Chapter (6)

LIFE INSURANCE

LIFE INSURANCE

6-1 Basic Types Of Life Insurance:

Life insurance companies have made available many types of life insurance contracts (policies) to meet individual needs. However, there are only three basic types of life insurance contracts: (1) whole life insurance, (2) term insurance, and (3) endowment insurance. Every life insurance contract is one of these three kinds or is a combination of them. The protection offered by any type of life insurance contract may also be deferred to a future date. Thus, in addition to the three basic types of insurance, a discussion of deferred life insurance is included in this chapter.

6-2 Whole Life Insurance:

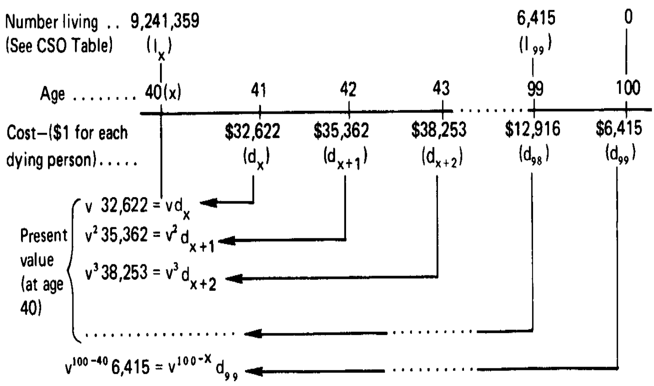
A whole life insurance contract provides that the insurance company will pay the face value of the policy to the beneficiary upon the death of the insured, regardless of when the death occurs. The premium for a whole life insurance policy (simply called a whole life policy) may be paid by a single amount or in periodic payments. When a premium is payable periodically, there are two plans under which the premium may be paid. Under the straight life plan, premiums are payable until death, while under the limited payment life plan, premiums are payable for a specified number of years. Net premiums are computed at a nominal interest rate of 4% in all of the life insurance problems in this chapters as explained before.

A. Finding the Net Single Premium

The following example is used to illustrate the method of finding the net single premium for a whole life policy.

EXAMPLE 1 A \$10,000 whole life policy is issued to a man aged 40. Find the net single premium.

First, find the net single premium for a \$1 whole life policy. The cost of the \$1 policy and its present value (at age 40) are diagrammed numerically and symbolically as follows:



Note: $v = (1 + i)^{-1}$

The cost in each year depends on the number of persons who die that year. According to the 1958 CSO Table, 32,622 persons will die between ages 40 and 41. For simplicity in this chapter, *the cost is computed with the assumption that all death benefits are paid at the end of the year of death.* Therefore, \$32,622 is required for the death benefits one year after the policies are issued (at the end of age 40 or the beginning of age 41).

The present value of the total cost is distributed among the people who are living at age 40. The cost of each policy, or the net single premium to each person at age 40, is denoted by A_{40} and is computed as follows:

$$A_{40} = \frac{v(32{,}622) + v^2(35{,}362) + v^3(38{,}253) + \cdots + v^{100-40}(6{,}415)}{9{,}241{,}359}$$

$$= .4671275 \quad \left(i = 2\frac{1}{2}\%\right)$$

The answer above may be calculated by using Table 6 [where $v^n = (1 + i)^{-n}$], but the calculation is quite laborious. However, it can be simplified by using the following formula and the commutation columns in Table 13.

Let A_x = the net single premium (or present value) of a $1 whole life insurance policy, issued at age x

The following formula can be obtained by a method similar to that followed in the above illustration:

$$A_x = \frac{M_x}{D_x} \qquad \textbf{(12-1)}^{1}$$

The values of M_x and D_x are listed in Table 13 (at a $2\frac{1}{2}\%$ interest rate).

The value of A_{40} in Example 1 may be computed by using the above formula, as follows:

$$A_{40} = \frac{M_{40}}{D_{40}} = \frac{1{,}607{,}743.17}{3{,}441{,}765.06} = .4671275$$

For a $10,000 whole life policy, the net single premium is:

$.4671275 \times 10{,}000 = \$4{,}671.28$

NOTE: **The symbol A has a subscript, such as A_x, when it is used to represent the net single premium of a $1 life insurance policy. When the symbol A has no subscript, it represents the net single premium of a life annuity of R per payment, such as A and A(due) in Chapter 19.**

Also, note that, again, the symbols used in this chapter are based on the statement of the "International Actuarial Notation," printed in *Transactions of the Actuarial Society of America.* (See footnote 3 of Chapter 19.)

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[1] *Proof—Formula (12-1):*

The value of A_x is obtained as follows (see the diagram in Example 1):

$$A_x = \frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + \cdots + v^{100-x}d_{99}}{l_x}$$

Multiply both the numerator and the denominator by v^x. Then

$$A_x = \frac{v^{x+1}d_x + v^{x+2}d_{x+1} + v^{x+3}d_{x+2} + \cdots + v^{100}d_{99}}{v^xl_x}$$

Let the commutation symbol $C_x = v^{x+1}d_x$. The above equation may be written

$$A_x = \frac{C_x + C_{x+1} + C_{x+2} + \cdots + C_{99}}{D_x}$$

Let the commutation symbol $M_x = C_x + C_{x+1} + C_{x+2} + \cdots + C_{99}$

The above equation may be further simplified as follows: $A_x = \frac{M_x}{D_x}$ (12-1)

Note: If we use $(1 + i)^{-1}$ in the derivation of A_x instead of the letter v and multiply both the numerator and the denominator by $(1 + i)^{100-x}$, we then have:

$C_x = (1 + i)^{100-(x+1)} \cdot d_x, \quad C_{x+1} = (1 + i)^{100-(x+2)} \cdot d_{x+1}$

and so on. Also, see footnote 5 in Chapter 19.

B. Finding the Net Annual Premium

I. STRAIGHT LIFE (ALSO CALLED ORDINARY LIFE)

Under the straight life plan, premiums are payable periodically until death.

Let P_x = the net annual premium that is payable to the insurance company for a \$1 policy each year for life, beginning at age x

The following formula can be obtained:

$$P_x = \frac{M_x}{N_x} \qquad (12\text{-}2)^2$$

EXAMPLE 2

Refer to Example 1. If the man wishes to pay the net premium annually, with the first annual premium payable at the date of purchase (at age 40), what should be the size of the net annual premium?

$x = 40$. Substituting the x value in Formula (20-2),

$$P_{40} = \frac{M_{40}}{N_{40}} = \frac{1{,}607{,}743.17}{75{,}194{,}899.17} = \$0.02138101$$

The annual premium for a \$10,000 policy is

$.02138101 \times 10{,}000 = \$213.81$

II. LIMITED PAYMENT LIFE

Under the limited payment life plan, premiums are payable periodically for a specified number of years. A whole life policy with a limited number of payments provides protection during the lifetime of the insured, but the premiums are payable for only a specified number of years. Some examples are the 20-payment life, 30-payment life, and life paid up at age 65.

Let ${}_nP_x$ = the net annual premium, beginning at age x, for an n payment life policy of \$1

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[2]*Proof—Formula* (12-2):

The insurance company collects only the annual premiums from the insured who is living. Thus, the annual premiums (P_x) form a *whole life annuity due*. The present value of the annual premiums can be obtained by using Formula (19-5).

$A(\text{due}) = R\ddot{a}_x$, or

$A(\text{due}) = P_x\ddot{a}_x$

The present value must be equal to the net single premium A_x, or $A(\text{due}) = A_x$. Substituting the value in the above equation,

$$A_x = P_x\ddot{a}_x, \quad P_x = A_x \div \ddot{a}_x = \frac{M_x}{D_x} \div \frac{N_x}{D_x} = \frac{M_x}{D_x} \cdot \frac{D_x}{N_x} = \frac{M_x}{N_x}, \quad \text{or} \quad P_x = \frac{M_x}{N_x} \qquad (20\text{-}2)$$

Note: This is identical to the type of problem of finding the unknown annual payment when the present value of a whole life annuity due is known. (See Example 6, p. 609.)

The following formula can be obtained:

$$ {}_nP_x = \frac{M_x}{N_x - N_{x+n}} \qquad (12\text{-}3)^3 $$

EXAMPLE 3 A \$10,000 whole life policy is issued to a man aged 25. Find (a) the net single premium, (b) the net annual premium if the policy is a straight life policy, (c) the net annual premium if the policy is a 20-payment life policy.

(a) $x = 25$

Substituting the value in Formula (20-1),

$$ A_{25} = \frac{M_{25}}{D_{25}} = \frac{1{,}754{,}288.51}{5{,}165{,}007.95} = \$.33964875 $$

The net single premium for a \$10,000 policy is:

$.33964875 \times 10{,}000 = \$3{,}396.49$

(b) $x = 25$

Substituting the value in Formula (20-2),

$$ P_{25} = \frac{M_{25}}{N_{25}} = \frac{1{,}754{,}288.51}{139{,}839{,}496.91} = \$.0125450145 $$

The net annual premium for the \$10,000 policy is:

$.012545 \times 10{,}000 = \$125.45$

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[3] *Proof—Formula (12-3):*

The n annual premiums (${}_nP_x$) form a *temporary life annuity due*. The present value of the annual premiums can be obtained by using Formula (19-11).

$A(\text{tem. due}) = R\ddot{a}_{x:\overline{n}|}$, or

$A(\text{tem. due}) = {}_nP_x \cdot \ddot{a}_{x:\overline{n}|}$

The present value must be equal to the net single premium A_x, or $A(\text{tem. due}) = A_x$. Substituting the value in the above equation, then, $A_x = {}_nP_x\ddot{a}_{x:\overline{n}|}$.

Since

$$ A_x = \frac{M_x}{D_x} \quad \text{and} \quad \ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x} $$

then

$$ \frac{M_x}{D_x} = {}_nP_x \cdot \frac{N_x - N_{x+n}}{D_x} $$

Solve for ${}_nP_x$.

$$ {}_nP_x = \frac{M_x}{D_x} \cdot \frac{D_x}{N_x - N_{x+n}} = \frac{M_x}{N_x - N_{x+n}} \qquad (12\text{-}3) $$

(c) $n = 20, x = 25$

Substituting these values in Formula (20-3),

$$ {}_{20}P_{25} = \frac{M_{25}}{N_{25} - N_{25+20}} = \frac{M_{25}}{N_{25} - N_{45}} = \frac{1{,}754{,}288.51}{139{,}839{,}496.91 - 58{,}927{,}803.08} $$

$$ = \$.02168152 $$

The net annual premium for the $10,000 policy for 20 payments is:

$.02168152 \times 10{,}000 = \$216.82$ (also see Example 3, Section 11.5)

NOTE: **In order to simplify mathematical operations, round the decimals of the values in the commutation columns (Table 12) when computing the problems in the exercises in this chapter.** Examples:

$C_{20} = 10{,}300.1828$, rounded to 10,300
$M_{25} = 1{,}754{,}288.5116$, rounded to 1,754,289

EXERCISE 12-1 REFERENCE: SECTION 20.2

A. For each of the following whole life policies, find (a) the net single premium, and (b) the net annual premium:

	Face Value of Policy	*Age of Insured on Purchase Date*	*Payment Plan*
1.	$ 5,000	26	straight life
2.	5,000	28	5-payment life
3.	3,000	34	10-payment life
4.	3,000	42	straight life
5.	10,000	50	straight life
6.	10,000	55	15-payment life
7.	100,000	65	20-payment life
8.	100,000	75	straight life

B. Statement problems:

9. Find the net single premium of a whole life policy of $100,000 issued to someone aged (a) 10, (b) 30, (c) 60, (d) 85.
10. Find the net single premium of a whole life policy of $100,000 issued to someone aged (a) 4, (b) 24, (c) 44, (d) 64.
11. A $1,000 whole life policy is issued to a person aged 35. Find the net single premium.
12. What is the net single premium of a $2,000 whole life policy issued to a person aged 45?
13. Refer to Problem 11. If the policy is a straight life policy, what is the net annual premium?
14. Refer to Problem 12. Find the net annual premium if the policy is a straight life policy.
15. Refer to Problem 11. If the policy is a 15-payment life policy, what is the net annual premium?
16. Refer to Problem 12. Find the net annual premium if the policy is a 10-payment life policy.

20.3 TERM INSURANCE

A *term insurance* policy provides that the insurance company will pay the face value of the policy to the beneficiary upon the death of the insured, if the insured dies during the term covered in the policy. The insurance company has no obligation for payment if the insured outlives the term. For example, if a five-year term policy is issued to someone aged 20, the insurance company is liable for payment of the policy if the insured dies within the five-year period, from the date of issuance until age 25.

A. Finding the Net Single Premium

Let n = the term in years
$A^1_{x:\,\overline{n}|}$ = the net single premium for a \$1, n-year term policy issued to a person aged x

The following formula can then be obtained:

$$A^1_{x:\,\overline{n}|} = \frac{M_x - M_{x+n}}{D_x} \qquad (\ \ -4)^4$$

B. Finding the Net Annual Premium

Let $P^1_{x:\,\overline{n}|}$ = the net annual premium for an n year term policy of \$1 issued at age x

The following formula can be obtained:

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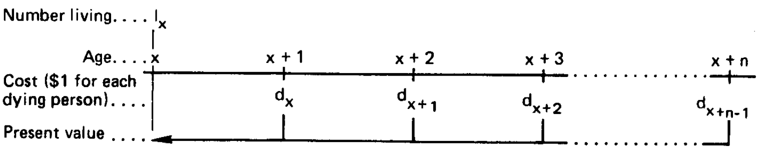
[4]*Proof—Formula (12-4):*

The cost of a \$1, n-year term policy and its net single premium (present value at age x) are diagramed and derived as follows:



The present value of the cost at age x is divided by the number of persons living at age x, as follows:

$$A^1_{x:\,\overline{n}|} = \frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + \cdots + v^nd_{x+n-1}}{l_x}$$

Multiply both the numerator and the denominator by v^x, and substitute the commutation symbols. Then

$$A^1_{x:\,\overline{n}|} = \frac{C_x + C_{x+1} + C_{x+2} + \cdots + C_{x+n-1}}{D_x}$$

Since $M_x = C_x + C_{x+1} + C_{x+2} + \cdots + C_{x+n-1} + C_{x+n} + \cdots + C_{99}$
and $M_{x+n} = C_{x+n} + \cdots + C_{99}$

the difference between M_x, and M_{x+n} equals the numerator in the above equation. Thus, the equation may be written in the following manner:

$$A^1_{x:\,\overline{n}|} = \frac{M_x - M_{x+4}}{D_x} \qquad (12\text{-}4)$$

$$P^{1}_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}} \qquad (12\text{-}5)^5$$

In Formula (12-5), it is assumed that the number of years in the term of the policy is the same as the number of the annual premium payments. If the term (n) is larger than the number of payments (y), the letter y should replace the letter n in the denominator of the fraction:

$${}_yP^{1}_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{N_x - N_{x+y}} \qquad (12\text{-}6)^6$$

EXAMPLE 1 A $1,000, five-year term policy is issued to a man aged 20. Find (a) the net single premium, and (b) the net annual premium.

$x = 20, n = 5, x + n = 25$

(a) Substituting the values in Formula (20-4),

$$\text{The net single premium} = 1{,}000A^{1}_{x:\overline{n|}} = 1{,}000 \cdot \frac{M_{20} - M_{25}}{D_{20}}$$

$$= 1{,}000 \cdot \frac{1{,}804{,}922.42 - 1{,}754{,}288.51}{5{,}898{,}264.97}$$

$$= 1{,}000 \cdot \frac{50{,}633.91}{5{,}898{,}264.97} = 1{,}000(.0085845)$$

$$= \$8.5845, \quad \text{or} \quad \$8.58$$

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[5]*Proof—Formula (12-5):*

The n annual premiums ($P^{1}_{x:\overline{n|}}$) form a temporary life annuity due. The present value of the annual premiums can be obtained by using Formula (19-11).

$A(\text{tem. due}) = R\ddot{a}_{x:\overline{n|}}$, or

$$A(\text{tem. due}) = P^{1}_{x:\overline{n|}}\ddot{a}_{x:\overline{n|}} = P^{1}_{x:\overline{n|}} \cdot \frac{N_x - N_{x+n}}{D_x} \quad \text{(see Section 19.5B)}$$

The present value must be equal to the net single premium $A^{1}_{x:\overline{n|}}$, or

$$A(\text{tem. due}) = A^{1}_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{D_x}$$

Substituting the value in the above equation,

$$P^{1}_{x:\overline{n|}} \cdot \frac{N_x - N_{x+n}}{D_x} = \frac{M_x - M_{x+n}}{D_x}$$

$$P^{1}_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}} \qquad (12\text{-}5)$$

[6]*Proof—Formula (20-6):*

Here the y annual premiums (${}_yP^{1}_{x:\overline{n|}}$) form a temporary life annuity due. Thus,

$$A(\text{tem. due}) = {}_yP^{1}_{x:\overline{n|}} \cdot \frac{N_x - N_{x+y}}{D_x} = \frac{M_x - M_{x+n}}{D_x}$$

$${}_yP^{1}_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{N_x - N_{x+y}} \qquad (12\text{-}6)$$

(b) Since the number of annual premium payments is not indicated in the problem, the number of years in the term of the policy is assumed to be the same as the number of payments.

Substituting the x and n values in Formula (20-5),

$$\text{The net annual premium} = 1{,}000P^{1}_{x:\overline{n|}} = 1{,}000 \cdot \frac{M_{20} - M_{25}}{N_{20} - N_{25}}$$

$$= 1{,}000 \cdot \frac{1{,}804{,}922.42 - 1{,}754{,}288.51}{167{,}827{,}045.88 - 139{,}839{,}496.91}$$

$$= 1{,}000 \cdot \frac{50{,}633.91}{27{,}987{,}548.97} = 1{,}000(.0018091584)$$

$$= \$1.8091584, \quad \text{or} \quad \$1.81$$

EXAMPLE 2 Refer to Example 1. Assume that the net premium is payable in three equal annual payments. Find the net annual premium.

$x = 20, n = 5$ (years, the term of the policy), $y = 3$ (annual payments), $x + n = 25$, $x + y = 23$

Substituting the values in Formula (20-6),

$$\text{The net annual premium} = 1{,}000\,{}_{y}P^{1}_{x:\overline{n|}} = 1{,}000 \cdot \frac{M_{20} - M_{25}}{N_{20} - N_{23}}$$

$$= 1{,}000 \cdot \frac{50{,}633.91}{167{,}827{,}045.88 - 150{,}590{,}926.74}$$

$$= 1{,}000(.002937663) = \$2.94$$

EXERCISE 12-2 REFERENCE: SECTION 20.3

A. For each of the following term insurance policies, find (a) the net single premium, (b) the net annual premium:

	Face Value of Policy	*Age of Insured on Purchase Date*	*Term of Policy*	*Payment Plan*
1.	$ 2,000	30	5 years	5 payments
2.	6,000	25	10 years	10 payments
3.	10,000	35	15 years	15 payments
4.	10,000	40	20 years	20 payments
5.	3,000	26	12 years	12 payments
6.	3,000	38	16 years	16 payments
7.	4,000	45	10 years	6 payments
8.	4,000	22	18 years	10 payments

B. Statement problems:

9. Find the net single premium and the net annual premium for a 10-year, \$5,000 term policy issued to someone aged (a) 18, (b) 28, (c) 50.
10. Find the net single premium and the net annual premium for a 20-year, \$10,000 term policy issued to someone aged (a) 16, (b) 36, (c) 56.
11. A \$1,000, 25-year term policy is issued to a person aged 24. Find (a) the net single premium, (b) the net annual premium.
12. A \$2,000, 30-year term policy is issued to a person aged 32. Find (a) the net single premium, (b) the net annual premium.
13. Refer to Problem 11. If the policy specifies that the net premium is payable in 20 equal annual payments, find the size of the net annual premium.
14. Refer to Problem 12. If the policy specifies that the net premium is payable in 15 equal annual payments, what is the size of the net annual premium?

20.4 ENDOWMENT INSURANCE

An *endowment insurance* policy combines the features of a pure endowment and a term insurance policy. The provisions of the combination are that (a) if the insured is living at the end of the term of the policy, the face value of the policy will be payable to the policyholder; and (b) if the insured dies during the term of the policy, the face value of the policy will be payable to the designated beneficiary. Thus, an n-year endowment insurance policy may be treated as a combined policy consisting of an n-year pure endowment and an n-year term insurance policy.

A. Finding the Net Single Premium

Let $A_{x:\,\overline{n}|}$ = the net single premium of an n-year endowment insurance policy of \$1, issued at age x

Then the value of $A_{x:\,\overline{n}|}$ is the sum of the net single premium of a \$1, n-year pure endowment policy and the net single premium of a \$1, n-year term insurance policy, which may be written as follows:

$$A_{x:\,\overline{n}|} = {}_nE_x + A^{1}_{x:\,\overline{n}|} = \frac{D_{x+n}}{D_x} + \frac{M_x - M_{x+n}}{D_x}, \quad \text{or}$$

$$\boldsymbol{A_{x:\,\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}} \qquad \textbf{(12-7)}$$

B. Finding the Net Annual Premium

Let $P_{x:\,\overline{n}|}$ = the net single premium of an n-year endowment insurance policy of \$1, issued at age x

Then the following formula may be obtained:

$$P_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} \qquad (12\text{-}8)^7$$

In Formula (12-8), it is assumed that the number of years in the term of the policy is the same as the number of the annual premium payments. If the term (n) is larger than the number of payments (y), the letter y should replace the letter n in the denominator of the fraction:

$$_yP_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+y}} \qquad (12\text{-}9)^8$$

EXAMPLE 1 A $1,000, 20-year endowment insurance policy was bought by someone aged 40. Find (a) the net single premium, (b) the net annual premium.

$x = 40, n = 20, x + n = 60$

(a) The net single premium is computed by using Formula (20-7), as follows:

$$A_{x:\overline{n}|} = A_{40:\overline{20}|} = \frac{M_{40} - M_{60} + D_{60}}{D_{40}} = \frac{1{,}607{,}743.17 - 1{,}187{,}445.15 + 1{,}749{,}787.72}{3{,}441{,}765.06}$$

$$= \frac{2{,}170{,}085.74}{3{,}441{,}765.06} = \$.630515$$

The net single premium of the 20-year endowment insurance of $1,000 is:

$.630515 \times 1{,}000 = \$630.515, \quad \text{or} \quad \$630.52$

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[7]*Proof—Formula (12-8):*

The n annual premiums ($P_{x:\overline{n}|}$) form a temporary life annuity due. The present value of the annual premiums can be obtained by using Formula (19-11):

$A(\text{tem. due}) = R\ddot{a}_{x:\overline{n}|}$, or

$$A(\text{tem. due}) = P_{x:\overline{n}|} \cdot \ddot{a}_{x:\overline{n}|} = P_{x:\overline{n}|} \cdot \frac{N_x - N_{x+n}}{D_x}$$

The present value must be equal to the net single premium $A_{x:\overline{n}|}$, or

$$A(\text{tem. due}) = A_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x} \quad \text{[see Formula (20-7)]}$$

Thus,

$$P_{x:\overline{n}|} \cdot \frac{N_x - N_{x+n}}{D_x} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}$$

$$P_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} \qquad (12\text{-}8)$$

[8]*Proof—Formula (20-9):*

Here, the y annual premiums ($_yP_{x:\overline{n}|}$) form a temporary life annuity due. Thus,

$$A(\text{tem. due}) = {}_yP_{x:\overline{n}|} \cdot \frac{N_x - N_{x+y}}{D_x} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}$$

$$_yP_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+y}} \qquad (12\text{-}9)$$

(b) Since the number of annual premium payments is not indicated in the problem, the number of years in the term of the policy is assumed to be the same as the number of payments.

Substituting the x and n values in Formula (20-8),

$$\text{Net annual premium} = 1{,}000P_{40:\overline{20|}} = 1{,}000 \cdot \frac{M_{40} - M_{60} + D_{60}}{N_{40} - N_{60}}$$

$$= 1{,}000 \cdot \frac{2{,}170{,}085.74}{75{,}194{,}899.17 - 23{,}056{,}044.97}$$

$$= 1{,}000(.04162) = \$41.62$$

EXAMPLE 2 Refer to Example 1. Assume that the net single premium is payable in 15 equal annual payments. Find the size of the annual premium payment.

$x = 40, n = 20, y = 15, x + n = 60, x + y = 55$

Substituting the values in Formula (20-9),

$$\text{Net annual premium} = 1{,}000\,{}_{15}P_{40:\overline{20|}} = 1{,}000 \cdot \frac{M_{40} - M_{60} + D_{60}}{N_{40} - N_{55}}$$

$$= \frac{1{,}000(2{,}170{,}085.74)}{75{,}194{,}899.17 - 32{,}978{,}578.86} = \$51.40$$

EXERCISE 12-3 REFERENCE: SECTION 12.4

A. For each of the following endowment insurance policies, find (a) the net single premium, (b) the net annual premium:

	Face Value of Policy	*Age of Insured on Purchase Date*	*Term of Policy*	*Payment Plan*
1.	\$ 2,000	30	5 years	5 payments
2.	6,000	25	10 years	10 payments
3.	10,000	35	15 years	15 payments
4.	10,000	45	20 years	20 payments
5.	3,000	22	25 years	25 payments
6.	3,000	32	18 years	18 payments
7.	4,000	50	20 years	15 payments
8.	5,000	20	10 years	8 payments

B. Statement problems:

9. A \$1,000, 30-year endowment insurance policy was purchased by someone aged 28. Find (a) the net single premium, (b) the net annual premium.
10. A \$1,500, 45-year endowment insurance policy was bought by someone aged 20. Find (a) the net single premium, (b) the net annual premium.
11. Refer to Problem 9. If the policy were purchased by a person aged 35, what would be the answers to (a) and (b)?

12. Refer to Problem 10. If the policy were purchased by a person aged 25, what would be the answers to (a) and (b)?
13. Refer to Problem 9. Assuming that the net single premium is payable in 25 equal annual payments, what is the size of the net annual premium?
14. Refer to Problem 10. What should be the size of the net annual premium if the premium of the policy were payable in 30 equal annual payments?

★ 12.5 DEFERRED LIFE INSURANCE

Life insurance may be deferred for a specified period of time. When the insurance is deferred for k years, the insurance protection does not begin at the time the policy is issued, but only after k years have passed. Thus, a 10-year deferred whole life insurance policy issued to someone aged 20 does not provide insurance protection until the age of 30. However, insurance premiums are paid beginning at the age of 20, or on the purchase date. An insurance company seldom sells a deferred insurance policy. More often, the company sells a combined insurance policy that includes a deferred provision.

The net single premium for an insurance policy may be derived by the same reasoning as in the preceding sections. However, the following methods involve less computation.

A. Deferred Whole Life Insurance

Let ${}_{k|}A_x$ = the net single premium for a whole life insurance policy of \$1 deferred k years, issued at age x

Then the value of ${}_{k|}A_x$ *equals* the net single premium for a \$1 whole life insurance policy *minus* the net single premium for a \$1, k-year term insurance policy. Thus,

$${}_{k|}A_x = A_x - A^{1}_{x:\overline{k|}} = \frac{M_x}{D_x} - \frac{M_x - M_{x+k}}{D_x}, \quad \text{or}$$

$$\mathbf{{}_{k|}A_x = \frac{M_{x+k}}{D_x}} \qquad \textbf{(12-10)}$$

B. Deferred Term Insurance

Let ${}_{k|}A^{1}_{x:\overline{n|}}$ = the net single premium for an n-year term insurance policy of \$1 deferred k years, issued at age x

Then the value of ${}_{k|}A^{1}_{x:\overline{n|}}$ *equals* the net single premium for a \$1, $k + n$ term insurance policy *minus* the net single premium for a \$1, k-year term insurance policy. Thus,

$${}_{k|}A^{1}_{x:\overline{n|}} = A^{1}_{x:\overline{k+n|}} - A^{1}_{x:\overline{k|}} = \frac{M_x - M_{x+k+n}}{D_x} - \frac{M_x - M_{x+k}}{D_x}, \quad \text{or}$$

$$\mathbf{{}_{k|}A^{1}_{x:\overline{n|}} = \frac{M_{x+k} - M_{x+k+n}}{D_x}} \qquad \textbf{(12-11)}$$

C. Deferred Endowment Insurance

Let $_{k|}A_{x:\overline{n}|}$ = the net single premium for an n-year endowment insurance policy of \$1 deferred k years, issued at age x

Then the value of $_{k|}A^{1}_{x:\overline{n}|}$ *equals* the net single premium for a $k + n$ year pure endowment policy of \$1 *plus* the net single premium for an n-year term insurance policy of \$1 deferred k years. Thus,

$$_{k|}A_{x:\overline{n}|} = {}_{k+n}E_x + {}_{k|}A^{1}_{x:\overline{n}|} = \frac{D_{x+k+n}}{D_x} + \frac{M_{x+k} - M_{x+k+n}}{D_x}, \quad \text{or}$$

$$_{k|}A_{x:\overline{n}|} = \frac{M_{x+k} - M_{x+k+n} + D_{x+k+n}}{D_x} \qquad \textbf{(12-12)}$$

EXAMPLE 1

A policy issued to someone aged 20 provides the following: (a) \$1,000 if the insured dies in 30 years, (b) \$3,000 if the insured dies after 30 years. Find the net annual premium for life.

The policy actually combines two types of insurance: (a) 30-year term insurance of \$1,000, and (b) whole life insurance of \$3,000 deferred for 30 years.

The net single premium for type (a) is $1{,}000A^{1}_{20:\overline{30}|}$ [Formula (20-4)].

The net single premium for type (b) is $3{,}000 \cdot {}_{30|}A_{20}$ [Formula (20-10)].

Let P = the annual premium of the policy payable for life.

The annual premiums (P) form a whole life annuity due. The present value of the annuity is $P\ddot{a}_x$ or $P\ddot{a}_{20}$ [Formula (19-5)].

The present value of the annuity must be equal to the two net single premiums, or $P\ddot{a}_{20} = 1{,}000A^{1}_{20:\overline{30}|} + 3{,}000 \cdot {}_{30|}A_{20}$

$$P \cdot \frac{N_{20}}{D_{20}} = 1{,}000 \cdot \frac{M_{20} - M_{20+30}}{D_{20}} + 3{,}000 \cdot \frac{M_{20+30}}{D_{20}}$$

$$P = \frac{D_{20}}{N_{20}} \cdot \frac{1{,}000(M_{20} - M_{50}) + 3{,}000M_{50}}{D_{20}} = \frac{1{,}000M_{20} + 2{,}000M_{50}}{N_{20}}$$

$$= \frac{1{,}000(1{,}804{,}922.42) + 2{,}000(1{,}454{,}100.51)}{167{,}827{,}045.88} = \$28.08$$

EXAMPLE 2

A policy, issued to a boy aged 10, promises to pay (a) \$1,000 if he dies before reaching age 25, (b) \$2,000 if he dies after reaching 25 but before 37, and (c) \$5,000 if he dies after reaching 37 but before 65, or if he is alive at 65. Find the net annual premium if the policy is a 20-payment policy.

The policy consists of the following types of insurance benefits: (a) a 15-year term insurance of \$1,000, (b) a 12-year term insurance of \$2,000, deferred 15 years—from age 10 to age 25, and (c) a 28-year endowment insurance of \$5,000, deferred 27 years—from age 10 to age 37.

As of the purchase date:

The net single premium for type (a) is $1{,}000A^{\,1}_{10:\overline{15}|}$ [Formula (12-4)].

The net single premium for type (b) is $2{,}000 \cdot {}_{15|}A^{\,1}_{10:\overline{12}|}$ [Formula (12-11)].

The net single premium for type (c) is $5{,}000 \cdot {}_{27|}A_{10:\overline{28}|}$ [Formula (12-12)].

The 20 annual premiums (P) form a temporary life annuity due. The present value of the annuity is $P\ddot{a}_{x:\overline{n}|}$ or $P\ddot{a}_{10:\overline{20}|}$ [Formula (19-11)].

The present value of the annuity must be equal to the three net single premiums, or

$$P\ddot{a}_{10:\overline{20}|} = 1{,}000A^{\,1}_{10:\overline{15}|} + 2{,}000 \cdot {}_{15|}A^{\,1}_{10:\overline{12}|} + 5{,}000 \cdot {}_{27|}A_{10:\overline{28}|}$$

$$P \cdot \frac{N_{10} - N_{30}}{D_{10}} = 1{,}000 \cdot \frac{M_{10} - M_{25}}{D_{10}} + 2{,}000 \cdot \frac{M_{25} - M_{37}}{D_{10}} + 5{,}000 \cdot \frac{M_{37} - M_{65} + D_{65}}{D_{10}}$$

$$P(N_{10} - N_{30}) = 1{,}000(M_{10} - M_{25}) + 2{,}000(M_{25} - M_{37}) + 5{,}000(M_{37} - M_{65} + D_{65})$$

$$P = \frac{1{,}000M_{10} + 1{,}000M_{25} + 3{,}000M_{37} - 5{,}000M_{65} + 5{,}000D_{65}}{N_{10} - N_{30}}$$

$$= \$86.19$$

★**EXERCISE** 12-4 REFERENCE: SECTION 12.5

1. Find the net single premium for a whole life insurance policy of $5,000 deferred 15 years, issued to someone now aged 25.
2. Find the net single premium for a whole life insurance policy of $3,000 deferred 22 years, issued to someone now aged 18.
3. What is the net single premium for a five-year term insurance policy of $1,000 deferred three years, issued to a person who is 30 years of age?
4. What is the net single premium for a 12-year term insurance policy of $2,000 deferred eight years, issued to a person who is 40 years of age?
5. Find the net single premium for a 20-year endowment insurance policy of $3,000 deferred 10 years, issued to someone who is 20 years of age?
6. What is the net single premium for a 30-year endowment insurance policy of $2,500 deferred 20 years, issued to someone now aged 35?
7. A policy issued to a person aged 25 provides the following: (a) $2,000 if the insured dies within 20 years, (b) $5,000 if the insured dies after 20 years. Find the net annual premium for life.
8. A policy issued to a person aged 30 provides the following: (a) $2,500 if the insured dies within 10 years, (b) $3,500 if the insured dies after 10 years. Find the net annual premium for life.
9. A policy issued to a child at age 8 promises to pay: (a) $1,000 if the insured dies before age 20, (b) $3,000 if death occurs after age 20 but before 25, and (c) $6,000 if death occurs after age 25 but before 50, or if the insured is living at age 50. Find the net annual premium if the policy is a 30-payment policy.
10. A policy issued to a woman aged 22 provides that the insurance company will pay: (a) $2,000 if she dies before age 40, (b) $4,000 if she dies after reaching age 40 but

before 65, and (c) \$1,000 if she dies after reaching age 65 but before 80, or if she is alive at age 80. What is the net annual premium if the policy is a 40-payment policy?

11. Refer to Problem 9. If item (c) reads: "\$6,000 if the insured dies after reaching age 25," what is the net annual premium?
12. Refer to Problem 10. Find the net annual premium if item (c) reads: "\$1,000 if she dies after reaching age 65."

12.6 NATURAL PREMIUM AND LEVEL PREMIUM

A. Computing the Natural Premium

The net single premium of a one-year term insurance policy is called the *natural premium.* According to the 1958 CSO Table, the death rate decreases from birth until age 10 and increases after the age of 10. Thus, if a person, aged 10, purchases a one-year term policy every year, the natural premium would increase from year to year. The natural premium for a particular age may be computed by using the following term insurance formula:

$$A^{1}_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{D_x}.\qquad \text{Here } n = 1. \text{ Thus,}$$

$$A^{1}_{x:\overline{1}|} = \frac{M_x - M_{x+1}}{D_x} = \frac{C_x}{D_x},\qquad \text{or it may be written}$$

$$c_x = \frac{C_x}{D_x} \qquad \textbf{(12-13)}^{9}$$

The values of C_x based on $2\frac{1}{2}\%$ interest are given in Table **12**.

The value of c_x is called the natural premium at age x, or the net single premium for a \$1, one-year term policy issued at age x.

EXAMPLE 1 Find the net single premium for a \$1,000, one-year term policy if the policy is issued at age (a) 20, (b) 21, (c) 22, (d) 23, (e) 24.

Since the policy is a one-year term policy, the natural premium formula is used to compute the value of the net single premium for each age as follows.

(a) $x = 20$

Substituting the value in Formula (20-13), the natural premium for a \$1 policy is:

$$c_x = \frac{C_x}{D_x} = \frac{C_{20}}{D_{20}} = \frac{10{,}300.18}{5{,}898{,}264.97} = .0017463$$

---

[9]*Proof—Formula (12-13):*

$$\begin{aligned} M_x &= C_x + C_{x+1} + C_{x+2} + \cdots + C_{99} \\ M_{x+1} &= \phantom{C_x + {}} C_{x+1} + C_{x+2} + \cdots + C_{99} \\ \hline M_x - M_{x+1} &= C_x \end{aligned}$$

Also see the proof for Formula (12-1) for the value of C_x.

The natural premium (or the net single premium) for a $1,000 policy is:

.0017463 × 1,000 = $1.7463, or $1.75

(b) $x = 21$

$$c_{21} = \frac{C_{21}}{D_{21}} = .0017853$$

Total net single premium = .0017853 × 1,000 = $1.7853, or $1.79

(c) $x = 22$

$$c_{22} = \frac{C_{22}}{D_{22}} = .0018146$$

Total net single premium = .0018146 × 1,000 = $1.8146, or $1.81

(d) $x = 23$

$$c_{23} = \frac{C_{23}}{D_{23}} = .0018439$$

Total net single premium = .0018439 × 1,000 = $1.8439, or $1.84

(e) $x = 24$

$$c_{24} = \frac{C_{24}}{D_{24}} = .0018634$$

Total net single premium = .0018634 × 1,000 = $1.8634, or $1.86

The division C_x/D_x may also be performed by using the values modified by the Committee for 1980 CSO Tables, as shown below. Here we use only $x = 20$ for illustration.

$$\frac{C_{20}}{D_{20}} = \frac{(1 + i)^{100-(x+1)} \cdot d_x}{(1 + i)^{100-x} \cdot l_x} = \frac{\left(1 + 2\frac{1}{2}\%\right)^{79} \cdot d_{20}}{\left(1 + 2\frac{1}{2}\%\right)^{80} \cdot l_{20}}$$

$$= \frac{17{,}300}{\left(1 + 2\frac{1}{2}\%\right)(9{,}664{,}994)} = \frac{17{,}300}{9{,}906{,}618.80} = 0.0017462$$

Also, see footnote 1 of this chapter and footnote 5 in Chapter 19.

B. Checking Natural Premium Against Cost

The natural premium represents the net cost of the death claim. The premium is collected by the insurance company at the beginning of each age and is invested at $2\frac{1}{2}\%$. The total amount at the end of each age should be enough to pay the death claim. The answer to (a) in Example 1 may be checked as follows:

At age 20, there are l_{20} or 9,664,994 persons living. The total premium collected from the group at the beginning of age 20 is:

$1.7463 \times 9{,}664{,}994 = \$16{,}877{,}979.02$

The interest on the total premium for one year at $2\frac{1}{2}\%$ is:

$16{,}877{,}979.02 \times 2\frac{1}{2}\% = \$421{,}949.48$

The total amount at the end of age 20 is:

$16{,}877{,}979.02 + 421{,}949.48 = \$17{,}299{,}928.50$

There are d_{20} or 17,300 persons dying during the period from the beginning of age 20 to the end of age 20. The death claim against the total amount for each death is:

$$\frac{17{,}299{,}928.50}{17{,}300} = \$999.996, \quad \text{or} \quad \$1{,}000 \text{ (policy value)}$$

C. Comparing Natural Premium with Level Premium

The answers in Example 1 indicate that the net cost of the death benefits (net single premium) to the insurance company for a $1,000, one-year term policy is $1.75 if the policyholder is 20 years old, $1.79 if the policyholder is 21 years old, $1.81 if the policyholder is 22 years old, and so on. The natural premium increases from year to year as the policyholder grows older. However, an insurance company usually sells policies having a term of more than one year and collects equal annual premiums for each policy. Since the net annual premium is a fixed amount for every year during the policy term, it is called a *level premium*.

EXAMPLE 2 A $1,000, five-year term policy is issued to a person aged 20. (a) Find the natural premium for each age during the term of the policy. (b) Find the level premium (net annual premium). (c) Construct a chart of the natural premiums and the level premium.

(a) The natural premium for each age is as follows:

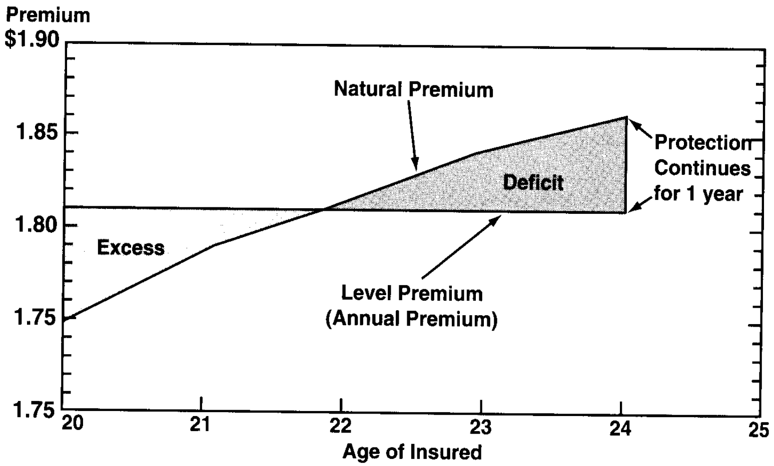
Age 20: $1.7463
Age 21: $1.7853
Age 22: $1.8146
Age 23: $1.8439
Age 24: $1.8634 (see Example 1)

(b) The level premium is:

$$1{,}000P^{1}_{20:\overline{5|}} = 1{,}000 \cdot \frac{M_{20} - M_{25}}{N_{20} - N_{25}} = \$1.8092 \qquad \text{[see Example 1(b), Section 12.3]}$$

I (c) Chart (see below):

Level Premium for a Five-Year Term Policy Issued at Age 20 and Natural Premiums for Ages 20 through 24 (Policy Face Value: $1,000)



The answers in Example 2 are also tabulated in the following schedule:

Comparison of Level Premium and Natural Premium

(1) *Age*	*(2)* *Natural Premium*	*(3)* *Excess (or Deficit) of Level Premium over Natural Premium: $1.8092 − (2)*
20	$1.7463	$ 0.0629
21	1.7853	0.0239
22	1.8146	−0.0054
23	1.8439	−0.0347
24	1.8634	−0.0542

In Example 2, during the first year the insurance company collects $.06 (or $1.8092 − $1.7463) more than the net cost of the insurance. [See Column (3) in the above schedule.] The company usually deposits the excess premium, $.06, in a *reserve fund,* which is invested to earn interest to meet future needs. (See Section 12.7.) On the other hand, when the policy holder reaches age 24, the insurance company collects $.05 (or $1.8092 − $1.8634) less than the net cost to the company. The company usually makes up the deficit premium from the reserve fund.

In general, for every insurance policy, the level premium is greater than the natural premium (the net cost to the company) in the earlier policy years. On the other hand, in the later policy years the level premium is less than the natural premium. If the policyholder wishes to cancel the contract before the entire amount of the reserve fund has been used, it is possible to recover a *cash surrender value,* which is paid by the life insurance company when the policyholder surrenders the policy to the company.

The following example illustrates the relationship between the natural premiums and the level premium for a straight whole life policy.

EXAMPLE 3

Find the natural premiums for a $1,000 policy if the policy is issued at the ages indicated in Column (1) of the schedule below. Also find the net annual premium (the level premium) for a $1,000 straight whole life policy issued to a person aged 50. Compare the natural premiums with the level premium and show the excess or deficit premiums.

The natural premiums for the required ages are computed by using Formula (12-13) and are arranged, respectively, in Column (2) of the following schedule.

Level Premium for a $1,000, Straight Whole Life Policy Issued at Age 50 and Natural Premiums for Various Ages

(1) Age (x)	*(2) Natural Premium:* $1{,}000 \cdot \frac{C_x}{D_x}$	*(3) Excess (or Deficit) of Level Premium over Natural Premium: \$32.38 − (2)*
50	$ 8.12	$ 24.26
55	12.68	19.70
60	19.84	12.54
65	30.98	1.40
70	48.58	− 16.20
75	71.58	− 39.20
80	107.30	− 74.92
85	157.21	−124.83
90	222.58	−190.20
95	342.67	−310.29
99	975.61	−943.23

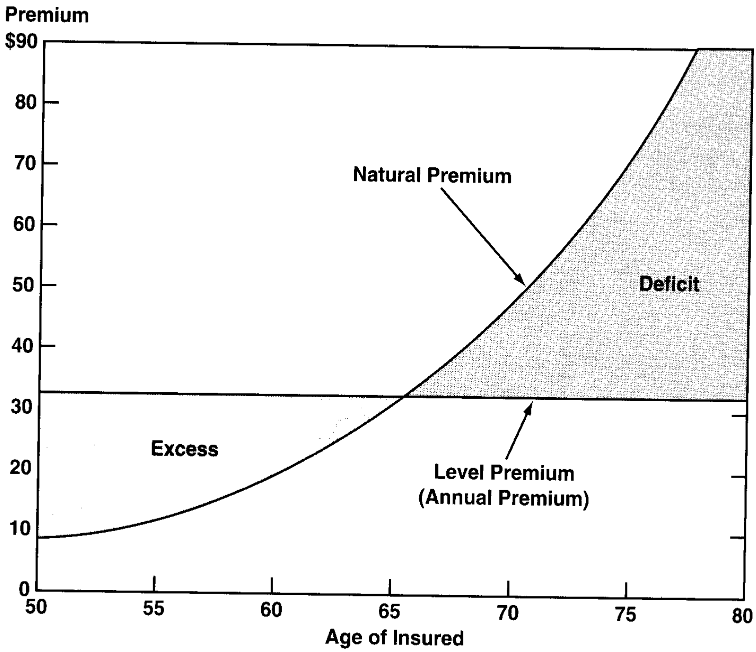
The net annual premium (the level premium) is:

$$1{,}000P_{50} = 1{,}000 \cdot \frac{M_{50}}{N_{50}} = \$32.38$$

The comparison between the natural premiums and the level premium is shown in Column (3) of the above schedule.

Example 3 is diagrammed in the following chart, which covers a portion of the information presented in the above schedule.

Level Premium for a $1,000 Straight Whole Life Policy for a Person Aged 50 and Natural Premiums for Various Ages



EXERCISE 12-5 REFERENCE: SECTION 20.6

1. Find the net single premium for a $1,000, one-year term policy, if the policy is issued at age (a) 8, (b) 9, (c) 10, (d) 11, (e) 12.
2. Find the net single premium for a $2,000, one-year term policy, if the policy is issued at age (a) 30, (b) 31, (c) 32, (d) 33, (e) 34.
3. Refer to the checking method described on pages 640–641. Use this method to check whether the natural premium obtained in Problem 1(a) is sufficient to pay the net cost of the death claim.
4. Refer to the checking method described on pages 640–641. Use this method to check whether the natural premium obtained in Problem 2(a) is sufficient to pay the net cost of the death claim.
5. A $1,000, five-year term policy is issued to someone aged 25. Construct a chart of the natural premiums and the level premium for the policy.
6. A $1,000, five-year term policy is issued to someone aged 40. Construct a chart of the natural premiums and the level premium for the policy.