 4.5^2}} \right) F_{DF} (4.5 \text{ ft}) = 0 \]
\[ F_{DF} = -3711 \text{ lb} \quad F_{DF} = 3710 \text{ lb} \]

\[ + \Sigma M_A = 0: \quad F_{EF} (18 \text{ ft}) - (400 \text{ lb})(6 \text{ ft}) - (400 \text{ lb})(12 \text{ ft}) = 0 \]
\[ F_{EF} = 400 \text{ lb} \quad F_{EF} = 400 \text{ lb} \]

\[ + \Sigma M_F = 0: \quad F_{EG} (4.5 \text{ ft}) - (1500 \text{ lb})(18 \text{ ft}) + (200 \text{ lb})(18 \text{ ft}) + (400 \text{ lb})(12 \text{ ft}) + (400 \text{ lb})(6 \text{ ft}) = 0 \]
\[ F_{EG} = 3600 \text{ lb} \quad F_{EG} = 3600 \text{ lb} \]
10- Determine the force in members $DE$ and $DF$ of the truss shown when $P = 20$ kips.

![Truss Diagram]

**SOLUTION**

Reactions:

$$C = K = 2.5P$$

$$\tan \beta = \frac{7.5}{18}$$

$$\beta = 22.62^\circ$$

**Member $DE$:**

$$\Sigma M_A = 0: \quad (2.5P)(6 \text{ ft}) - F_{DE}(12 \text{ ft}) = 0$$

$$F_{DE} = +1.25P$$

For $P = 20$ kips,

$$F_{DE} = +1.25(20) = +25 \text{ kips}$$

$$F_{DE} = 25.0 \text{ kips} \quad T$$

**Member $DF$:**

$$\Sigma M_K = 0: \quad P(12 \text{ ft}) - (2.5P)(6 \text{ ft}) - F_{DF} \cos 22.62^\circ(5 \text{ ft}) = 0$$

$$12P - 15P - F_{DF} \cos 22.62^\circ(5 \text{ ft}) = 0$$

$$F_{DF} = -0.65P$$

For $P = 20$ kips,

$$F_{DF} = -0.65(20) = -13 \text{ kips}$$

$$F_{DF} = 13.00 \text{ kips} \quad C$$

11- Determine the force in members $EH$ and $GI$ of the truss shown. *(Hint: Use section $aa$.)*
SOLUTION

Reactions:

\[ \Sigma F_x = 0: \quad A_x = 0 \]
\[ \Sigma M_P = 0: \quad \begin{align*}
12(45) + 12(30) + 12(15) - A_y(90) &= 0 \\
A_y &= 12 \text{kips} \\
P &= 24 \text{kips}
\end{align*} \]
\[ \Sigma F_y = 0: \quad 12 - 12 - 12 + P = 0 \quad \begin{align*}
P &= 24 \text{kips} \\
F_{EH} &= -22.5 \text{kips} \quad \begin{align*}
F_{EH} &= 22.5 \text{kips} \\
C &= \downarrow \quad \begin{align*}
F_{Gl} &= 22.5 \text{kips} \quad \begin{align*}
F_{Gl} &= 22.5 \text{kips} \\
T &= \downarrow
\end{align*}
\end{align*}
\end{align*} \]
12- The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as counters. Determine the forces in the counters that are acting under the given loading.

**SOLUTION**

**Free body: Truss:**

\[ \Sigma F_x = 0: \quad F_x = 0 \]
\[ + \Sigma M_H = 0: \quad 4.8(3a) + 4.8(2a) + 4.8a - 2.4a - F_y(2a) = 0 \]

\[ F_y = +13.20 \text{ kips} \]

\[ \Sigma F_y = 0: \quad H + 13.20 \text{ kips} - 3(4.8 \text{ kips}) - 2(2.4 \text{ kips}) = 0 \]

\[ H = +6.00 \text{ kips} \]

**Free body: ABE:**

We assume that counter BG is acting.

\[ + \Sigma F_y = 0: \quad -\frac{9.6}{14.6} F_{BG} + 13.20 - 2(4.8) = 0 \]

\[ F_{BG} = +5.475 \text{ kips} \]

Since BG is in tension, our assumption was correct.

**Free body: DEH:**

We assume that counter DG is acting.

\[ + \Sigma F_y = 0: \quad -\frac{9.6}{14.6} F_{DG} + 6.00 - 2(2.4) = 0 \]

\[ F_{DG} = +1.825 \text{ kips} \]

Since DG is in tension, O.K.
13- Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)

**SOLUTION**

**Structure (a)**
Number of members: \( m = 12 \)
Number of joints: \( n = 8 \)
Reaction components: \( r = 3 \)
\[ m + r = 15, \quad 2n = 16 \]
\[ m + r < 2n \]
Thus, Structure is partially constrained.

**Structure (b)**
\[ m = 13, \quad n = 8 \]
\[ r = 3 \]
\[ m + r = 16, \quad 2n = 16 \]
Thus, \( m + r = 2n \)
To verify that the structure is actually completely constrained and determinate, we observe that it is a simple truss (follow lettering to check this) and that it is simply supported by a pin-and-bracket and a roller. Thus, structure is completely constrained and determinate.

**Structure (c)**
\[ m = 13, \quad n = 8 \]
\[ r = 4 \]
\[ m + r = 17, \quad 2n = 16 \]
Thus, \( m + r > 2n \)
Structure is completely constrained and indeterminate.

This result can be verified by observing that the structure is a simple truss (follow lettering to check this), therefore it is rigid, and that its supports involve four unknowns.
14- Determine the force in member BD and the components of the reaction at C.

**SOLUTION**

We note that BD is a two-force member. The force it exerts on ABC, therefore, is directed along line BD.

Free body: ABC:

\[ BD = \sqrt{(24)^2 + (10)^2} = 26 \text{ in.} \]

\[ \sum F_y = 0: \quad C_y - 160 \text{ lb} + \frac{10}{26} (780 \text{ lb}) = 0 \]

\[ C_y = -140.0 \text{ lb} \]

\[ \sum F_x = 0: \quad C_x + \frac{24}{26} (780 \text{ lb}) = 0 \]

\[ C_x = 720 \text{ lb} \]

\[ \sum M_C = 0: \quad (160 \text{ lb})(30 \text{ in.}) - \left( \frac{10}{26} F_{BD} \right) (16 \text{ in.}) = 0 \]

\[ F_{BD} = +780 \text{ lb} \quad F_{BD} = 780 \text{ lb} \]

\[ C_y = 140.0 \text{ lb} \]

\[ C_x = 720 \text{ lb} \]

\[ F_{BD} = 780 \text{ lb} \]
15- Determine the components of all forces acting on member $ABCD$ of the assembly shown.

**SOLUTION**

**Free body: Entire assembly:**

\[ + \Sigma M_B = 0: \quad D(120 \text{ mm}) - (480 \text{ N})(80 \text{ mm}) = 0 \]

\[ D = 320 \text{ N} \]

\[ + \Sigma F_x = 0: \quad B_x + 480 \text{ N} = 0 \]

\[ B_x = 480 \text{ N} \]

\[ + \Sigma F_y = 0: \quad B_y + 320 \text{ N} = 0 \]

\[ B_y = 320 \text{ N} \]

**Free body: Member $ABCD$:**

\[ + \Sigma M_A = 0: \quad (320 \text{ N})(200 \text{ mm}) - C(160 \text{ mm}) - (320 \text{ N})(80 \text{ mm}) - (480 \text{ N})(40 \text{ mm}) = 0 \]

\[ C = 120.0 \text{ N} \]

\[ + \Sigma F_x = 0: \quad A_x - 480 \text{ N} = 0 \]

\[ A_x = 480 \text{ N} \]

\[ + \Sigma F_y = 0: \quad A_y - 320 \text{ N} - 120 \text{ N} + 320 \text{ N} = 0 \]

\[ A_y = 120.0 \text{ N} \]
1- Determine the force in member $BD$ and the components of the reaction at $C$.

**SOLUTION**

We note that $BD$ is a two-force member. The force it exerts on $ABC$, therefore, is directed along line $BD$.

**Free body: $ABC$:**

\[ BD = \sqrt{(24)^2 + (10)^2} = 26 \text{ in.} \]

\[ + \sum M_C = 0: \quad (160 \text{ lb})(30 \text{ in.}) - \left(\frac{10}{26} F_{BD}\right)(16 \text{ in.}) = 0 \]

\[ F_{BD} = 780 \text{ lb} \quad F_{BD} = 780 \text{ lb} \quad T \uparrow \]

\[ + \sum F_x = 0: \quad C_x + \frac{24}{26} (780 \text{ lb}) = 0 \]

\[ C_x = -720 \text{ lb} \quad C_x = 720 \text{ lb} \leftarrow \]

\[ + \sum F_y = 0: \quad C_y - 160 \text{ lb} + \frac{10}{26} (780 \text{ lb}) = 0 \]

\[ C_y = -140.0 \text{ lb} \quad C_y = 140.0 \text{ lb} \uparrow \]
2- For the frame and loading shown, draw the free-body diagram(s) needed to determine the force in member $BD$ and the components of the reaction at $C$.

**SOLUTION**

We note that $BD$ is a two-force member. The force it exerts on $ABC$, therefore, is directed along line $BD$.

Free body: $ABC$:

Attaching $F_{BD}$ at $D$ and resolving it into components, we write

\[ + \sum F_y = 0: \quad C_y - 400 \text{ N} + \frac{450}{510} (-255 \text{ N}) = 0 \]

\[ C_y = 4625 \text{ N} \]

\[ + \sum F_x = 0: \quad C_x + \frac{240}{510} (-255 \text{ N}) = 0 \]

\[ C_x = 120.0 \text{ N} \]

\[ - \sum M_C = 0: \quad (400 \text{ N})(135 \text{ mm}) + \left( \frac{450}{510} F_{BD} \right)(240 \text{ mm}) = 0 \]

\[ F_{BD} = -255 \text{ N} \quad F_{BD} = 255 \text{ N} \]
4- For the frame and loading shown, draw the free-body diagram(s) needed to determine the components of all forces acting on member ABC.

\[ + \Sigma F_y = 0: \quad A_y = 20 \text{ kips} \]
\[ A_y = 20.0 \text{ kips} \]

\[ + \Sigma M_E = 0: \quad -A_x(4) - (20 \text{ kips})(5) = 0 \]
\[ A_x = -25 \text{ kips}, \quad A_x = 25.0 \text{ kips} \]

Free body: Entire frame:

Free body: Member ABC:

\[ + \Sigma M_C = 0: \quad (25 \text{ kips})(4 \text{ ft}) - (20 \text{ kips})(10 \text{ ft}) + B_x(2 \text{ ft}) + B_y(5 \text{ ft}) = 0 \]
\[ -100 \text{ kip} \cdot \text{ft} + B_x(2 \text{ ft}) + \frac{2}{5}B_y(5 \text{ ft}) = 0 \]
\[ B_x = 25 \text{ kips} \]
\[ B_y = \frac{2}{5}B_x = \frac{2}{5}(25) = 10 \text{ kips} \]
\[ B_y = 10.00 \text{ kips} \]

\[ + \Sigma F_x = 0: \quad C_x - 25 \text{ kips} - 25 \text{ kips} = 0 \]
\[ C_x = 50 \text{ kips} \]
\[ C_x = 50.0 \text{ kips} \]

\[ + \Sigma F_y = 0: \quad C_y + 20 \text{ kips} - 10 \text{ kips} = 0 \]
\[ C_y = -10 \text{ kips} \]
\[ C_y = 10.00 \text{ kips} \]

5- Determine the components of all forces acting on member ABCD when $\theta = 0$. 
6- Determine the components of all forces acting on member ABCD when \( \theta = 90^\circ \).
SOLUTION

Free body: Entire assembly:

\[ \sum M_B = 0: \quad A(200) - (150 \text{ N})(200) = 0 \]
\[ A = +150.0 \text{ N} \quad A = 150.0 \text{ N} \uparrow \]

\[ \sum F_x = 0: \quad B_x + 150 - 150 = 0 \]
\[ B_x = 0 \]

\[ \sum F_y = 0: \quad B_y = 0 \]
\[ B = 0 \uparrow \]

Free body: Member ABCD:

We note that D is directed along DE, since DE is a two-force member.

\[ \sum M_C = 0: \quad D(300) + (150 \text{ N})(200) = 0 \]
\[ D = -100.0 \text{ N} \quad D = 100.0 \text{ N} \downarrow \]

\[ \sum F_x = 0: \quad C_x + 150 N = 0 \]
\[ C_x = -150 \text{ N} \]

\[ \sum F_y = 0: \quad C_y - 100 N = 0 \]
\[ C_y = +100.0 \text{ N} \]
\[ C_y = 100.0 \text{ N} \uparrow \]
7- Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (a) at B, (b) at D.

\[ \Sigma M_E = 0: \quad -(750 \, \text{N})(80 \, \text{mm}) - A_x(200 \, \text{mm}) = 0 \]
\[ A_x = -300 \, \text{N} \quad A_x = 300 \, \text{N} \]

\[ \Sigma F_x = 0: \quad E_x - 300 \, \text{N} = 0 \quad E_x = 300 \, \text{N} \]
\[ \Sigma F_y = 0: \quad A_y + E_y - 750 \, \text{N} = 0 \]

(a) Load applied at B.

Free body: Member CE:

CE is a two-force member. Thus, the reaction at E must be directed along CE.

\[ \frac{E_y}{300 \, \text{N}} = \frac{75 \, \text{mm}}{250 \, \text{mm}} \quad E_y = 90 \, \text{N} \]

From Eq. (1): \[ A_y + 90 \, \text{N} - 750 \, \text{N} = 0 \quad A_y = 660 \, \text{N} \]

Thus, reactions are

\[ A_x = 300 \, \text{N} \quad A_y = 660 \, \text{N} \]
\[ E_x = 300 \, \text{N} \quad E_y = 90.0 \, \text{N} \]

(b) Load applied at D.

Free body: Member AC:

AC is a two-force member. Thus, the reaction at A must be directed along AC.

\[ \frac{A_y}{300 \, \text{N}} = \frac{125 \, \text{mm}}{250 \, \text{mm}} \quad A_y = 150 \, \text{N} \]
From Eq. (1):

\[ A_y + E_y - 750 \text{ N} = 0 \]

\[ 150 \text{ N} + E_y - 750 \text{ N} = 0 \]

\[ E_y = 600 \text{ N} \quad E_y = 600 \text{ N} \]

Thus, reactions are

\[ A_x = 300 \text{ N} \quad A_y = 150.0 \text{ N} \]

\[ E_x = 300 \text{ N} \quad E_y = 600 \text{ N} \]
8- Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (a) at B, (b) at D.

**SOLUTION**

**Free-body: Entire frame:**
The following analysis is valid for both parts (a) and (b) since position of load on its line of action is immaterial.

\[ \sum M_E = 0: \quad -(750 \text{ N})(240 \text{ mm}) - A_x(400 \text{ mm}) = 0 \]
\[ A_x = -450 \text{ N} \quad A_y = 450 \text{ N} \rightarrow \]
\[ \sum F_x = 0: \quad E_x - 450 \text{ N} = 0 \quad E_x = 450 \text{ N} \quad E_x = 450 \text{ N} \rightarrow \]
\[ \sum F_y = 0: \quad A_y + E_y - 750 \text{ N} = 0 \]

(1)

(a) **Load applied at B.**

**Free body: Member CE:**

CE is a two-force member. Thus, the reaction at E must be directed along CE.

\[ \frac{E_y}{450 \text{ N}} = \frac{240 \text{ mm}}{480 \text{ mm}}; \quad E_y = 225 \text{ N} \uparrow \]

From Eq. (1):
\[ A_y + 225 - 750 = 0; \quad A_y = 525 \text{ N} \uparrow \]

Thus, reactions are
\[ A_x = 450 \text{ N} \rightarrow, \quad A_y = 525 \text{ N} \uparrow \]
\[ E_x = 450 \text{ N} \rightarrow, \quad E_y = 225 \text{ N} \uparrow \]

(b) **Load applied at D.**

**Free body: Member AC:**

AC is a two-force member. Thus, the reaction at A must be directed along AC.

\[ \frac{A_y}{450 \text{ N}} = \frac{160 \text{ mm}}{480 \text{ mm}}; \quad A_y = 150.0 \text{ N} \uparrow \]

From Eq. (1):
\[ A_y + E_y - 750 = 0 \]
\[ 150 \text{ N} + E_y - 750 = 0 \]
\[ E_y = 600 \text{ N} \quad E_y = 600 \text{ N} \uparrow \]

Thus, reactions are
\[ A_x = 450 \text{ N} \rightarrow, \quad A_y = 150.0 \text{ N} \uparrow \]
\[ E_x = 450 \text{ N} \rightarrow, \quad E_y = 600 \text{ N} \uparrow \]
9- Draw the free-body diagram(s) needed to determine all the forces exerted on member AI if the frame is loaded by a clockwise couple of magnitude 1200 lb?in. applied at point D.

![Free-body diagram of the frame](image)

**SOLUTION**

*Free body: Entire frame:*

Location of couple is immaterial.

\[ + \sum M_H = 0: \quad I(48 \text{ in.}) - 1200 \text{ lb\cdot in.} = 0 \]

\[ I = +25.0 \text{ lb} \]

\( (a) \) and \( (b) \) \quad I = 25.0 \text{ lb} \]

We note that \( AB, BC, \) and \( FG \) are two-force members.

*Free body: Member AI:*

\[ \tan \alpha = \frac{20}{48} = \frac{5}{12} \quad \alpha = 22.6^\circ \]

\( (a) \) **Couple applied at D.**

\[ + \sum F_y = 0: \quad -\frac{5}{13} A + 25 \text{ lb} = 0 \]

\[ A = +65.0 \text{ lb} \quad A = 65.0 \text{ lb } \overrightarrow{22.6^\circ} \]

\[ + \sum M_G = 0: \quad \frac{12}{13} (65 \text{ lb})(40 \text{ in.}) - C(20 \text{ in.}) = 0 \]

\[ C = +120 \text{ lb} \quad C = 120 \text{ lb } \rightarrow \]

\[ + \sum F_x = 0: \quad -\frac{12}{13} (65 \text{ lb}) + 120 \text{ lb} + G = 0 \]

\[ G = -60.0 \text{ lb} \quad G = 60 \text{ lb } \leftarrow \]
(b) Couple applied at E.

\[ + \sum F_y = 0: \quad - \frac{5}{13} A + 25 \text{ lb} = 0 \]
\[ A = 65.0 \text{ lb} \quad A = 65.0 \text{ lb} \rightarrow 22.6^\circ \]

\[ + \sum M_G = 0: \quad \frac{12}{13} (65 \text{ lb}) + 40 \text{ in.} - C(20 \text{ in.}) - 1200 \text{ lb \cdot in.} = 0 \]
\[ C = 60.0 \text{ lb} \quad C = 60.0 \text{ lb} \rightarrow \]

\[ \pm \sum F_x = 0: \quad - \frac{12}{13} (65 \text{ lb}) + 60 \text{ lb} + G = 0 \]
\[ G = 0 \]
10- A 3-ft-diameter pipe is supported every 16 ft by a small frame like that shown. Knowing that the combined weight of the pipe and its contents is 500 lb/ft and assuming frictionless surfaces, determine the components (a) of the reaction at E, (b) of the force exerted at C on member CDE.

**SOLUTION**

Free body: 16-ft length of pipe:

\[ W = (500 \text{ lb/ft})(16 \text{ ft}) = 8 \text{ kips} \]

\[
\begin{align*}
\frac{B}{3} &= \frac{D}{5} = \frac{8 \text{ kips}}{4} \\
B &= 6 \text{ kips} \\
D &= 10 \text{ kips}
\end{align*}
\]

Determination of CB = CD

We note that horizontal projection of \( \overline{BO} + \overline{OD} = \) horizontal projection of \( \overline{CD} \)

\[ r + \frac{3}{5} \cdot r = \frac{4}{5} (CD) \]

Thus,

\[ CB = CD = \frac{8}{4} r = 2(1.5 \text{ ft}) = 3 \text{ ft} \]

Free body: Member ABC:

\[ + \Delta \Sigma M_A = 0: \quad C_A h - (6 \text{ kips})(h - 3) \]

\[ C_A = \frac{h - 3}{h} (6 \text{ kips}) \]

For \( h = 9 \text{ ft}, \)

\[ C_A = \frac{9 - 3}{9} (6 \text{ kips}) = 4 \text{ kips} \]
Free body: Member CDE:

From above, we have

\[ \Sigma F_x = 0: \quad 4 \text{kips} + \frac{3}{5} \times 10 \text{kips} + E_x = 0 \]
\[ E_x = -2 \text{kips}, \quad E_x = 2.00 \text{kips} \]

\[ \Sigma F_y = 0: \quad 5.75 \text{kips} - \frac{4}{5} \times 10 \text{kips} + E_y = 0, \]
\[ E_y = 2.25 \text{kips} \]

\[ \Sigma M_E = 0: \quad (10 \text{kips})(7 \text{ ft}) - (4 \text{kips})(6 \text{ ft}) - C_y(8 \text{ ft}) = 0 \]
\[ C_y = +5.75 \text{kips}, \quad C_y = 5.75 \text{kips} \]

\[ C_x = 4.00 \text{kips} \]
11- Knowing that the pulley has a radius of 50 mm, determine the components of the reactions at \( B \) and \( E \).

**SOLUTION**

**Free body: Entire assembly:**

\[ +\sum M_E = 0: \quad -(300 \text{ N})(350 \text{ mm}) - B_x(150 \text{ mm}) = 0 \]

\[ B_x = -700 \text{ N} \quad B_x = 700 \text{ N} \]

\[ \pm \sum F_x = 0: \quad -700N + E_x = 0 \]

\[ E_x = 700 \text{ N} \]

\[ +\sum F_y = 0: \quad B_y + E_y - 300 \text{ N} = 0 \]

\[ (1) \]

**Free body: Member \textit{ACE}:**

\[ +\sum M_C = 0: \quad (700 \text{ N})(150 \text{ mm}) - (300 \text{ N})(50 \text{ mm}) - E_y(180 \text{ mm}) = 0 \]

\[ E_y = 500 \text{ N} \quad E_y = 500 \text{ N} \]

From Eq. (1):

\[ B_y + 500 \text{ N} - 300 \text{ N} = 0 \]

\[ B_y = -200 \text{ N} \]

\[ B_y = 200 \text{ N} \]

Thus, reactions are

\[ B_x = 700 \text{ N} \quad B_y = 200 \text{ N} \]

\[ E_x = 700 \text{ N} \quad E_y = 500 \text{ N} \]
12- A trailer weighing 2400 lb is attached to a 2900-lb pickup truck by a ball-and-socket truck hitch at D. Determine (a) the reactions at each of the six wheels when the truck and trailer are at rest, (b) the additional load on each of the truck wheels due to the trailer.

**SOLUTION**

(a) **Free body: Trailer:**
(We shall denote by A, B, C the reaction at one wheel.)
\[ + \Sigma M_A = 0: \quad -(2400 \text{ lb})(2 \text{ ft}) + D(11 \text{ ft}) = 0 \]
\[ D = 436.36 \text{ lb} \]
\[ + \Sigma F_y = 0: \quad 2A - 2400 \text{ lb} + 436.36 \text{ lb} = 0 \]
\[ A = 981.82 \text{ lb} \]

Free body: Truck.
\[ + \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) - (2900 \text{ lb})(5 \text{ ft}) + 2C(9 \text{ ft}) = 0 \]
\[ C = 732.83 \text{ lb} \]
\[ + \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2900 \text{ lb} + 2(732.83 \text{ lb}) = 0 \]
\[ B = 935.35 \text{ lb} \]

(b) **Additional load on truck wheels.**
Use free body diagram of truck without 2900 lb.
\[ + \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) + 2C(9 \text{ ft}) = 0 \]
\[ C = -72.73 \text{ lb} \]
\[ \Delta C = -72.73 \text{ lb} \]
\[ + \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2(72.73 \text{ lb}) = 0 \]
\[ B = 290.9 \text{ lb} \]
\[ \Delta B = +291 \text{ lb} \]
13- In order to obtain a better weight distribution over the four wheels of the pickup truck of Prob. 12, a compensating hitch of the type shown is used to attach the trailer to the truck. The hitch consists of two bar springs (only one is shown in the figure) that fit into bearings inside a support rigidly attached to the truck. The springs are also connected by chains to the trailer frame, and specially designed hooks make it possible to place both chains in tension. (a) Determine the tension $T$ required in each of the two chains if the additional load due to the trailer is to be evenly distributed over the four wheels of the truck. (b) What are the resulting reactions at each of the six wheels of the trailer-truck combination?

**SOLUTION**

(a) We shall first find the additional reaction $A$ at each wheel due to the trailer.

Free body diagram: (Same $A$ at each truck wheel)

\[ +\sum M_A = 0: \quad -(2400 \text{ lb})(2 \text{ ft}) + 2A(14 \text{ ft}) + 2A(23 \text{ ft}) = 0 \]

\[ \sum F_y = 0: \quad 2A - 2400 \text{ lb} + 4(64.86 \text{ lb}) = 0; \]

\[ A = 1070 \text{ lb}; \]

Free body: Truck:
(Trailer loading only)

\[ +\sum M_D = 0: \quad 2A(12 \text{ ft}) + 2A(3 \text{ ft}) - 2T(1.7 \text{ ft}) = 0 \]

\[ T = \frac{8.824A}{2} = \frac{8.824(64.86 \text{ lb})}{2} = 572.3 \text{ lb} \]

Free body: Truck:
(Truck weight only)

\[ -\sum M_B = 0: \quad -(2900 \text{ lb})(5 \text{ ft}) + 2C'(9 \text{ ft}) = 0 \]

\[ C' = 805.6 \text{ lb} \]

\[ C' = 805.6 \text{ lb} \]
\[ \Sigma F_y = 0: \quad 2B' - 2900 \text{ lb} + 2(805.6 \text{ lb}) = 0 \]

\[ B' = 644.4 \text{ lb} \quad \text{B'} = 644.4 \text{ lb} \uparrow \]

**Actual reactions:**

\[ B = B' + \Delta = 644.4 \text{ lb} + 64.86 = 709.2 \text{ lb} \quad \text{B} = 709 \text{ lb} \uparrow \]

\[ C = C' + \Delta = 805.6 \text{ lb} + 64.86 = 870.46 \text{ lb} \quad \text{C} = 870 \text{ lb} \uparrow \]

**From part a:**

\[ \text{A} = 1070 \text{ lb} \uparrow \]
14- The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300-kN motor unit is located at \( G_m \), while the centers of gravity of the 100-kN cab and 75-kN load are located, respectively, at \( G_c \) and \( G_l \). Knowing that the machine is at rest with its brakes released, determine (a) the reactions at each of the four wheels, (b) the forces exerted on the motor unit at \( C \) and \( D \).

**SOLUTION**

(a) **Free body: Entire machine:**

\[
\begin{align*}
&\Sigma M_A = 0: \quad 75(3.2 \text{ m}) - 100(1.2 \text{ m}) + 2B(4.8 \text{ m}) - 300(5.6 \text{ m}) = 0 \\
&\quad 2B = 325 \text{ kN} \\
&\Sigma F_y = 0: \quad 2A + 325 - 75 - 100 - 300 = 0 \\
&\quad 2A = 150 \text{ kN} \\
&\Sigma F_x = 0: \quad A = 75.0 \text{ kN} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
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&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
&2A = 150 \text{ kN} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 100 \times 1.2 = 120 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 300 \times 0.8 = 240 \text{ kN} \cdot \text{m} \\
&2B = 325 \text{ kN} \\
&\Sigma M_B = 0: \quad 75 \times 3.2 = 240 \text{ kN} \cdot \text{m} \\
Free body: Motor unit:

\[ + \Sigma M_D = 0: \quad C(1 \text{ m}) + 2B(2.8 \text{ m}) - 300(3.6 \text{ m}) = 0 \]

\[ C = 1080 - 5.6B \quad (1) \]

Recalling \( B = 162.5 \text{ kN}, \quad C = 1080 - 5.6(162.5) = 170 \text{ kN} \)

\[ \pm \Sigma F_x = 0: \quad D_1 - 170 = 0 \quad \Rightarrow \quad D_1 = 170.0 \text{ kN} \]

\[ \pm \Sigma F_y = 0: \quad 2(162.5) - D_y - 300 = 0 \quad \Rightarrow \quad D_y = 25.0 \text{ kN} \]
16- For the frame and loading shown, determine the components of the forces acting on member CDE at C and D.

**SOLUTION**

**Free body: Entire frame:**

\[ + \sum M_y = 0: \quad A_y - 25 \text{ lb} = 0 \]
\[ A_y = 25 \text{ lb} \]
\[ \sum M_F = 0: \quad A_y(6.928 + 2 \times 3.464) - (25 \text{ lb})(12 \text{ in.}) = 0 \]
\[ A_y = 21.651 \text{ lb} \]
\[ \sum F_x = 0: \quad F - 21.651 \text{ lb} = 0 \]
\[ F = 21.651 \text{ lb} \]

**Free body: Member CDE:**

\[ + \sum M_C = 0: \quad D_x(4 \text{ in.}) - (25 \text{ lb})(10 \text{ in.}) = 0 \]
\[ D_x = +62.5 \text{ lb} \]
\[ \sum F_y = 0: \quad -C_y + 62.5 \text{ lb} - 25 \text{ lb} = 0 \]
\[ C_y = 37.5 \text{ lb} \]

**Free body: Member ABD:**

\[ + \sum M_B = 0: \quad D_x(3.464 \text{ in.}) + (21.651 \text{ lb})(6.928 \text{ in.}) \]
\[ -(25 \text{ lb})(4 \text{ in.}) - (62.5 \text{ lb})(2 \text{ in.}) = 0 \]
\[ D_x = +21.651 \text{ lb} \]

**Return to free body: Member CDE:**

From above:

\[ D_x = +21.65 \text{ lb} \]
\[ \sum F_x = 0: \quad C_x - 21.65 \text{ lb} = 0 \]
\[ C_x = +21.65 \text{ lb} \]
1- The shear shown is used to cut and trim electronic circuit board laminates. For the position shown, determine (a) the vertical component of the force exerted on the shearing blade at $D$, (b) the reaction at $C$.

**SOLUTION**

We note that $BD$ is a two-force member.

**Free body: Member $ABC$**

We have the components:

- $P_x = (400 \text{ N}) \sin 30^\circ = 200 \text{ N}$
- $P_y = (400 \text{ N}) \cos 30^\circ = 346.41 \text{ N}$

We get:

- $(F_{BD})_x = \frac{25}{65} F_{BD}$
- $(F_{BD})_y = \frac{60}{65} F_{BD}$

\[ 
\sum_{C} = 0: \quad (F_{BD})_x (45) + (F_{BD})_y (30) - P_x (45 + 300 \sin 30^\circ) 
= -P_x (30 + 300 \cos 30^\circ) = 0 
\]

\[ 
\left( \frac{25}{65} F_{BD} \right) (45) + \left( \frac{60}{65} F_{BD} \right) (30) = (200)(195) + (346.41)(289.81) 
\]

\[ 
45 F_{BD} = 139.39 \times 10^3 
\]

\[ 
F_{BD} = 3097.6 \text{ N} 
\]

(a) **Vertical component of force exerted on shearing blade at $D$**

\[ 
(F_{BD})_y = \frac{60}{65} F_{BD} = \frac{60}{65} (3097.6 \text{ N}) = 2859.3 \text{ N} 
\]

\[ 
(F_{BD})_y = 2860 \text{ N} 
\]
Returning to $FB$ diagram of member $ABC$,

$$\sum F_x = 0: \quad (F_{BD})_x - P_x - C_x = 0$$

$$C_x = (F_{BD})_x - P_x = \frac{25}{65} F_{BD} - P_x$$

$$= \frac{25}{65} (3097.6) - 200$$

$$C_x = +991.39 \quad \text{N} \quad C_x = 991.39 \text{ N} \quad \quad \downarrow$$

$$\sum F_y = 0: \quad (F_{BD})_y - P_y - C_y = 0$$

$$C_y = (F_{BD})_y - P_y = \frac{60}{65} F_{BD} - P_y = \frac{60}{65} (3097.6) - 346.41$$

$$C_y = +2512.9 \text{ N} \quad C_y = 2512.9 \text{ N} \quad \quad \downarrow$$

$$C = 2295 \text{ N} \quad C = 2700 \text{ N} \quad \searrow 68.5^\circ$$
2- Water pressure in the supply system exerts a downward force of 135 N on the vertical plug at A. Determine the tension in the fusible link \( DE \) and the force exerted on member \( BCE \) at \( B \).

**SOLUTION**

Free body: Entire linkage:

\[ + \sum F_y = 0: \quad C - 135 = 0 \]
\[ C = +135 \text{ N} \]

Free body: Member \( BCE \):

\[ + \sum F_x = 0: \quad B_x = 0 \]
\[ + \sum M_B = 0: \quad (135 \text{ N})(6 \text{ mm}) - T_{DE}(10 \text{ mm}) = 0 \]
\[ T_{DE} = 81.0 \text{ N} \]
\[ + \sum F_y = 0: \quad 135 + 81 - B_y = 0 \]
\[ B_y = +216 \text{ N} \]

\[ B = 216 \text{ N} \]
3- An 84-lb force is applied to the toggle vise at C. Knowing that $\theta = 90^\circ$, determine (a) the vertical force exerted on the block at D, (b) the force exerted on member ABC at B.

**SOLUTION**

We note that $BD$ is a two-force member.

**Free body: Member ABC:**

We have

$$BD = \sqrt{(7)^2 + (24)^2} = 25 \text{ in.}$$

$$F_{BD} = \frac{7}{25} F_{BD}, \quad (F_{BD})_y = \frac{24}{25} F_{BD}$$

$$\sum M_A = 0: \quad (F_{BD})_x (24) + (F_{BD})_y (7) - 84(16) = 0$$

$$\left( \frac{7}{25} F_{BD} \right)(24) + \left( \frac{24}{25} F_{BD} \right)(7) = 84(16)$$

$$\frac{336}{25} F_{BD} = 1344$$

$$F_{BD} = 100 \text{ lb}$$

$$\tan \alpha = \frac{24}{7} \quad \alpha = 73.7^\circ$$

(b) **Force exerted at B.**

(a) **Vertical force exerted on block.**

$$F_{BD} = 100.0 \text{ lb} \angle 73.7^\circ$$

$$F_{BD} = 24 F_{BD} = \frac{24}{25} (100 \text{ lb}) = 96 \text{ lb}$$

$$(F_{BD})_y = 96.0 \text{ lb}$$
4- For the system and loading shown, determine (a) the force \( P \) required for equilibrium, (b) the corresponding force in member \( BD \), (c) the corresponding reaction at \( C \).

**SOLUTION**

**Member FBDs:**

**I:**

\[ \Sigma M_C = 0: \quad R(F_{BD} \sin 30^\circ) - [R(1 - \cos 30^\circ)](100 \text{ N}) - R(50 \text{ N}) = 0 \]

\[ F_{BD} = 126.795 \text{ N} \quad (b) \quad F_{BD} = 126.8 \text{ N} \quad T \uparrow \]

\[ \Sigma F_x = 0: \quad -C_x + (126.795 \text{ N}) \cos 30^\circ = 0 \quad C_x = 109.808 \text{ N} \leftarrow \]

\[ \Sigma F_y = 0: \quad C_y + (126.795 \text{ N}) \sin 30^\circ - 100 \text{ N} - 50 \text{ N} = 0 \]

\[ C_y = 86.603 \text{ N} \quad \text{so (c) } C = 139.8 \text{ N} \uparrow 38.3^\circ \]

**II:**

\[ \Sigma M_A = 0: \quad aP - a[(126.795 \text{ N}) \cos 30^\circ] = 0 \]

\[ (a) \quad P = 109.8 \text{ N} \leftarrow \]
5- The Whitworth mechanism shown is used to produce a quick-return motion of Point $D$. The block at $B$ is pinned to the crank $AB$ and is free to slide in a slot cut in member $CD$. Determine the couple $M$ that must be applied to the crank $AB$ to hold the mechanism in equilibrium when (a) $\alpha = 0$, (b) $\alpha = 30^\circ$.

**SOLUTION**

(a) Free body: Member $CD$:

\[ + \sum M_C = 0: \quad B(0.5 \text{ m}) - (1200 \text{ N})(0.7 \text{ m}) = 0 \]

\[ B = 1680 \text{ N} \]

Free body: Crank $AB$:

\[ + \sum M_A = 0: \quad M - (1680 \text{ N})(0.1 \text{ m}) = 0 \]

\[ M = 168.0 \text{ N} \cdot \text{m} \]

(b) Geometry:

\[ AB = 100 \text{ mm}, \quad \alpha = 30^\circ \]

\[ \tan \theta = \frac{50}{486.6} \quad \theta = 5.87^\circ \]

\[ BC = \sqrt{(50)^2 + (486.6)^2} = 489.16 \text{ mm} \]

Free body: Member $CD$:

\[ + \sum M_C = 0: \quad B(0.48916) - (1200 \text{ N})(0.7 \text{ m}) \cos 5.87^\circ = 0 \]

\[ B = 1708.2 \text{ N} \]
Free body: Crank $AB$:

\[ + \sum M_A = 0 : \quad M - (B \sin 65.87^\circ)(0.1 \text{ m}) = 0 \]

\[ M = (1708.2 \text{ N})(0.1 \text{ m})\sin 65.87^\circ \]

\[ M = 155.90 \text{ N} \cdot \text{m} \]

\[ M = 155.9 \text{ N} \cdot \text{m} \]
6- A couple \( \mathbf{M} \) of magnitude 1.5 kN \( \cdot \) m is applied to the crank of the engine system shown. For each of the two positions shown, determine the force \( \mathbf{P} \) required to hold the system in equilibrium.

**SOLUTION**

(a) FBDs:

Note:

\[
\tan \theta = \frac{50 \text{ mm}}{175 \text{ mm}} = \frac{2}{7}
\]

FBD whole:

\[
\Sigma M_A = 0: \quad (0.250 \text{ m})C_y - 1.5 \text{ kN} \cdot \text{m} = 0 \quad C_y = 6.00 \text{ kN}
\]

FBD piston:

\[
\Sigma F_y = 0: \quad C_y - F_{BC} \sin \theta = 0 \quad F_{BC} = \frac{C_y}{\sin \theta} = \frac{6.00 \text{ kN}}{\sin \theta}
\]

\[
\Sigma F_x = 0: \quad F_{BC} \cos \theta - P = 0
\]

\[
P = F_{BC} \cos \theta = \frac{6.00 \text{ kN}}{\tan \theta} = 7 \text{ kips}
\]

\[
P = 21.0 \text{ kN} \quad \downarrow
\]
(b) FBDs:

Note: \( \tan \theta = \frac{2}{7} \) as above

FBD whole:

\[ \Sigma M_A = 0: \quad (0.100 \text{ m})C_y - 1.5 \text{ kN} \cdot \text{m} = 0 \quad C_y = 15 \text{ kN} \]

\[ \Sigma F_y = 0: \quad C_y - F_{BC} \sin \theta = 0 \quad F_{BC} = \frac{C_y}{\sin \theta} \]

\[ \rightarrow \Sigma F_x = 0: \quad F_{BC} \cos \theta - P = 0 \]

\[ P = F_{BC} \cos \theta = \frac{C_y}{\tan \theta} = \frac{15 \text{ kN}}{2/7} \quad P = 52.5 \text{ kN} \]
7- The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple $M$ required to hold the system in equilibrium when $\theta = 30^\circ$.

**SOLUTION**

Free body: Member ABC:

\[
+ \sum M_C = 0: \quad (25 \text{ lb})(13.856 \text{ in.}) - B(3 \text{ in.}) = 0 \\
\Rightarrow B = +115.47 \text{ lb}
\]

\[
+ \sum F_y = 0: \quad -25 \text{ lb} + C_y = 0 \\
\Rightarrow C_y = +25 \text{ lb}
\]

\[
\pm \sum F_x = 0: \quad 115.47 \text{ lb} - C_x = 0 \\
\Rightarrow C_x = +115.47 \text{ lb}
\]

Free body: Member CD:

\[
\beta = \sin^{-1} \frac{5.196}{8}; \quad \beta = 40.505^\circ
\]

\[
CD \cos \beta = (8 \text{ in.}) \cos 40.505^\circ = 6.083 \text{ in.}
\]

\[
+ \sum M_D = 0: \quad M - (25 \text{ lb})(5.196 \text{ in.}) - (115.47 \text{ lb})(6.083 \text{ in.}) = 0
\]

\[
M = +832.3 \text{ lb} \cdot \text{in.}
\]

\[
M = 832 \text{ lb} \cdot \text{in.}
\]
8- Rod $CD$ is attached to the collar $D$ and passes through a collar welded to end $B$ of lever $AB$. Neglecting the effect of friction, determine the couple $M$ required to hold the system in equilibrium when $\theta = 30^\circ$.

**SOLUTION**

**FBD DC:**

\[
\sum F_y = 0: \quad D_y \sin 30^\circ - (150 \text{ N})\cos 30^\circ = 0
\]

\[
D_y = (150 \text{ N}) \csc 30^\circ = 259.81 \text{ N}
\]

**FBD machine:**

\[
\sum M_A = 0: \quad (0.100 \text{ m})(150 \text{ N}) + d(259.81 \text{ N}) - M = 0
\]

\[
d = b - 0.040 \text{ m}
\]

\[
b = \frac{0.030718 \text{ m}}{\tan 30}
\]

\[
b = 0.053210 \text{ m}
\]

\[
d = 0.0132100 \text{ m}
\]

\[
M = 18.4321 \text{ N} \cdot \text{m}
\]

\[
M = 18.43 \text{ N} \cdot \text{m}
\]
9- Rod $CD$ is attached to the collar $D$ and passes through a collar welded to end $B$ of lever $AB$. Neglecting the effect of friction, determine the couple $M$ required to hold the system in equilibrium when $\theta = 30^\circ$.

**SOLUTION**

Note: $B \perp CD$

**FBD $DC$:**

$\sum F_x = 0: \quad D_y \sin 30^\circ - (300 \text{ N}) \cos 30^\circ = 0$

$D_y = \frac{300 \text{ N}}{\tan 30^\circ} = 519.62 \text{ N}$

**FBD machine:**

$\sum M_A = 0: \quad \frac{0.200 \text{ m}}{\sin 30^\circ} (519.62 \text{ N} - M) = 0$

$M = 207.85 \text{ N} \cdot \text{m}$

$\text{M} = 208 \text{ N} \cdot \text{m}$
10- Two rods are connected by a frictionless collar $B$. Knowing that the magnitude of the couple $MA$ is 500 lb · in., determine (a) the couple $MC$ required for equilibrium, (b) the corresponding components of the reaction at $C$.

![Image of the problem setup]

**SOLUTION**

(a) **Free body: Rod $AB$ & collar:**

\[
\sum M_A = 0: \quad (B \cos \alpha)(6 \text{ in.}) + (B \sin \alpha)(8 \text{ in.}) - M_A = 0
\]

\[
B = (6 \cos 21.8^\circ + 8 \sin 21.8^\circ) - 500 = 0
\]

\[
B = 58.535 \text{ lb}
\]

(b) **Free body: Rod $BC$:**

\[
\sum M_C = 0: \quad M_C - Bl = 0
\]

\[
M_C = Bl = (58.535 \text{ lb})(21.541 \text{ in.}) = 1260.9 \text{ lb-in.} \quad (M_C = 1261 \text{ lb-in.})
\]

\[
\sum F_x = 0: \quad C_x + B \cos \alpha = 0
\]

\[
C_x = -B \cos \alpha = -(58.535 \text{ lb}) \cos 21.8^\circ = -54.3 \text{ lb}
\]

\[
C_x = 54.3 \text{ lb}
\]

\[
\sum F_y = 0: \quad C_y - B \sin \alpha = 0
\]

\[
C_y = B \sin \alpha = (58.535 \text{ lb}) \sin 21.8^\circ = +21.7 \text{ lb}
\]

\[
C_y = 21.7 \text{ lb}
\]