★ 12.7 TERMINAL RESERVE

A. General Computation

Terminal reserve is the value accumulated from the excess premiums and the interest on the premiums. It represents the value of an insurance policy at the end of any policy year; the premium then due is not included. For example, the fourth terminal reserve for an insurance policy is the value of the policy at the end of the fourth year after the policy is issued, but it does not include the net annual premium for the fifth year. The value of the terminal reserve at the end of a particular policy year may be obtained in various ways. Two methods are illustrated below.

I. RETROSPECTIVE METHOD

The *retrospective method* is based on the premiums collected in the *past* and the death benefits paid in the *past*. Premiums are collected by the insurance company at the *beginning* of each policy year and are invested at a given interest rate for accumulation. The death benefits are paid from this accumulated sum. It is assumed here, as in the previous discussion, that the death benefits are paid at the *end* of the year of death. The remaining part (premiums and interest less death benefit payments) is the terminal reserve, which is divided by the number of surviving policyholders in the 1958 CSO Table to determine the terminal reserve per policyholder for the year.

EXAMPLE 1

The CSO Table shows that l_{20} = 9,664,994. Assume that each of the 9,664,994 persons aged 20 is issued a $1,000, five-year term insurance policy and the annual premiums are invested at $2\frac{1}{2}$%. Find the terminal reserve per surviving policyholder at the end of each policy year.

The terminal reserves are computed in the following table:

(1) Policy Year	(2) Surviving Policy-holders at Beginning of Year	(3) Premiums Paid at Beginning of Year: (2) × 1.8091584*	(4) Reserve Fund at Beginning of Year: (3) + (7)**	(5) One Year's Interest for the Reserve Fund: (4) × $2\frac{1}{2}$%	(6) Death Claims at End of Year: $1,000 Each (of d_x)	(7) Reserve Fund at End of Year: (4) + (5) − (6)	(8) Terminal Reserve per Survivor: (7) ÷ (2)***
1	9,664,994	$17,485,505	$17,485,505	$437,138	$17,300,000	$622,643	$.064538
2	9,647,694	17,454,207	18,076,850	451,921	17,655,000	873,771	.090734
3	9,630,039	17,422,266	18,296,037	457,401	17,912,000	841,438	.087539
4	9,612,127	17,389,860	18,231,298	455,782	18,167,000	520,080	.054209
5	9,593,960	17,356,993	17,877,073	446,927	18,324,000	—0–	—0—

*Net annual premium. See Section 20.6, Example 2(b).
**Of the preceding year. For example, $18,076,850 = 17,454,207 + 622,643.
***Of the succeeding year. For example, $.064538 = 622,643 ÷ 9,647,694.

The preceding method may be used to compute the terminal reserves for any type of insurance. As shown below, the procedure used in this method may also be expressed symbolically to obtain a formula. The formula can then be used to find the terminal reserve per surviving policyholder at the end of any given policy year without constructing a table.

Let P = the net annual premium (or level premium) for a policy of \$1
t = the number of years the policy is in force: the terminal reserve is desired at the end of the tth policy year
V = tth terminal reserve per surviving policyholder for a \$1 policy
V' = $V \times$ Policy value = the total tth terminal reserve per surviving policyholder

If the tth terminal reserve is on a date that is before or on the date of the last payment of the n annual premiums, the formula for the value V is as follows:

$$V = \frac{P(N_x - N_{x+t}) - (M_x - M_{x+t})}{D_{x+t}}, \quad \text{where} \quad t \leq n \qquad \textbf{(12-14)}^{10}$$

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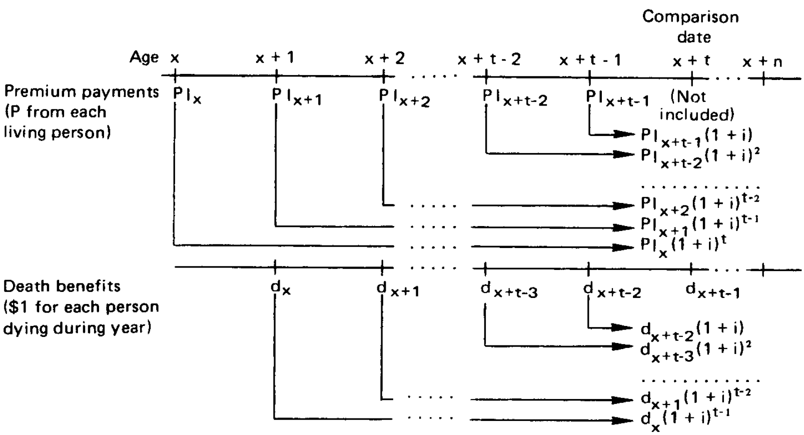
[10]*Proof—Formula (12-14):*

The number of years that the policy has been in force (t) is smaller than or equal to the number of annual premium payments (n) (or, $t \leq n$).

Let age $x + t$ be the comparison date and i be the interest rate compounded annually. The accumulated values of the *past* annual premiums and the *past* death benefits at age $x + t$ are diagrammed, respectively, as follows:



At the end of the tth policy year, the comparison date, the total accumulated annual premiums (not considering the payments of the death benefits) are:

$$Pl_x(1 + i)^t + Pl_{x+1}(1 + i)^{t-1} + \cdots + Pl_{x+t-1}(1 + i) \qquad \text{(I)}$$

The total accumulated annual death benefits (if they had been withheld and invested by the insurance company) at the end of the tth policy year are:

$$d_x(1 + i)^{t-1} + d_{x+1}(1 + i)^{t-2} + \cdots + d_{x+t-1} \qquad \text{(II)}$$

The total accumulated annual death benefits are subtracted from the total accumulated annual premiums to obtain the total terminal reserve at the end of the tth policy year. The total terminal reserve is divided by the l_{x+t}

(continued)

If the *t*th terminal reserve is on a date that is after the last payment of the *n* annual premiums [this may occur in a limited payment life insurance policy as in Example 5(b) below], the formula for the value of *V* is as follows:

$$V = \frac{P(N_x - N_{x+n}) - (M_x - M_{x+t})}{D_{x+t}}, \quad \text{where} \quad t > n \qquad \textbf{(12-15)}^{11}$$

---

[10] *Proof—(continued):*

policyholders who are living at age $x + t$. The result is the terminal reserve per surviving policyholder and is as follows:

$$V = \frac{(\mathrm{I}) - (\mathrm{II})}{l_{x+t}}, \quad \text{or}$$

$$V = \frac{\{P[l_x(1+i)^t + l_{x+1}(1+i)^{t-1} + \cdots + l_{x+t-1}(1+i)] - [d_x(1+i)^{t-1} + d_{x+1}(1+i)^{t-2} + \cdots + d_{x+t-1}]\}}{l_{x+t}}$$

Substitute v for $(1 + i)^{-1}$, or $v^{-1} = (1 + i)$.

$$V = \frac{\{P[l_x v^{-t} + l_{x+1}v^{-(t-1)} + \cdots + l_{x+t-1}v^{-1}] - [d_x v^{-(t-1)} + d_{x+1}v^{-(t-2)} + \cdots + d_{x+t-1}]\}}{l_{x+t}}$$

Multiply both the numerator and the denominator of the fraction by v^{x+t}.

$$V = \frac{\{P(v^x l_x + v^{x+1}l_{x+1} + \cdots + v^{x+t-1}l_{x+t-1}) - (v^{x+1}d_x + v^{x+2}d_{x+1} + \cdots + v^{x+t}d_{x+t-1})\}}{v^{x+t}l_{x+t}}$$

Substitute the appropriate commutation symbols.

$$V = \frac{P(D_x + D_{x+1} + \cdots + D_{x+t-1}) - (C_x + C_{x+1} + \cdots + C_{x+t-1})}{D_{x+t}}$$

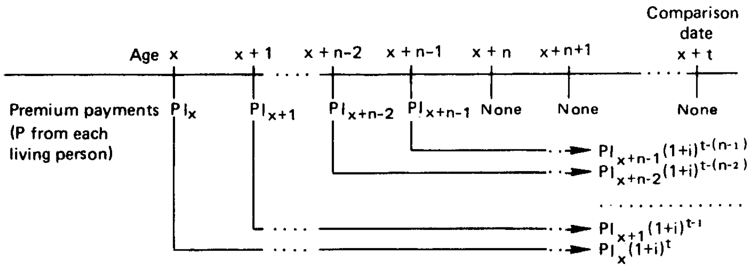
$$= \frac{P(N_x - N_{x+t}) - (M_x - M_{x+t})}{D_{x+t}}, \quad \text{where} \quad t \leq n \qquad (12\text{-}14)$$

[11] *Proof—Formula (20-15):*

The number of years that the policy has been in force (t) is larger than the number of annual premium payments (n), (or $t > n$).

If the *t*th terminal reserve is on a date after the last payment of the *n* annual premiums is made, the total accumulated annual premiums at the end of the *t*th policy year (the comparison date) would be:

$$Pl_x(1+i)^t + Pl_{x+1}(1+i)^{t-1} + \cdots + Pl_{x+n-2}(1+i)^{t-(n-2)} + Pl_{x+n-1}(1+i)^{t-(n-1)}$$
$$= P(l_x v^{-t} + l_{x+1}v^{-(t-1)} + \cdots + l_{x+n-2}v^{-[t-(n-2)]} + l_{x+n-1}v^{-[t-(n-1)]})$$
$$= P(l_x v^{-t} + l_{x+1}v^{-t+1} + \cdots + l_{x+n-2}v^{-t+n-2} + l_{x+n-1}v^{-t+n-1}) \qquad (\mathrm{IA})$$



(continued)

EXAMPLE 2 What is the third terminal reserve for a \$1,000, five-year term policy issued to a person aged 20?

$P = .0018091584$ [for a 1\$ policy, see Section 12.6, Example 2(b)], $x = 20$, $n = 5$, $t = 3$, $x + t = 23$

Since t is smaller (<) than n, the above values are substituted in Formula (20-14) as follows:

$$V = \frac{.0018091584(N_{20} - N_{23}) - (M_{20} - M_{23})}{D_{23}} = .000087539$$

The third terminal reserve for a \$1,000 policy is:

$V' = 1{,}000V = 1{,}000(.000087539) = \$.087539$

The answer may be compared with that given in column (8) of the table in Example 1 of this section.

EXAMPLE 3 Find the seventh terminal reserve for a 10-year endowment policy of \$1,000 issued to a man aged 20.

$x = 20$, $n = 10$, $t = 7$, $x + t = 27$, t is smaller than n

The net annual premium of a 10-year endowment policy of \$1 issued to a man aged 20 is:

$$P = P_{20:\overline{10|}} = \frac{M_{20} - M_{30} + D_{30}}{N_{20} - N_{30}} = .087980555 \qquad \text{[Formula (12-8)]}$$

Substituting the above values in Formula (20-14),

$$V = \frac{.087980555(N_{20} - N_{27}) - (M_{20} - M_{27})}{D_{27}} = .67169$$

The seventh terminal reserve for the \$1,000 policy is:

$V' = 1{,}000V = 1{,}000(.67169) = \$671.69$

EXAMPLE 4 Find the tenth terminal reserve for a \$1,000 straight whole life policy issued to a person aged 25.

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[11]*Proof—(continued):*

The total accumulated annual death benefits are the same as shown in the diagram for equation (II) in the proof of Formula (12-14). Thus, the terminal reserve per surviving policyholder is as follows:

$$V = \frac{(IA) - (II)}{l_{x+t}}$$

Multiply both the numerator and the denominator of the fraction by v^{x+t}, and substitute the appropriate commutation symbols, as was done in the proof of Formula (12-14). Then

$$V = \frac{P(N_x - N_{x+n}) - (M_x - M_{x+t})}{D_{x+t}}, \quad \text{where} \quad t > n \qquad (12\text{-}15)$$

$P = .0125450145$ [for a $1 policy, see Example 3(b), Section 20.2], $x = 25$, $t = 10$, $x + t = 35$; t is smaller than the number of payments (n), since the premiums are payable until the death of the insured.

Substituting the values in Formula (12-14),

$$V = \frac{.0125450145(N_{25} - N_{35}) - (M_{25} - M_{35})}{D_{35}} = .12187$$

The tenth terminal reserve for the $1,000 policy is:

$V' = 1{,}000V = 1{,}000(.12187) = \$121.87$

EXAMPLE 5 A $1,000 whole life policy is issued to a person aged 25. If the policy is a 20-payment life policy, find the (a) 15th and (b) 30th terminal reserves.

The net annual premium for a policy of $1 is $.02168152 (see Section 12.2, Example 3(c)], or $P = .02168152$

(a) $x = 25$; $n = 20$; $t = 15$, which is smaller than the value of n;
$x + t = 25 + 15 = 40$

The 15th terminal reserve is computed by using Formula (12-14) as follows:

$$V = \frac{P(N_x - N_{x+t}) - (M_x - M_{x+t})}{D_{x+t}}$$

$$= \frac{.02168152(N_{25} - N_{40}) - (M_{25} - M_{40})}{D_{40}} = .36465$$

The terminal reserve for a $1,000 policy is:

$V' = 1{,}000V = 1{,}000(.36465) = \$364.65$

(b) $x = 25$; $n = 20$; $t = 30$, which is greater (>) than the value of n;
$x + n = 25 + 20 = 45$; $x + t = 25 + 30 = 55$

Substituting the values in Formula (12-15),

$$V = \frac{P(N_x - N_{x+n}) - (M_x - M_{x+t})}{D_{x+t}}$$

$$= \frac{.02168152(N_{25} - N_{45}) - (M_{25} - M_{55})}{D_{55}} = .62455$$

The terminal reserve for a $1,000 policy is:

$V' = 1{,}000V = 1{,}000(.62455) = \$624.55$

★**EXERCISE 12-6** REFERENCE: SECTION 20.7A

1. A 15-year term insurance policy of $1,000 was issued to a person aged 25. Find (a) the 5th and (b) the 15th terminal reserves.

2. What are (a) the 8th and (b) the 12th terminal reserves for a 20-year term insurance policy of \$2,000 issued to a person aged 30?
3. Find (a) the 6th and (b) the 25th terminal reserve for a 25-year endowment policy of \$1,000 issued to someone aged 22.
4. A 15-year endowment policy of \$2,000 was issued to someone aged 35. Find (a) the 4th and (b) the 10th terminal reserves.
5. What are (a) the 12th and (b) the 50th terminal reserves for a \$1,000 straight life policy issued to a person aged 30?
6. A \$1,000 straight life policy was issued to a person aged 20. Find (a) the 15th and (b) the 65th terminal reserves.
7. A \$1,000 whole life policy is issued to someone aged 26. If the policy is a 25-payment life policy, find (a) the 10th and (b) the 34th terminal reserves.
8. A \$1,000 whole life policy is issued to someone aged 30. If the policy is a 15-payment life policy, find (a) the 5th and (b) the 25th terminal reserves.

II. PROSPECTIVE METHOD

The *prospective method* is based on the annual premiums to be collected in the *future* and the annual death benefits to be paid in the *future.*

Let age $x + t$ be the comparison date. Values at age $x + t$ may be written as follows:

$$\begin{bmatrix}\textbf{\textit{t}th terminal}\\ \textbf{reserve } (V')\end{bmatrix} = \begin{bmatrix}\textbf{Present value}\\ \textbf{(at age } x + t\textbf{) of}\\ \textbf{future death}\\ \textbf{benefits}\end{bmatrix} - \begin{bmatrix}\textbf{Present value}\\ \textbf{(at age } x + t\textbf{) of}\\ \textbf{future net annual}\\ \textbf{premiums}\end{bmatrix} \qquad \textbf{(12-16)}$$

At age $x + t$, the present value of the future death benefits is the net single premium for the benefits. The present value (at age $x + t$) of the future net annual premiums is the present value of a life annuity due formed by the future annual premiums.

EXAMPLE 6

Find the third terminal reserve for a \$1,000, five-year term policy issued to a person aged 20.

$x = 20, t = 3, x + t = 20 + 3 = 23$. The future or remaining term from age $x + t$, or 23, is $(5 - 3) = 2$ (years).

At age 23, the future death benefits of the policy are equal to a two-year term insurance policy of \$1,000. The present value, or the net single premium, of the two-year policy is:

$1{,}000A^{1}_{23:\overline{2}|}$

The net annual premium of the five-year term policy is computed as follows:

$1{,}000P^{1}_{20:\overline{5}|} = 1.8091584$ [see Section 12.6, Example 2(b)]

At age 23, the two future annual premium payments of \$1.8091584 per year form a two-year life annuity due. The present value of the annuity is:

$A(\text{tem. due}) = 1.8091584\ddot{a}_{23:\overline{2}|}$

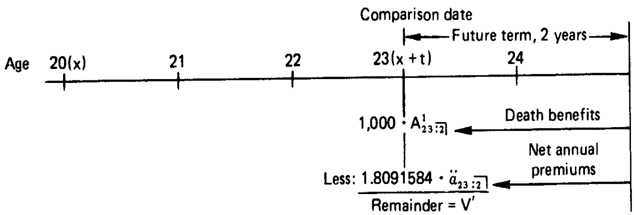
Thus,

$$V' = 1{,}000A^{1}_{23:\overline{2}|} - 1.8091584\ddot{a}_{23:\overline{2}|}$$

$$= 1{,}000 \cdot \frac{M_{23} - M_{23+2}}{D_{23}} - 1.8091584 \cdot \frac{N_{23} - N_{23+2}}{D_{23}}$$

$$= \$.087539$$

The above answer may be compared with that given in Section 12.7, Example 2. Example 6 is diagrammed as follows:



EXAMPLE 7 Find the seventh terminal reserve for a 10-year endowment policy of $1,000 issued to a person aged 20.

$x = 20$, $t = 7$, $x + t = 20 + 7 = 27$. The future (remaining term) $= 10 - 7 = 3$ (years).

The future death benefits of the policy at age 27 are equivalent to a three-year endowment insurance policy of $1,000 for a person aged 27. The present value, or net single premium, of the three-year policy is:

$1{,}000A_{27:\overline{3}|}$

The net annual premium of the 10-year endowment policy is:

$1{,}000P_{20:\overline{10}|} = 1{,}000(.087980555) = \$87.980555$ (See Section 12.7, Example 3.)

The future net annual premiums at age 27 form a three-year temporary life annuity due of $87.980555 per year. The present value of the annuity at age 27 is:

$87.980555\ddot{a}_{27:\overline{3}|}$

Thus,

$$V' = 1{,}000A_{27:\overline{3}|} - 87.980555\ddot{a}_{27:\overline{3}|}$$

$$= 1{,}000 \cdot \frac{M_{27} - M_{27+3} + D_{27+3}}{D_{27}} - 87.980555 \cdot \frac{N_{27} - N_{30}}{D_{27}}$$

$$= \$671.69$$

The above answer may be compared with that given in Section 12.7, Example 3.

EXAMPLE 8 Find the tenth terminal reserve for a \$1,000 straight whole life policy issued to a person aged 25.

$x = 25$; $t = 10$; $x + t = 35$; the future (remaining term) = age 35 to 99 (the end of the CSO Table)

The future death benefits of the policy at age 35 are equivalent to a whole life policy of \$1,000 for a person aged 35. The net single premium (present value) of the new whole life policy is

$1{,}000A_{35}$

The net annual premium of the original whole life policy is:

$1{,}000P_{25} = \$12.545$ [See Example 3(b), Section 12.2.]

The future net annual premiums at age 35 form a whole life annuity due of \$12.545 per year. The present value of this annuity is:

$12.545\ddot{a}_{35}$

Thus,

$$V' = 1{,}000A_{35} - 12.545\ddot{a}_{35} = 1{,}000 \cdot \frac{M_{35}}{D_{35}} - 12.545\frac{N_{35}}{D_{35}}$$

$$= \$121.87$$

The above answer may be compared with that given in Example 4.

EXAMPLE 9 Find the 30th terminal reserve for a \$1,000, 20-payment life policy issued to a person aged 25.

$x = 25$; $t = 30$; $x + t = 55$; the future (remaining term of the life policy) = age 55 to 99 (the end of the CSO Table)

The future death benefits of the policy at age 55 are equivalent to a whole life policy of \$1,000 for age 55. The net single premium of the policy for a person aged 55 is:

$$1{,}000A_{55} = 1{,}000 \cdot \frac{M_{55}}{D_{55}} = \$624.55$$

There are no future net annual premiums at age 55, since the original policy is paid up in 20 annual premium payments. Thus,

$$V' = 624.55 - 0 = \$624.55$$

The above answer may be compared with that given in Example 5(b).

★**EXERCISE 12-7** REFERENCE: SECTION 12.7A

Use the prospective method to compute the following problems:

1. A life insurance policy was issued to a person aged 30. The value of future death benefits at age 40 is \$467.13. Find the tenth terminal reserve by assuming that the net annual premium is \$14.80 payable for life.

2. A life insurance policy was issued to a person aged 20. The value of future death benefits at the end of the 18th policy year is $895.72. If the net annual premium is $21.51 payable for life, what is the 18th terminal reserve?
3. Compute Problems 1, 3, 5, and 7 in Exercise 12-6.
4. Compute Problems 2, 4, 6, and 8 in Exercise 12-6.

B. Uses of the Terminal Reserve

The value of an insurance policy in any policy year is based on the terminal reserve. A policyholder may use the reserve for any one of the following purposes:

1. The policyholder may surrender the policy to the insurance company and receive the cash value of the reserve.
2. A policyholder who has no intention of surrendering the policy may borrow money on the reserve at a low interest rate from the insurance company, the reserve being regarded as security for the loan.
3. The policyholder may also use the reserve as a single premium payment to extend the original policy for a period of time, or to convert the original policy to a reduced amount of paid-up insurance. In fact, if a person stops paying the annual premium and fails to do anything about the policy, the company usually continues to keep the policy in force for an extended period of time based on the amount of the terminal reserve.

EXAMPLE 10 The annual premium payment on a $1,000 straight whole life policy issued to a person aged 25 is discontinued at age 35. The policyholder wishes to use the reserve as the premium payment. Find (a) the extended term of the policy with the original face value ($1,000), and (b) the new policy face value for the same type of policy (whole life policy).

First, find the tenth terminal reserve of the original policy. In Example 4 of this section, the tenth terminal reserve for a $1,000 straight life policy issued to a person aged 25 is $121.87.

(a) The net single premium on a $1,000, n-year policy issued at age 35 is now equal to the tenth terminal reserve, or

$$1{,}000A^{\,1}_{35:\overline{n}|} = 121.87$$

$$A^{\,1}_{35:\overline{n}|} = \frac{121.87}{1{,}000} = .12187$$

Thus,

$$A^{\,1}_{35:\overline{n}|} = \frac{M_{35} - M_{35+n}}{D_{35}} = .12187$$

$$\frac{1{,}659{,}440.36 - M_{35+n}}{3{,}949{,}851.09} = .12187$$

$$M_{35+n} = 1{,}178{,}072.01$$

The following nearest values are obtained from Table 12:

$$M_{60} = 1{,}187{,}445.01$$
$$M_{35+n} = 1{,}178{,}072.01$$
$$M_{61} = 1{,}152{,}722.42$$

When the interpolation method is used, the value of 1,178,072.01 is found to be $M_{60.26994}$, as follows:

	x(age)	M_x	
	60	1,187,445.15	(1)
	35 + n	1,178,072.01	(2)
	61	1,152,722.42	(3)
(2) − (3)	(35 + n) − 61 =	25,349.59	(4)
(1) − (3)	−1	34,722.73	(5)

Solve for n from the proportion formed by the differences on lines (4) and (5).

$$(35 + n) - 61 = (-1) \cdot \frac{25{,}349.59}{34{,}722.73}$$

$$35 + n = -.73006 + 61 = 60.26994$$

$$n = 60.26994 - 35$$

$$= 25.26994, \quad \text{or 25 years and 99 days}$$
$$[.26994 \times 365 \text{ (days a year)} = 99 \text{ days}]$$

(b) The net single premium of a $1 whole life policy at age 35 is:

$$A_{35} = \frac{M_{35}}{D_{35}} = \$.420127$$

The face value of the new whole life policy at age 35 is:

$$\frac{121.87}{.420127} = \$290.08$$

★EXERCISE 12-8 REFERENCE: SECTION 20.7B

1. A 15-year term insurance policy of $1,000 was issued to a person at age 25. The policyholder stopped making annual premium payments at age 30. (a) What is the term of the new policy if the policyholder wishes to use the reserve to buy a term insurance policy with the same face value? (b)What is the amount of the new policy if the policyholder wishes to use the reserve to buy a 10-year term insurance policy? (Also see Problem 1, Exercise 12-6.)
2. Refer to Problem 1. Assume that the policyholder stopped making annual premium payments at age 32. What is the answer to (a)? What is the answer to (b) if the policyholder now wishes to use the reserve to buy a five-year term insurance policy?
3. A person now aged 28 is unable to maintain the annual premium payments on a $1,000, 25-year endowment insurance policy, which was issued at age 22. If the policyholder wishes to use the reserve as the premium payment, find (a) the term of the new policy for a term insurance policy with a face value of $1,000, and (b) the face value of the new policy for a 15-year endowment insurance policy. (Also see Problem 3, Exercise 12-6.)

4. Refer to Problem 3. If the policyholder is now 35 and is unable to maintain the annual premium payments, what is the answer to (a)? What is the answer to (b) for a 10-year endowment insurance policy?
5. A straight life policy of $1,000 issued to someone aged 30 is discontinued at age 42. (a) Find the term of the new policy if the policyholder wishes to use the reserve to purchase an extended insurance policy of the same face value. (b) Find the amount of the whole new policy if the policyholder wishes to keep the same type of policy, the whole life policy. (Also see Problem 5, Exercise 12-6.)
6. Refer to Problem 5. Find the answers to (a) and (b) if the policy is discontinued at age 50.

★ 12.8 COMPUTING COMMUTATION FUNCTIONS BY COMPUTERS

As stated before a properly written computer program can extend its applications to any mortality table at any interest rate. Example 1 in that section illustrated the computation of commutation functions D_x and N_x by computers. Here we shall illustrate the computation of C_x and M_x.

EXAMPLE 1 A $1 whole life policy is issued to a man aged 95. Find the net single premium. Use (a) the 1958 CSO Table at $2\frac{1}{2}$% interest, and (b) the 1980 CSO Table at 4% interest.

SOLUTION The formula for finding the net single premium of a $1 whole life insurance policy, issued at age x is [Formula (20-1)]:

$$A_x = \frac{M_x}{D_x} = \frac{M_{95}}{D_{95}} = \frac{C_{95} + C_{96} + C_{97} + C_{98} + C_{99}}{D_{95}}$$

$$C_x = (1 + i)^{100-(x+1)} \cdot d_x \qquad \text{(footnote 1, Chapter 12)}$$

$$D_x = (1 + i)^{100-x} \cdot l_x$$

We shall use the spreadsheet to compute.

COMPUTER USING SPREADSHEET

(a) Based on the 1958 CSO Table (Table 12) at $2\frac{1}{2}$%

I. Entering Data and Formulas

Spreadsheet Display (Find the net single premium)

C17: +C15/C16

	A	B	C
1	i		0.025
2	x	$100-(x+1)$	d_x
3	95	4	34128
4	96	3	25250
5	97	2	18456

6	98	1	12916
7	99	0	6415
8	(100−x), L_{95}	5	97165
9	- - - - - - - -		
10	C95		+C3*(1+C1)∧B3
11	C96		+C4*(1+C1)∧B4
12	C97		+C5*(1+C1)∧B5
13	C98		+C6*(1+C1)∧B6
14	C99		+C7*(1+C1)∧B7
15	M95		@SUM(C10...C14)
16	D95		+C8*(1+C1)∧B8
17	Ax		+C15/C16

II. Finding the Answer

Print the spreadsheet display: $A_x = A_{95} = 0.945179$

i		0.025
x	100−(x+1)	d_x
95	4	34128
96	3	25250
97	2	18456
98	1	12916
99	0	6415
(100−x), L_{95}	5	97165
C95		37670.93
C96		27191.49
C97		19390.34
C98		13238.9
C99		6415
M95		103906.6
D95		109933.3
Ax =		0.945179

Note: When the definition $C_x = v^{x+1}d_x$ is used in the computer program, the C_x values will be different from the above answers. For example, $C_{95} = (1 + 2\frac{1}{2}\%)^{-96}d_{95} = 0.09343486$ (34128) = 3188.7449 (Table 13), not 37670.93 as above.

(b)1 Based on the 1980 CSO Table—Male (Table 14) at 4%

I. Entering Data and Formulas

Spreadsheet Display (Find the net single premium)

C17: +C15/C16

	A	B	C	
				Note: The d_x values are computed from Table 14:
1	*i*		0.04	
2	*x*	100−(x+1)	d_x	
3	95	4	900	= 2728 − 1828
4	96	3	703	= 1828 − 1125

5	97	2	540	= 1125 − 585
6	98	1	385	= 585 − 200
7	99	0	200	= 200
8	(100−x), L_{95}	5	2728	
9				

The entries in Columns A and C in rows 10 to 17 are identical to those in (*a*)I.

II. Finding the Answer

Print the spreadsheet display: $A_x = A_{95} = 0.91235$

i		0.04
x	100−(x+1)	d_x
95	4	900
96	3	703
97	2	540
98	1	385
99	0	200
(100−x), L_{95}	5	2728
C95		1052.873
C96		790.7794
C97		584.064
C98		400.4
C99		200
M95		3028.116
D95		3319.029
A_x =		0.91235

(b)2 Based on the 1980 CSO Table—Female (Table 14) at 4%

I. Entering Data and Formulas

Spreadsheet Display (Find the net single premium)

C17: +C15/C16

	A	B	C	Note: The d_x values are computed from Table 14:
1	*i*	100	0.04	
2	*x*	100−(x+1)	d_x	
3	95	4	824	= 2597 − 1773
4	96	3	666	= 1773 − 1107
5	97	2	526	= 1107 − 581
6	98	1	381	= 581 − 200
7	99	0	200	= 200
8	(100−x), L_{95}	5	2597	
9				

The entries in Columns A and C in rows 10 to 17 are identical to those in (*a*)I.

II. Finding the Answer

Print the spreadsheet display: $A_x = A_{95} = 0.910951$

i		0.04
$x\cdot$	$100-(x+1)$	d_x
95	4	824
96	3	666
97	2	526
98	1	381
99	0	200
$(100-x)$, L_{95}	5	2597
C95		963.9635
C96		749.1594
C97		568.9216
C98		396.24
C99		200
M95		2878.284
D95		3159.648
$A_x =$		0.910951

12.9 SUMMARY OF LIFE INSURANCE FORMULAS

Symbols: A = the net single premium (or present value) of a \$1 life insurance policy (see note for symbol A on page 626)

P = the net annual premium of a \$1 life insurance policy

V = terminal reserve for tth policy year of a \$1 life insurance policy

V' = total terminal reserve for tth policy year of a life insurance policy

x = age of the insured on the date the policy is issued

n = number of net annual premium payments, also number of years in the length of a term or an endowment insurance policy

y = number of net annual premium payments for a term insurance or an endowment insurance policy; y is used only when the number of premium payments (y) does not equal the number of years in the length of a policy (n), or $y \neq n$

k = number of years that a life policy is deferred

$D_x = v^x l_x$

$N_x = D_x + D_{x+1} + D_{x+2} + \cdots + D_{99}$

$C_x = v^{x+1} d_x$

$M_x = C_x + C_{x+1} + C_{x+2} + \cdots + C_{99}$

Note: As modified by the Society of Actuaries in 1983 for the use of 1980 CSO Table:

$D_x = (1 + i)^{100-x} \cdot l_x$

$C_x = (1 + i)^{100-(x+1)} \cdot d_x$

Also, see footnote 1 of this chapter.

Application	*Formula*	*Formula Number*	*Reference Page*
Whole Life Insurance			
Net Single Premium	$A_x = \frac{M_x}{D_x}$	(12-1)	
Net Annual Premium	$P_x = \frac{M_x}{N_x}$	(12-2)	
	${}_nP_x = \frac{M_x}{N_x - N_{x+n}}$	(12-3)	
Term Insurance			
Net Single Premium	$A^{1}_{x:\overline{n}\rceil} = \frac{M_x - M_{x+n}}{D_x}$	(12-4)	
Net Annual Premium	$P^{1}_{x:\overline{n}\rceil} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}}$	(12-5)	
	${}_yP^{1}_{x:\overline{n}\rceil} = \frac{M_x - M_{x+n}}{N_x - N_{x+y}}$	(12-6)	
Endowment Insurance			
Net Single Premium	$A_{x:\overline{n}\rceil} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}$	(12-7)	
Net Annual Premium	$P_{x:\overline{n}\rceil} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}}$	(12-8)	
	${}_yP_{x:\overline{n}\rceil} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+y}}$	(12-9)	
Deferred Life Insurance			
Deferred Whole Life Insurance	${}_{k\mid}A_x = \frac{M_{x+k}}{D_x}$	(12-10)	
Deferred Term Insurance	${}_{k\mid}A^{1}_{x:\overline{n}\rceil} = \frac{M_{x+k} - M_{x+k+n}}{D_x}$	(12-11)	
Deferred Endowment Insurance	${}_{k\mid}A_{x:\overline{n}\rceil} = \frac{M_{x+k} - M_{x+k+n} + D_{x+k+n}}{D_x}$	(12-12)	
Natural Premium	$A^{1}_{x:\overline{1}\rceil} = c_x = \frac{C_x}{D_x}$	(12-13)	

Note: *In general* (either the insurance protection beginning on the date the policy is issued or deferred for a number of years), the above formulas may be summarized in the following manner:

The *net single premium*
of a $1 whole life or term insurance policy is:

$$A = \frac{M_{\text{age when insurance protection begins}} - M_{\text{age when insurance protection ends}}}{D_{\text{age on the policy date}}}$$

Application	*Formula*	*Formula Number*	*Reference Page*
of a \$1 endowment insurance policy is:	$A = \dfrac{M_{\text{age when insurance protection begins}} - M_{\text{age when insurance protection ends}} + D_{\text{age when insurance protection ends}}}{D_{\text{age on the policy date}}}$		
The *net annual premium* of a \$1 whole life or term insurance policy is:	$P = \dfrac{M_{\text{age when insurance protection begins}} - M_{\text{age when insurance protection ends}}}{N_{\text{age on the policy date}} - N_{\text{age when annual premium payment ends}}}$		
of a \$1 endowment insurance policy is:	$P = \dfrac{M_{\text{age when insurance protection begins}} - M_{\text{age when insurance protection ends}} + D_{\text{age when insurance protection ends}}}{N_{\text{age on the policy date}} - N_{\text{age when annual premium payment ends}}}$		

The value of *M, D,* and *N* for the age larger than the highest age (99) given in Table 13 is zero. Thus, the first and third general formulas above are also valid for whole life insurance policies.

Terminal Reserves

Retrospective Method	$V = \dfrac{P(N_x - N_{x+t}) - (M_x - M_{x+t})}{D_{x+t}}, \quad (t \le n)$	(12-14)	
	$V = \dfrac{P(N_x - N_{x+n}) - (M_x - M_{x+t})}{D_{x+t}}, \quad (t > n)$	(12-15)	

Prospective Method

$$\begin{bmatrix}t\text{th terminal}\\ \text{reserve } (V')\end{bmatrix} = \begin{bmatrix}\text{Present value (at age } x + t)\\ \text{of future death benefits}\end{bmatrix} - \begin{bmatrix}\text{Present value (at age } x + t) \text{ of}\\ \text{future net annual premiums}\end{bmatrix} \quad (12\text{-}16)$$

EXERCISE 12-9 REVIEW OF CHAPTER 12

1. A \$20,000 whole life policy is issued to a person aged 32. Find (a) the net single premium, (b) the net annual premium if the policy is a straight life policy, and (c) the net annual premium if the policy is a 25-payment life policy.
2. A \$100,000 whole life policy is issued to a person at age 20. What are (a) the net single premium, (b) the net annual premium if the policy is a straight life policy, and (c) the net annual premium if the policy is a 30-payment life policy?
3. Refer to Problem 1. If the policy is issued to someone at age 22, what are the answers to (a), (b), and (c)?
4. Refer to Problem 2. If the policy is issued to a child at age 10, what are the answers to (a), (b), and (c)?

5. A \$1,000, 15-year term policy is issued to someone aged 30. Find (a) the net single premium, (b) the net annual premium if the policy is a 15-payment policy, and (c) the net annual premium if the policy is a 10-payment policy.
6. A \$1,000, 20-year term policy is issued to someone aged 24. Find (a) the net single premium, (b) the net annual premium if the policy is a 20-payment policy, and (c) the net annual premium if the policy is a 15-payment policy.
7. Refer to Problem 5. Assuming that the policy is issued at age 40, what are the answers to (a), (b), and (c)?
8. Refer to Problem 6. Assuming that the policy is issued at age 14, what are the answers to (a), (b), and (c)?
9. A \$1,000, 40-year endowment insurance policy was bought by a person aged 25. Find (a) the net single premium, (b) the net annual premium if the policy is a 40-payment policy, and (c) the net annual premium if the policy is a 35-payment policy.
10. A \$1,000, 25-year endowment insurance policy was issued to a person at age 30. Find (a) the net single premium, (b) the net annual premium if the policy is a 25-payment policy, and (c) the net annual premium if the policy is a five-payment policy.
11. Refer to Problem 9. If the policy is bought by a person at age 18, what are the answers to (a), (b), and (c)?
12. Refer to Problem 10. If the policy is issued to a person aged 35, what are the answers to (a), (b), and (c)?

★13. A policy issued to a man at age 22 promises that the insurance company will pay \$1,000 if the insured dies before reaching age 40, \$2,000 if he dies after reaching 40 but before 50, \$3,000 if he dies after reaching 50 but before 65 or if he is alive at 65, and \$1,000 if he dies after 65. Find the net annual premiums if (a) the premium of the policy is payable annually for life, and (b) the premium is payable in 15 equal annual payments.

★14. Refer to Problem 13. Assume that the policy is issued to a boy at age 12. (a) What is the annual premium if the premium of the policy is payable annually for life? (b) What is the annual premium if the premium of the policy is payable in 25 equal annual payments?

15. A \$1,000, 10-year term policy is issued to a woman aged 30. (a) What is the level premium? (b) What are the natural premiums for age 30, 33, 36, and 39?
16. A \$1,000, 15-year term policy is issued to a woman at age 50. (a) What is the level premium? (b) Find the natural premiums for ages 50, 55, 60, and 64.

★17. Find the 15th terminal reserve for a \$1,000 policy issued at age 22. Assume that the policy is (a) a 25-year term insurance policy, (b) a 20-year endowment insurance policy, (c) a straight life policy, (d) a 40-payment life policy.

★18. Find the 10th terminal reserve for a \$1,000 policy issued at age 28. Assume that the policy is (a) a 30-year term insurance policy, (b) a 25-year endowment insurance policy, (c) a straight life policy, (d) a 30-payment life policy.

★19. Refer to Problem 17. Suppose that the insured stops paying the annual premium at age 37 but wishes to buy a new term insurance policy of \$1,000. What is the term of the new policy if the insured buys the policy with the reserve obtained from assumption (a)? (b)? (c)? (d)?

★20. Refer to Problem 18. Suppose that the insured is unable to maintain the annual premium payment at age 38, but wants to buy a new term insurance policy of \$1,000. What is the term of the new policy if the insured buys the policy with the 10th terminal reserve obtained from assumption (a)? (b)? (c)? (d)?

★21. Refer to Problem 17. Suppose that the insured stops paying the annual premium at age 37 but wishes to buy a new policy of reduced face value. What is the face value of the new policy if the reserve obtained from assumption (a) is used to buy a 10-year term insurance policy? If the reserve obtained from assumption (b) is used to buy a five-year endowment insurance policy? If the reserve obtained from assumption (c) is used to buy a whole life insurance policy? If the reserve obtained from assumption (d) is used to buy a whole life insurance policy?

★22. Refer to Problem 18. Suppose that the insured is unable to maintain the annual premium payment at age 38, but wants to buy a new policy of reduced face value. What is the face value of the new policy if the reserve obtained from assumption (a) is used to buy a 20-year term insurance policy? If the reserve obtained from assumption (b) is used to buy a 15-year endowment insurance policy? If the reserve obtained from assumption (c) is used to buy a whole life insurance policy? If the reserve obtained from assumption (d) is used to buy a whole life insurance policy?

Age, x	Number Living l_x	Number Dying d_x	Deaths per 1,000	Age, x	Number Living l_x	Number Dying d_x	Deaths per 1,000
0	10,000,000	70,800	7.08	50	8,762,306	72,902	8.32
1	9,929,200	17,475	1.76	51	8,689,404	79,160	9.11
2	9,911,725	15,066	1.52	52	8,610,244	85,758	9.96
3	9,896,659	14,449	1.46	53	8,524,488	92,832	10.89
4	9,882,210	13,835	1.40	54	8,431,654	100,337	11.90
5	9,868,375	13,322	1.35	55	8,331,317	108,307	13.00
6	9,855,053	12,812	1.30	56	8,223,010	116,848	14.21
7	9,842,241	14,401	1.26	57	8,106,161	125,970	15.54
8	9,829,840	12,091	1.23	58	7,980,191	135,663	17.00
9	9,817,749	11,879	1.21	59	7,844,528	145,830	18.59
10	9,805,870	11,665	1.21	60	7,698,698	156,592	20.34
11	9,794,005	12,047	1.23	61	7,542,106	167,736	22.24
12	9,781,958	12,325	1.26	62	7,374,370	179,271	24.31
13	9,769,633	12,896	1.32	63	7,195,099	191,174	26.57
14	9,756,737	13,562	1.39	64	7,003,925	203,394	29.04
15	9,743,175	14,225	1.46	65	6,800,531	215,917	31.75
16	9,728,950	14,983	1.54	66	6,584,614	228,749	34.74
17	9,713,967	15,737	1.62	67	6,355,865	241,777	38.04
18	9,698,230	16,390	1.69	68	6,114,088	254,835	41.68
19	9,681,840	16,846	1.74	69	5,859,253	267,241	45.61
20	9,664,994	17,300	1.79	70	5,592,012	278,426	49.79
21	9,647,694	17,655	1.83	71	5,313,588	287,731	54.15
22	9,630,039	17,912	1.86	72	5,025,855	294,766	58.65
23	9,612,127	18,167	1.89	73	4,731,089	299,289	63.26
24	9,593,960	18,324	1.91	74	4,431,800	301,894	68.12
25	9,575,636	17,481	1.93	75	4,129,906	303,011	73.37
26	9,557,155	18,732	1.96	76	3,826,895	303,014	79.18
27	9,538,423	18,981	1.99	77	3,523,881	301,997	85.70
28	9,519,442	19,324	2.03	78	3,221,884	299,829	93.06
29	9,500,118	19,760	2.08	79	2,922,055	295,683	101.10
30	9,480,358	20,193	2.13	80	2,626,372	288,848	108.98
31	9,460,165	20,718	2.19	81	2,337,524	278,983	119.35
32	9,439,447	21,239	2.25	82	2,058,541	265,902	129.17
33	9,418,208	21,850	2.32	83	1,792,639	249,858	139.38
34	9,396,358	22,551	2.40	84	1,542,781	231,433	150.01
35	9,373,807	23,628	2.51	85	1,311,348	211,311	161.14
36	9,350,279	24,865	2.64	86	1,100,037	190,108	172.82
37	9,325,594	26,112	2.80	87	909,929	168,455	185.13
38	9,299,482	27,991	3.01	88	741,474	146,997	199.25
39	9,271,491	30,132	3.25	89	594,477	126,303	212.46
40	9,241,359	32,622	3.53	90	468,174	106,609	228.14
41	9,208,737	35,362	3.84	91	361,365	88,813	245.77
42	9,173,375	38,253	4.17	92	272,522	72,480	265.93
43	9,135,122	41,382	4.53	93	200,072	57,881	289.30
44	9,093,740	44,741	4.92	94	142,191	45,026	316.66
45	9,046,899	46,412	5.35	95	97,165	34,128	351.24
46	9,000,587	52,473	5.83	96	63,037	25,250	400.56
47	8,948,114	56,910	6.36	97	37,787	18,456	488.42
48	8,891,204	61,794	6.95	98	19,331	12,916	666.15
49	8,829,410	67,104	7.60	99	6,415	6,415	1,000.00

Age. x	D_x	N_x	C_x	M_x
0	10,000,000.0000	324,850,104.9680	69,073.1710	2,076,826.7172
1	9,687,025.4290	314,850,104.9880	16,632.9566	2,007,753.5462
2	9,434,122.5838	305,163,080.5390	13,990.2787	1,991,120.5896
3	9,190,031.7084	295,728,957.9552	13,090.0608	1,977,130.3109
4	8,952,794.4741	286,538,926.2468	12,228.1241	1,964,040.2301
5	8,722,205.5791	277,588,131.7727	11,487.5169	1,951,812.1060
6	8,497,981.3556	268,863,926.1936	10,778.2903	1,940,324.5871
7	8,279,935.2370	260,365,944.8380	10,178.0762	1,929,546.2968
8	8,067,807.4636	252,086,009.6010	9,681.6066	1,919,368.2186
9	7,881,350.0557	244,016,202.1374	9,279.6558	1,909,686.6120
10	7,660,329.9546	236,158,852.0817	9,042.6476	1,900,406.7562
11	7,464,449.7660	228,496,522.1271	8,957.6178	1,891,363.9084
12	7,273,432.4866	221,032,072.3411	8,940.8062	1,882,406.2906
13	7,067,090.8833	213,758,639.8545	9,126.8500	1,873,465.4844
14	6,905,108.1561	206,671,548.9712	9,364.0939	1,664,338.6344
15	6,727,326.7626	199,766,440.8131	9,582.3146	1,854,974.5405
16	6,553,663.2627	193,039,114.0305	9,846.7536	1,845,392.2259
17	6,383,971.1254	186,485,450.7676	10,090.0279	1,835,545.4723
18	6,218,174.4633	180,101,479.6424	10,252.3993	1,825,455.4444
19	6,056,259.2006	173,883,305.1791	10,280.6243	1,815,203.0451
20	5,898,264.9735	167,627,045.6765	10,300.1628	1,804,922.4208
21	5,744,104.7377	161,928,780.8050	10,255.1657	1,794,622.2380
22	5,593,749.4258	156,184,676.1673	10,150.6810	1,784,367.0723
23	5,447,165.8414	150,590,926.7415	10,044.0865	1,774,216.3913
24	5,304,263.9929	145,143,760.9001	9,883.7932	1,764,172.3048
25	5,165,007.9517	139,839,496.9072	9,725.3439	1,754,288.5116
26	5,029,306.7854	134,674,768.9555	9,617.0037	1,744,563.1677
27	4,897,023.7928	129,645,182.1701	9,507.1611	1,734,946.1640
28	4,768,076.9758	124,748,158.3773	9,442.8900	1,725,439.0029
29	4,642,339.5370	119,980,081.4015	9,420.4356	1,715,996.1129
30	4,519,891.3751	115,337,741.8645	9,392.0634	1,706,575.6773
31	4,400,062.8465	110,818,050.4894	9,401.2163	1,697,183.6139
32	4,263,343.0569	106,417,987.6429	9,402.5686	1,667,762.3956
33	4,169,468.7479	102,134,644.5660	9,437.1317	1,678,379.6270
34	4,056,337.1966	78,965,175.8381	9,502.3390	1,668,942.6953
35	3,949,651.0856	93,906,838.6413	9,672.2130	1,659,440.3563
36	3,643,640.9771	89,956,997.5557	9,900.3401	1,649,768.1433
37	3,740,168.4751	86,113,146.5766	10,217.2318	1,639,667.8032
38	3,638,747.0704	82,372,958.1035	10,665.3232	1,629,650.5714
39	3,539,311.8617	70,743,211.0331	11,222.0794	1,628,965.2482
40	3,441,765.0620	75,194,899.1714	11,653.1042	1,607,743.1688
41	3,345,966.5023	71,753,134.1094	12,535.2926	1,595,890.0646
42	3,251,822.2774	68,407,167.6071	13,229.3739	1,583,354.7720
43	3,159,280.1764	65,155,345.3297	13,962.4424	1,570,125.3981
44	3,068,262.0685	61,996,065.1513	14,727.5918	1,556,162.9557
45	2,976,696.8164	58,927,803.0828	15,547.3065	1,541,435.3639
46	2,890,500.3526	55,949,104.2664	16,440.4699	1,525,888.0554
47	2,803,559.9046	53,058,603.9138	17,395.7457	1,509,447.5855
48	2,717,784.6405	50,255,044.0090	18,427.9447	1,492,051.8398
49	2,633,069.2135	47,537,259.3685	19,523.3861	1,473,623.8951

Age, x	D_x	N_x	C_x	M_x
50	2,549,324.6723	44,904,190.1550	20,692.9455	1,454,100.5090
51	2,466,453.0661	42,354,665.4827	21,921.2231	1,433,407.5635
52	2,384,374.4270	39,888,412.3936	23,169.1325	1,411,486.3404
53	2,303,049.6123	37,504,037.9666	24,468.5917	1,388,317.2079
54	2,222,409.2905	35,200,988.1543	25,801.7117	1,363,848.6162
55	2,142,402.4988	32,978,576.8638	27,171.9031	1,338,046.9045
56	2,062,976.8254	30,636,176.3650	28,599.9098	1,310,875.0014
57	1,964,060.3996	28,773,199.5396	30,080.3536	1,282,275.0916
58	1,905,588.3725	26,769,139.1400	31,604.6230	1,252,194.7380
59	1,827,505.7998	24,883,550.7675	33,144.7659	1,220,589.9150
60	1,749,787.7198	23,056,044.9677	34,722.7242	1,187,445.1491
61	1,672,387.2632	21,306,257.2479	36,286.6291	1,152,722.4249
62	1,595,310.6622	19,633,869.9847	37,836.1144	1,116,435.7958
63	1,518,564.5694	18,038,559.3225	39,364.2006	1,078,599.6814
64	1,442,162.1578	16,519,994.7531	40,858.9196	1,039,235.4808
65	1,366,128.5462	15,077,832.5953	42,316.6940	998,376.5612
66	1,290,491.6985	13,711,704.0491	43,736.1310	956,059.8672
67	1,215,278.1249	12,421,212.3506	45,101.6207	912,321.7362
68	1,140,535.6099	11,205,934.2257	46,378.0358	867,220.1155
69	1,066,339.5743	10,065,398.6158	47,449.5963	820,842.0797
70	992,881.7500	8,999,059.0415	48,229.7870	773,392.4634
71	920,435.3077	8,006,177.2915	48,625.9779	725,162.6964
72	849,359.6946	7,085,741.9838	48,599.8832	676,536.7185
73	780,043.7396	6,236,382.2892	48,142.0663	627,936.8353
74	712,876.2140	5,456,338.5496	47,376.6752	579,794.7690
75	646,112.3021	4,743,462.3356	46,392.1628	532,416.0938
76	585,912.5111	4,095,350.0335	45,261.0951	486,025.9310
77	526,360.8716	3,509,437.5224	44,008.9628	440,764.8359
78	469,513.8465	2,983,076.6508	42,627.3426	396,755.8731
79	415,434.9294	2,513,562.8043	41,012.5834	354,128.5305
80	364,289.7969	2,098,127.8749	39,087.3533	313,115.9471
81	316,317.3241	1,733,838.0760	36,831.6174	274,028.5938
82	271,770.6619	1,417,520.7519	34,248.4382	237,196.9764
83	230,893.6600	1,145,750.0900	31,397.0289	202,946.5382
84	193,865.0736	914,858.4300	28,372.4430	171,551.5093
85	160,764.2229	720,991.3564	25,273.7507	143,179.0663
86	131,569.3974	560,227.1335	22,183.1949	117,905.3156
87	106,177.1855	428,657.7361	19,177.1362	95,722.1207
88	84,410.3641	322,480.5506	16,326.1748	76,544.9845
89	66,025.3978	238,070.1865	13,685.6613	60,218.8097
90	50,729.3636	172,044.7887	11,291.0955	46,533.1484
91	38,200.9638	121,315.4251	9,159.6932	35,242.0529
92	28,109.5416	83,114.4613	7,292.8738	26,082.3597
93	20,131.0686	55,004.9196	5,681.8884	18,789.4859
94	13,958.1795	34,873.8512	4,312.1729	13,107.5975
95	9,305.5630	20,915.6717	3,188.7449	8,795.4246
96	5,889.8533	11,610.1087	2,301.6660	5,606.6797
97	3,444.5103	5,720.2554	1,641.3406	3,304.9917
98	1,719.1569	2,275.7451	1,120.6380	1,663.6509
99	556.5882	556.5882	543.0129	543.0129

Age, x	Male: Number Living l_x	Male: Deaths per 1,000	Female: Number Living l_x	Female: Deaths per 1,000
0	185,890	4.18	65,135	2.89
1	183,113	1.07	64,947	0.87
2	184,915	0.99	64,890	0.81
3	184,732	0.98	64,837	0.79
4	184,551	0.95	64,786	0.77
5	184,376	0.90	64,736	0.76
6	184,210	0.86	64,687	0.73
7	184,052	0.80	64,640	0.72
8	183,905	0.76	64,593	0.70
9	183,765	0.74	64,548	0.69
10	183,629	0.73	64,503	0.68
11	183,495	0.77	64,459	0.69
12	183,354	0.85	64,415	0.72
13	183,198	0.99	64,369	0.75
14	183,017	1.15	64,321	0.80
15	182,807	1.33	64,270	0.85
16	182,564	1.51	64,215	0.90
17	182,288	1.67	64,157	0.95
18	181,984	1.78	64,096	0.98
19	181,660	1.86	64,033	1.02
20	181,322	1.90	63,968	1.05
21	180,977	1.91	63,901	1.07
22	180,631	1.89	63,833	1.09
23	180,290	1.86	63,763	1.11
24	179,955	1.82	63,692	1.14
25	179,627	1.77	63,619	1.16
26	179,309	1.73	63,545	1.19
27	178,999	1.71	63,469	1.22
28	178,693	1.70	63,392	1.26
29	178,389	1.71	63,312	1.30
30	178,084	1.73	63,230	1.35
31	177,776	1.78	63,145	1.40
32	177,460	1.83	63,057	1.45
33	177,135	1.91	62,966	1.50
34	176,797	2.00	62,872	1.58
35	176,443	2.11	62,773	1.65
36	176,071	2.24	62,669	1.76
37	175,677	2.40	62,559	1.89
38	175,255	2.58	62,441	2.04
39	174,803	2.79	62,314	2.22
40	174,315	3.02	62,176	2.42
41	173,789	3.29	62,026	2.64
42	173,217	3.56	61,862	2.87
43	172,600	3.87	61,684	3.09
44	171,932	4.19	61,493	3.32
45	171,212	4.55	61,289	3.56
46	170,433	4.92	61,071	3.80
47	169,594	5.32	60,839	4.05
48	168,692	5.74	60,593	4.33
49	167,724	5.21	60,331	4.63
50	166,682	6.71	60,052	4.96
51	165,564	7.30	59,756	5.31
52	164,355	7.96	59,437	5.70
53	163,047	8.71	59,098	6.15
54	161,627	9.56	58,735	6.61
55	160,082	10.47	58,347	7.09
56	158,406	11.46	57,933	7.57
57	156,591	12.49	57,494	8.03
58	154,635	13.59	57,032	8.47
59	152,534	14.77	56,549	8.94
60	150,281	16.08	56,043	9.47
61	147,864	17.54	55,512	10.13
62	145,270	19.19	54,950	10.96
63	142,482	21.06	54,348	12.02
64	139,481	23.14	53,695	13.25
65	136,253	25.42	52,984	14.59
66	132,789	27.85	52,211	16.00
67	129,091	30.44	51,376	17.43
68	125,161	33.19	50,481	18.84
69	121,007	36.17	49,530	20.36
70	116,630	39.51	48,522	22.11
71	112,022	43.30	47,449	24.23
72	107,171	47.65	46,299	26.87
73	102,064	52.64	45,055	30.11
74	96,691	58.19	43,698	33.93
75	91,065	64.19	42,215	38.24
76	85,220	70.53	40,601	42.97
77	79,209	77.12	38,856	48.04
78	73,100	83.90	36,989	53.45
79	66,967	91.05	35,012	59.35
80	60,870	98.84	32,934	65.99
81	54,854	107.48	30,761	73.00
82	48,958	117.25	28,497	82.40
83	43,218	128.26	26,149	92.53
84	37,675	140.25	23,729	103.81
85	32,391	152.95	21,266	116.10
86	27,437	166.09	18,797	129.29
87	22,880	179.55	16,367	143.32
88	18,772	193.27	14,021	158.18
89	15,144	207.29	11,803	173.94
90	12,005	221.77	9,750	190.75
91	9,343	236.98	7,890	208.87
92	7,129	253.45	6,242	228.81
93	5,322	272.11	4,814	251.51
94	3,874	295.90	3,603	279.31
95	2,728	329.96	2,597	317.32
96	1,828	384.55	1,773	375.74
97	1,125	480.20	1,107	474.97
98	585	657.98	581	655.85
99	200	1,000.00	200	1,000.00