	DAMEITTA UNIVERSITY Faculty of ENGINEERING			
Program/ Year	First Year Mechanical Depts. 2019-20	semester	Second	
Course title:	Material strength & stresses analysis	Sheet No.	4	

1. A hollow shaft of 40mm outer diameter and 25mm inner diameter is subjected to a twisting moment of 120 N-m, simultaneously; it is subjected to an axial thrust of 10 KN and a bending moment of 80 N-m. Calculate the maximum compressive and shear stresses.

Solution. Given: $d_o = 40$ mm ; $d_i = 25$ mm ; $T = 120$ N-m $= 120 \times 10^3$ N-mm ; $P = 10$ kN $= 10 \times 10^3$ N ; $M = 80$ N-m $= 80 \times 10^3$ N-mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] = \frac{\pi}{4}\left[(40)^2 - (25)^2\right] = 766 \text{ mm}^2$$

$\therefore$ Direct compressive stress due to axial thrust,

$$\sigma_o = \frac{P}{A} = \frac{10 \times 10^3}{766} = 13.05 \text{ N/mm}^2 = 13.05 \text{ MPa}$$

Section modulus of the shaft,

$$Z = \frac{\pi}{32}\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{32}\left[\frac{(40)^4 - (25)^4}{40}\right] = 5325 \text{ mm}^3$$

$\therefore$ Bending stress due to bending moment,

$$\sigma_b = \frac{M}{Z} = \frac{80 \times 10^3}{5325} = 15.02 \text{ N/mm}^2 = 15.02 \text{ MPa (compressive)}$$

and resultant compressive stress,

$$\sigma_c = \sigma_b + \sigma_o = 15.02 + 13.05 = 28.07 \text{ N/mm}^2 = 28.07 \text{ MPa}$$

We know that twisting moment (T),

$$120 \times 10^3 = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \left[\frac{(40)^4 - (25)^4}{40}\right] = 10\ 650\ \tau$$

$$\therefore \quad \tau = 120 \times 10^3/10\ 650 = 11.27 \text{ N/mm}^2 = 11.27 \text{ MPa}$$

Maximum compressive stress

We know that maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

$$= \frac{28.07}{2} + \frac{1}{2}\left[\sqrt{(28.07)^2 + 4\,(11.27)^2}\right]$$

$$= 14.035 + 18 = 32.035 \text{ MPa} \quad \textbf{Ans.}$$

Maximum shear stress

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(28.07)^2 + 4\,(11.27)^2}\right] = 18 \text{ MPa} \quad \textbf{Ans.}$$

2. A propeller shaft for a launch transmits 75 KW at 150 rpm and is subjected to a maximum bending moment of 1KN-m and an axial thrust of 70 KN. Find the shaft diameter based on maximum principal stress if the shear strength of the shaft material is limited to 100 MPa

$$\text{Torque, } T = \frac{75x10^3}{\left(\frac{2\pi x150}{60}\right)} = 4775\,\text{Nm; then, } \tau = \frac{24.3}{d^3}\,\text{KPa}$$

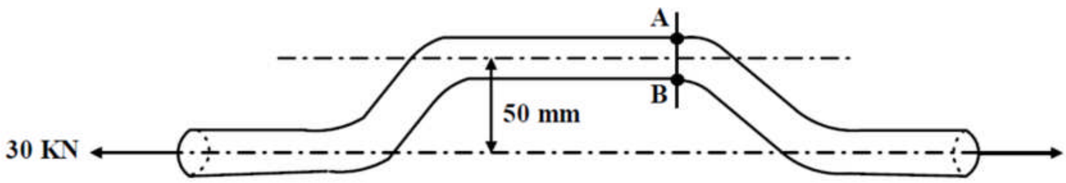
$$\text{Maximum bending moment} = 1\,\text{KNm; then, } \sigma_b = \frac{10.19}{d^3}\,\text{KPa}$$

$$\text{Axial force} = 70\,\text{KN; then, } \sigma = \frac{70}{\frac{\pi d^2}{4}}\,\text{KPa} = \frac{89.12}{d^2}\,\text{KPa}$$

$$\text{Maximum shear stress} = \sqrt{\left(\frac{89.12}{2d^2} - \frac{10.19}{2d^3}\right)^2 + \left(\frac{24.3}{d^3}\right)^2} = 100x10^3$$

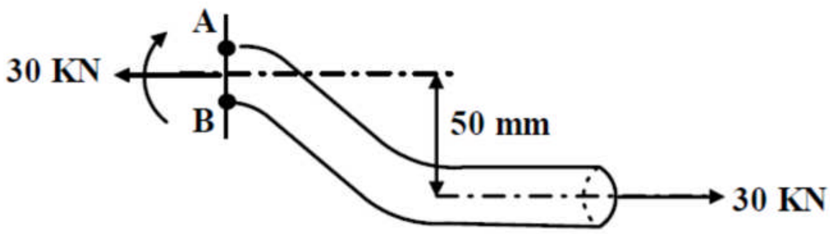
Solving we get the value of shaft diameter d = 63.4 mm.

3. A 100 mm diameter off-set link is transmitting an axial pull of 30 KN as shown in the following figure. Find the stresses at points A and B.

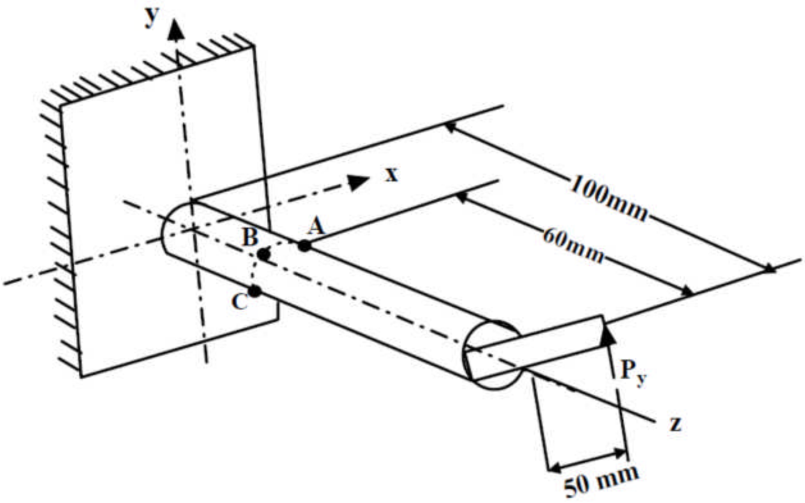


$$\sigma_A = -\frac{30x10^3 x0.05x0.05}{\frac{\pi}{64}(0.1)^4} + \frac{30x10^3}{\frac{\pi}{4}(0.1)^2} = -11.46\,\text{MPa}$$

$$\sigma_B = \frac{30x10^3 x0.05x0.05}{\frac{\pi}{64}(0.1)^4} + \frac{30x10^3}{\frac{\pi}{4}(0.1)^2} = 19.1\,\text{MPa}$$

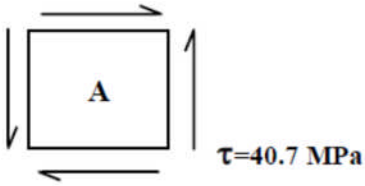


4. A vertical load Py = 20 KN is applied at the free end of a cylindrical bar of radius 50 mm as shown in the following fig. Determine the principal and maximum shear stresses at the points A, B and C.



At section ABC a bending moment of 1.2 KN-m and a torque of 1KN-m act.On elements A and C there is no bending stress. Only torsional shear stress acts and

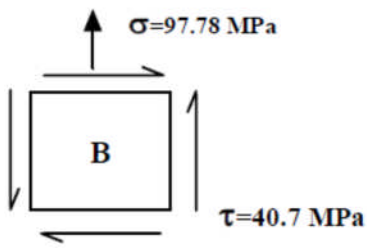
$$\tau = \frac{16T}{\pi d^3} = 40.7 \text{ MPa}$$



On element B both bending (compressive) and torsional shear stress act.

$$\sigma_B = \frac{32M}{\pi d^3} = 97.78\,\text{MPa}$$

$$\tau = 40.7\,\text{MPa}$$



Principal stresses at B = $\left(\frac{97.78}{2} \pm \sqrt{\left(\frac{97.78}{2}\right)^2 + (40.7)^2} \right)$

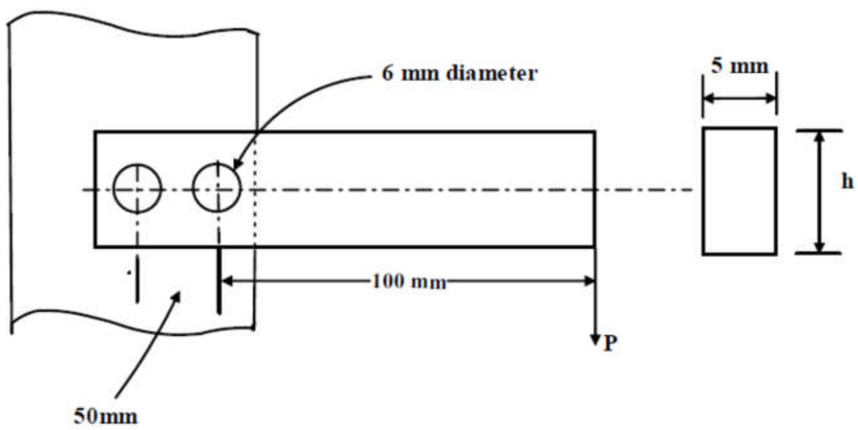
$$\sigma_{B1} = 112.5\text{MPa}; \quad \sigma_{B2} = -14.72\text{MPa}$$

Maximum shear stress at B = $\sqrt{\left(\frac{97.78}{2}\right)^2 + (40.7)^2} = 63.61 \text{ MPa}$

5. A 5mm thick steel bar is fastened to a ground plate by two 6 mm diameter pins as shown in figure-2.2.7.1. If the load P at the free end of the steel bar is 5 KN, find

(a) The shear stress in each pin

(b) The direct bearing stress in each pin.



Due to the application of force P the bar will tend to rotate about point 'O' causing shear and bearing stresses in the pins A and B. Let the forces at pins A and B be FA and FB and equating moments about 'O' ,

$$5\times10^3\times0.125 = (F_A+F_B)\times 0.025 \qquad (1)$$

Also, from force balance, $F_A+P = F_B$ (2)

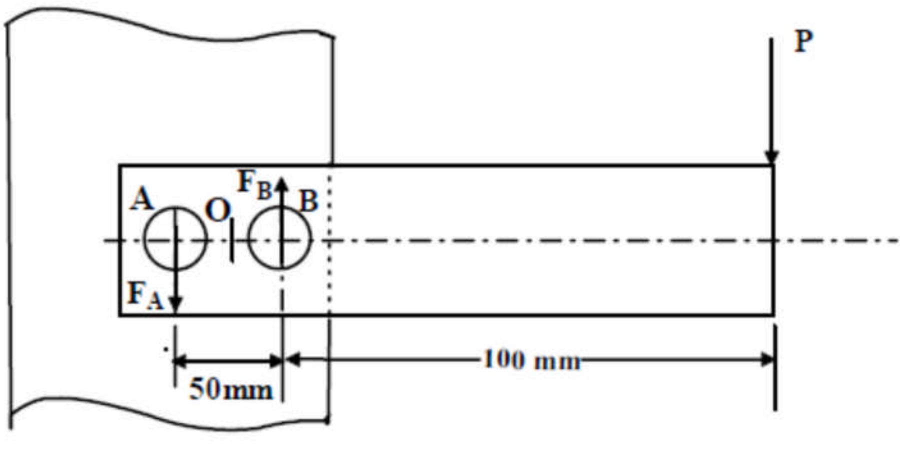
Solving equations-1 and 2 we have, $F_A = 10$ KN and $F_B = 15$ KN.

(a) Shear stress in pin A = $\dfrac{10\times10^3}{\left(\dfrac{\pi\times0.006^2}{4}\right)} = 354\,\text{MPa}$

Shear stress in pin B = $\dfrac{15\times10^3}{\left(\dfrac{\pi\times0.006^2}{4}\right)} = 530.5\,\text{MPa}$

(b) Bearing stress in pin A = $\dfrac{10\times10^3}{(0.006\times0.005)} = 333\,\text{MPa}$

Bearing stress in pin B = $\dfrac{15\times10^3}{(0.006\times0.005)} = 500\,\text{MPa}$


P
F_B
A
O
B
F_A
50mm
100 mm