Answer the following questions

Question (1): Find the value of each of the following:

1) $\lim_{x\to 3} \frac{\sqrt{12-x}-x}{\sqrt{6+x}-3}$

2) $\lim_{x\to 0} \frac{sin\ 3\ x}{x}$

3) $\lim_{x\to 0} \frac{1-cos\ x}{2x^2}$

4) $\lim_{x\to \pi} \frac{\sin x}{\pi - x}$

5) $\lim_{x\to 1} \frac{e^{-x}-e^{-1}}{x-1}$

6) $\lim_{x\to 1} \frac{e-e^{x}}{x-1}$

7) $\lim_{x\to 0} \frac{x^3 cot\ x}{1-cos\ x}$ $\quad s.T \quad cot\ x = \frac{1}{\tan x}$

8) $\lim_{x\to 0} \frac{cosec\ x - cot\ x}{x}$ $\quad s.T \quad cosec\ x = \frac{1}{\sin x}$

Question (2): Prove that

a) $\lim_{x\to a} \frac{x^n - a^n}{x-a} = n\ a^{n-1}$

b) $\lim_{x\to 0} sin\ x = 0 \quad and \quad \lim_{x\to 0} cos\ x = 1$

Note : $\begin{bmatrix} \because cos\ 2x = 1 - 2\ sin^2\ x, \\ \therefore 1 - cos\ 2x = 2\ sin^2\ x \\ or\ \ 1 - cos\ x = 2\ sin^2 \frac{x}{2} \end{bmatrix}$

Question (3): Prove that:

1) $\frac{d}{dx}(sin\ x) = cos\ x$

2) $\frac{d}{dx}(ln\ x) = \frac{1}{x}$

3) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

4) $\frac{d}{dx} ln\ (cos\ x) = -cot\ x$

Question (4):find $\frac{dy}{dx}$ if

1) $y = \sin(x^2 + 1)$

2) $y = \left(\frac{4}{x} + \frac{2}{x^4}\right) \sin\ x$

3) $y = \sin^3(x)$

4) $y = \sin(x)^3$

5) $y = e^x - \ ln\ (3x - 2)$

6) $x^3 + y^3 = 6x$

Question (5):

a) Prove that:

1) $D^n\left(\frac{1}{ax+b}\right) = \frac{-(-1)^n\ \ n!}{(ax+b)^{n+1}}$

2) $D^n(e^{ax}) = a^n\ \ e^{ax}$

b) **Find D^{11} if:**

1) $y = x^{10}$

2) $y = \ x^3\ \ e^{2x}$

Question (6):

a)Prove the following formula by induction:

1) $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

2) $2\ +\ 5\ +\ 8\ + \cdots + (3n\ -\ 1)\ =\ \frac{1}{2}\ n(3n\ +\ 1)$.

b) Evaluate each of the following integrals:

1) $\int \frac{x^2}{x^3-3}\ dx$

2) $\int cot\ x\ \ dx$

Question (7):

Evaluate each of the following integrals:

1) $\int \frac{ln\ x}{x}\ dx$

2) $\int x^2 \sqrt[3]{x^3 + 2}\ dx$

3) $I = \int e^{5x}\ sin\ x\ dx$

4) $\int \frac{x^2}{(x+1)(x-1)^2}\ dx$

QUESTIONS END
GOOD LUCK

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