$$z_c = \frac{\max(n_p, n_n) - 0.5(n_p + n_n) - 0.5}{0.5\sqrt{n_p + n_n}} \tag{3.8}$$

Formula 3.8 approximates a binomial distribution to the normal distribution. However, the binomial distribution is a discrete distribution, while the normal distribution is continuous. More to the point, discrete values deal with heights but not widths, while the continuous distribution deals with both heights and widths. The correction adds or subtracts 0.5 of a unit from each discrete X-value to fill the gaps and make it continuous.

The one sided p-value is $p_1 = 1 - \Phi|z_c|$, where $\Phi|z_c|$ is the area under the respective tail of the normal distribution at z_c. The two-sided p-value is $p = 2p_1$.

3.4.1 Sample Sign Test (Small Data Samples)

To present the process for performing the sign test, we are going to use the data from Section 3.3.1, which used the Wilcoxon signed rank test. Recall that the sample involves 12 members of the counseling staff from Clear Creek County School District who are working on a program to improve response to bullying in the schools. The data from Table 3.1 are being reduced to a binomial distribution for use with the sign test. The relatively small sample size warrants a nonparametric procedure.

3.4.1.1 State the Null and Research Hypotheses The null hypothesis states that the counselors reported no difference between positive or negative interventions between last year and this year. In other words, the changes in responses produce a balanced number of positive and negative differences. The research hypothesis states that the counselors observed some differences between this year and last year. Our research hypothesis is a two-tailed, nondirectional hypothesis because it indicates a difference, but in no particular direction.

The null hypothesis is

H_O: $p = 0.5$

The research hypothesis is

H_A: $p \neq 0.5$

3.4.1.2 Set the Level of Risk (or the Level of Significance) Associated with the Null Hypothesis The level of risk, also called an alpha (α), is frequently set at 0.05. We will use $\alpha = 0.05$ in our example. In other words, there is a 95% chance that any observed statistical difference will be real and not due to chance.

3.4.1.3 Choose the Appropriate Test Statistic Recall from Section 3.3.1 that the data are obtained from 12 counselors, or participants, who are using a new program designed to reduce bullying among students in the elementary schools. The participants reported the percentage of successful interventions last year and the percentage this year. We are comparing last year's percentages with this year's percentages. Therefore, the data samples are related or paired. In addition, sample sizes are relatively small. Since we are comparing two related samples, we will use the sign test.

3.4.1.4 *Compute the Test Statistic*

First, decide if there is a difference in intervention score from year 1 to year 2. Determine if the difference is positive or negative and put the sign of the difference in the sign column. If we count the number of ties or "0" differences among the group, we find only two with no difference from last year to this year. Ties are discarded.

Now, we count the number of positive and negative differences between last year and this year. Count the number of "+" or positive differences. When we look at Table 3.7, we see that eight participants showed positive differences, $n_p = 8$. Count the number of "−" or negative differences. When we look at Table 3.7, we see only two negative differences, $n_n = 2$.

TABLE 3.7

	Percentage of successful intervention		
Participant	Last year	This year	Sign of difference
1	31	31	0
2	14	14	0
3	53	50	−
4	18	30	+
5	21	28	+
6	44	48	+
7	12	35	+
8	36	32	−
9	22	23	+
10	29	34	+
11	17	27	+
12	40	42	+

Next, we find the X-score at and beyond where the area under our binomial probability function is $\alpha = 0.05$. Since we are performing a two-tailed test, we use 0.025 for each tail. We will calculate the probabilities associated with the binomial distribution for $p = 0.5$ and $n = 10$. We will demonstrate one of the calculations, but list the results for each value. To simplify calculation, use the table of factorials in Appendix B, Table B.9:

$$P(X) = \frac{n!}{(n-X)!X!} \cdot p^X \cdot (1-p)^{n-X}$$

$$P(0) = \frac{10!}{(10-0)!0!} \cdot 0.5^0 \cdot (1-0.5)^{10-0}$$

$$P(0)=\frac{3{,}628{,}800}{(3{,}628{,}800)(0)}\cdot 1\cdot 0.000977$$

$$P(0)=0.0010$$

$$P(1)=0.0098$$

$$P(2)=0.0439$$

$$P(3)=0.1172$$

$$P(4)=0.2051$$

$$P(5)=0.2461$$

$$P(6)=0.2051$$

$$P(7)=0.1172$$

$$P(8)=0.0439$$

$$P(9)=0.0098$$

$$P(10)=0.0010$$

Notice that the values form a symmetric distribution with the median at $P(5)$, as shown in Figure 3.1. Using this distribution, we find the p-values for each tail. To do that, we sum the probabilities for each tail until we find a probability equal to or greater than $\alpha/2 = 0.025$. First, calculate P for pluses:

$$P(8, 9, \text{or } 10)=0.0439+0.0098+0.0010=0.0547$$

Second, calculate P for minuses:

$$P(0, 1, \text{or } 2)=0.0010+0.0098+0.0439=0.0547$$

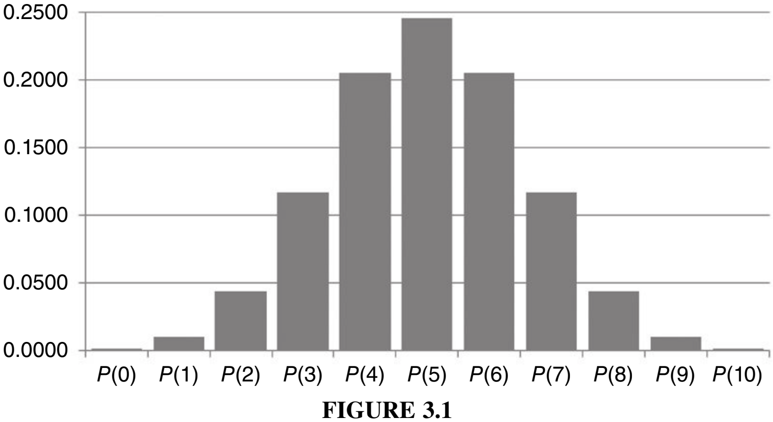


FIGURE 3.1

Finally, calculate the obtained value p by combining the two tails:

$$p = P(8, 9, \text{or } 10) + P(0, 1, \text{or } 2) = 0.0547 + 0.0547$$

$$p = 0.1094$$

3.4.1.5 Determine the Critical Value Needed for Rejection of the Null Hypothesis In the example in this chapter, the two-tailed probability was computed and is compared with the level of risk specified earlier, $\alpha = 0.05$.

3.4.1.6 Compare the Obtained Value with the Critical Value The critical value for rejecting the null hypothesis is $\alpha = 0.05$ and the obtained p-value is $p = 0.1094$. If the critical value is greater than the obtained value, we must reject the null hypothesis. If the critical value is less than the obtained value, we do not reject the null hypothesis. Since the critical value is less than the obtained value ($p > \alpha$), we do not reject the null hypothesis.

3.4.1.7 Interpret the Results We did not reject the null hypothesis, suggesting that no real difference exists between last year's and this year's percentages. There was no evidence of positive or negative intervention by counselors. These results differ from the data's analysis using the Wilcoxon signed rank test. A discussion about statistical power addresses those differences toward the end of this chapter.

3.4.1.8 Reporting the Results When reporting the findings for the sign test, you should include the sample size, the number of pluses, minuses, and ties, and the probability of getting the obtained number of pluses and minuses.

For this example, the obtained value, $p = 0.1094$, was greater than the critical value, $\alpha = 0.05$. Therefore, we did not reject the null hypothesis, suggesting that the new bullying program is not providing evidence of a change in student behavior as perceived by the school counselors.

3.4.2 Sample Sign Test (Large Data Samples)

We are going to demonstrate a sign test with large samples using the data from the Wilcoxon signed rank test for large samples in Section 3.3.3. The data from the implementation of the bullying program in the Jonestown School District are presented in Table 3.8. The data are used to determine the effect of the bullying program from year 1 to year 2. If there is an increase in successful intervention, we will use a "+" to identify the positive difference in response. If there is a decrease in successful intervention in the response, we will identify a negative difference with a "−." There are 25 participants in this study.

3.4.2.1 State the Null and Alternate Hypotheses The null hypothesis states that there was no positive or negative effect of the bullying program on successful intervention. The research hypothesis states that either a positive or negative effect exists from the bullying program.

TABLE 3.8

	Percentage of successful interventions	
Participant	Last year	This year
1	53	50
2	18	43
3	21	28
4	44	48
5	12	35
6	36	32
7	22	23
8	29	34
9	17	27
10	10	42
11	38	44
12	37	16
13	19	33
14	37	50
15	28	20
16	15	27
17	25	27
18	38	30
19	40	51
20	30	50
21	23	45
22	41	20
23	31	49
24	28	43
25	14	30

The null hypothesis is

$$H_O\colon p = 0.5$$

The research hypothesis is

$$H_A\colon p \neq 0.5$$

3.4.2.2 Set the Level of Risk (or the Level of Significance) Associated with the Null Hypothesis The level of risk, also called an alpha (α), is frequently set at 0.05. We will use $\alpha = 0.05$ in our example. In other words, there is a 95% chance that any observed statistical difference will be real and not due to chance.

3.4.2.3 Choose the Appropriate Test Statistic Recall from Section 3.3.3 that the data were obtained from 25 counselors, or participants, who were using a new program designed to reduce bullying among students in the elementary schools. The

TABLE 3.9

	Percentage of successful interventions		
Participant	Last year	This year	Sign of difference
1	53	50	−
2	18	43	+
3	21	28	+
4	44	48	+
5	12	35	+
6	36	32	−
7	22	23	+
8	29	34	+
9	17	27	+
10	10	42	+
11	38	44	+
12	37	16	−
13	19	33	+
14	37	50	+
15	28	20	−
16	15	27	+
17	25	27	+
18	38	30	−
19	40	51	+
20	30	50	+
21	23	45	+
22	41	20	−
23	31	49	+
24	28	43	+
25	14	30	+

participants reported the percentage of successful interventions last year and the percentage this year. We are comparing last year's percentages with this year's percentages. Therefore, the data samples are related or paired. Since we are making dichotomous comparisons of two related samples, we will use the sign test.

3.4.2.4 Compute the Test Statistic First, we determine the sign of the differences between last year and this year. Table 3.9 includes the column for the sign of the difference for each participant. Next, we count the numbers of positive and negative differences. We find six negative differences, $n_n = 6$, and 19 positive differences, $n_p = 19$.

Since the sample size is $n \geq 25$, we will use a z-score approximation of the binomial distribution. The binomial distribution becomes an approximation of the normal distribution as n becomes large and p is not too close to the 0 or 1 values. If this approximation is used, $P(Y \leq k)$ is obtained by computing the corrected z-score for the given data that are as extreme or more extreme than the data given:

$$z_c = \frac{\max(n_p, n_n) - 0.5(n_p + n_n) - 0.5}{0.5\sqrt{n_p + n_n}} = \frac{19 - (0.5)(19+6) - 0.5}{(0.5)(\sqrt{19+6})}$$

$$= \frac{19 - 12.5 - 0.5}{(0.5)(5)} = \frac{6}{2.5}$$

$$z_c = 2.4$$

Next, we find the one-sided p-value. Table B.1 is used to establish $\Phi|z_c|$.

$$p_1 = 1 - \Phi|z_c| = 1 - 0.9918$$

$$p_1 = 0.0082$$

We now multiply two times the one-sided p-value to find the two-sided p-value:

$$p = 2p_1 = (2)(0.0082)$$

$$p = 0.016$$

3.4.2.5 Determine the Critical Value Needed for Rejection of the Null Hypothesis In the example in this chapter, the two-tailed probability was computed and compared with the level of risk specified earlier, $\alpha = 0.05$.

3.4.2.6 Compare the Obtained Value with the Critical Value The critical value for rejecting the null hypothesis is $\alpha = 0.05$ and the obtained p-value is $p = 0.016$. If the critical value is greater than the obtained value, we must reject the null hypothesis. If the critical value is less than the obtained value, we do not reject the null hypothesis. Since the critical value is greater than the obtained value ($p < \alpha$), we reject the null hypothesis.

3.4.2.7 Interpret the Results We rejected the null hypothesis, suggesting that there is a real difference between last year's and this year's degree of successful intervention for the 25 counselors who were in the study.

Analysis was limited to the identification of the presence of positive "+" or negative "–" differences between year 1 and year 2 for each participant. The level of significance does not describe the strength of the test's level of significance.

3.4.2.8 Reporting the Results When reporting the findings for the sign test, you should include the sample size, the number of pluses, minuses, and ties, and the probability of getting the obtained number of pluses and minuses.

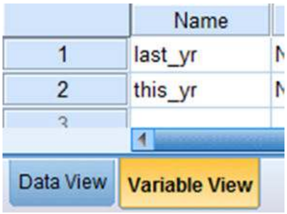
For this example, the obtained significance, $p = 0.016$, was less than the critical value, $\alpha = 0.05$. Therefore, we rejected the null hypothesis, suggesting that the number of successful interventions was significantly different from year 1 to year 2.

3.5 PERFORMING THE WILCOXON SIGNED RANK TEST AND THE SIGN TEST USING SPSS

We will analyze the small sample examples for the Wilcoxon signed rank test and the sign test using SPSS.

3.5.1 Define Your Variables

First, click the "Variable View" tab at the bottom of your screen. Then, type the names of your variables in the "Name" column. As shown in Figure 3.2, we have named our variables "last_yr" and "this_yr."

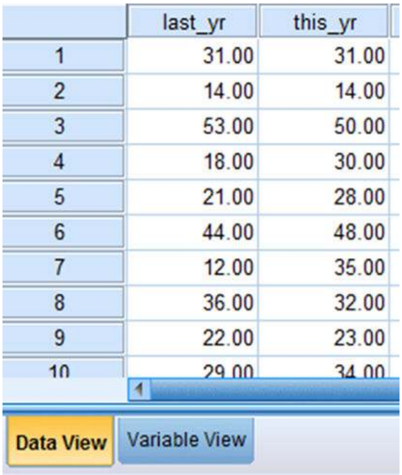


	Name
1	last_yr
2	this_yr
3	

FIGURE 3.2

3.5.2 Type in Your Values

Click the "Data View" tab at the bottom of your screen and type your data under the variable names. As shown in Figure 3.3, we are comparing "last_yr" with "this_yr."



	last_yr	this_yr
1	31.00	31.00
2	14.00	14.00
3	53.00	50.00
4	18.00	30.00
5	21.00	28.00
6	44.00	48.00
7	12.00	35.00
8	36.00	32.00
9	22.00	23.00
10	29.00	34.00

FIGURE 3.3

3.5.3 Analyze Your Data

As shown in Figure 3.4, use the pull-down menus to choose “Analyze,” “Nonparametric Tests,” “Legacy Dialogs,” and “2 Related Samples . . .”

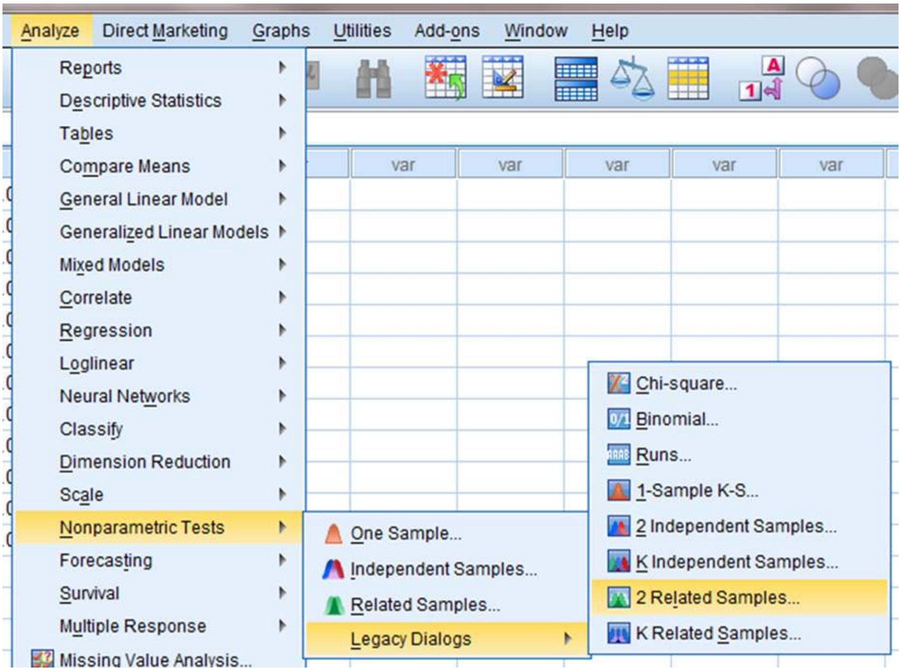


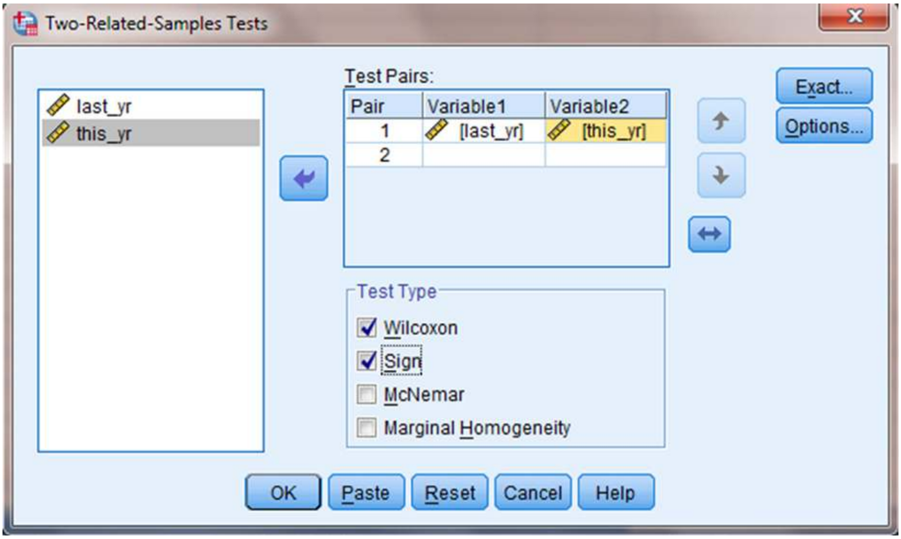
FIGURE 3.4

In the upper left box, select both variables that you want to compare. Then, use the arrow button to place your variable pair in the box labeled “Test Pairs:”. Next, check the “Test Type” you wish to perform. In Figure 3.5, we have checked “Wilcoxon” and “Sign” to perform both tests. Finally, click “OK” to perform the analysis.

3.5.4 Interpret the Results from the SPSS Output Window

SPSS Output 3.1 begins by reporting the results from the Wilcoxon signed rank test. The first output table (called “Ranks”) provides the Wilcoxon T or obtained value. From the “Sum of Ranks” column, we select the smaller of the two values. In our example, $T = 7.5$. The second output table (called “Test Statistics”) returns the critical z-score for large samples. In addition, SPSS calculates the two-tailed significance ($p = 0.041$).

Based on the results from SPSS, the number of successful interventions was significantly different ($T = 7.5$, $n = 12$, $p < 0.05$). In addition, the sum of the positive difference ranks ($\Sigma R_+ = 47.5$) was larger than the sum of the negative difference ranks ($\Sigma R_- = 7.5$), demonstrating a positive impact from the program.



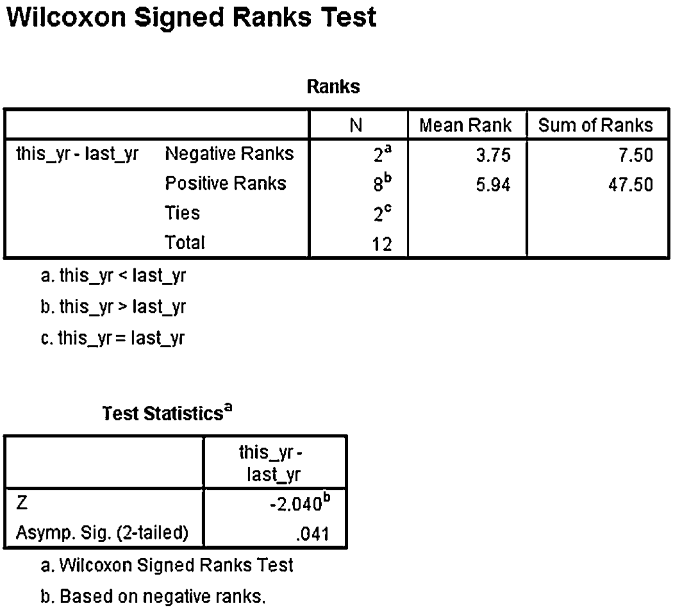
Test Pairs:

Pair	Variable1	Variable2
1	[last_yr]	[this_yr]
2		

Test Type

☑ Wilcoxon
☑ Sign
□ McNemar
□ Marginal Homogeneity

FIGURE 3.5



Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
this_yr - last_yr	Negative Ranks	2[a]	3.75	7.50
	Positive Ranks	8[b]	5.94	47.50
	Ties	2[c]		
	Total	12		

a. this_yr < last_yr

b. this_yr > last_yr

c. this_yr = last_yr

Test Statistics[a]

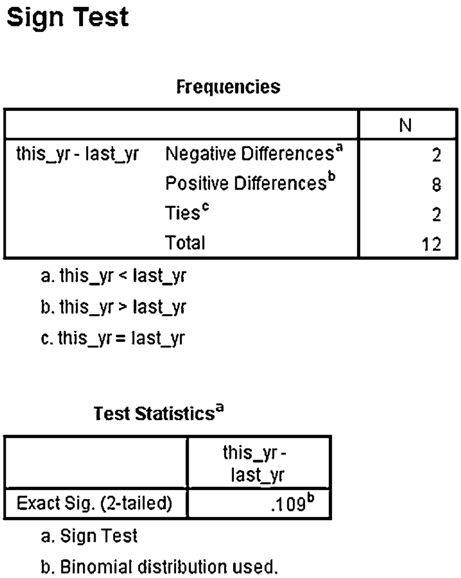
	this_yr - last_yr
Z	-2.040[b]
Asymp. Sig. (2-tailed)	.041

a. Wilcoxon Signed Ranks Test

b. Based on negative ranks.

SPSS OUTPUT 3.1

Next, SPSS Output 3.2 reports the results from the sign test. The first output table (called “Frequencies”) provides the negative differences, positive differences, ties, and total comparisons. The second output table (called “Test Statistics”) returns the two-tailed significance ($p = 0.109$). Based on the results of the sign test using SPSS, the number of successful interventions was not significantly different ($0.109 > 0.05$).



Sign Test

Frequencies

		N
this_yr - last_yr	Negative Differences[a]	2
	Positive Differences[b]	8
	Ties[c]	2
	Total	12

a. this_yr < last_yr

b. this_yr > last_yr

c. this_yr = last_yr

Test Statistics[a]

	this_yr - last_yr
Exact Sig. (2-tailed)	.109[b]

a. Sign Test

b. Binomial distribution used.

SPSS OUTPUT 3.2

The notion that the Wilcoxon signed rank test produced significant results while the sign test did not is addressed next in a brief discussion about statistical power.

3.6 STATISTICAL POWER

Comparing our conflicting results from the small sample Wilcoxon signed rank test with the sign test presents an opportunity to discuss statistical power. That difference is especially visible when comparing the results from the sample problems in Sections 3.3.1 and 3.4.1 of this chapter. Both sections analyzed the same data; however, one section demonstrated a Wilcoxon signed rank test and the other demonstrated the sign test.

Notice that the result from the Wilcoxon signed rank test was significant, yet the result from the sign test was not significant. In other words, one test produced significant results and the other test did not. The reason involves differences in statistical power.

Nonparametric methods generally have less statistical power compared with their parametric equivalents, especially when used in small samples. For instance, a test with less statistical power has a smaller chance of detecting a true effect where one might actually exist. This difference in statistical power is especially true for the sign test (Siegel and Castellan, 1988).

A statistical test's power depends on several factors: the size of the effect (discussed later), level of desired significance (α), and sample size. Researchers use this information to perform a statistical power analysis before performing the experiment. This allows the researcher to determine the needed sample size. A quick search returns a variety of online power analysis tools. Currently, *G*Power* is a free tool. In addition, Cohen (1988) has provided several tables for finding sample sizes based on level of power.

3.7 EXAMPLES FROM THE LITERATURE

To be shown are varied examples of the nonparametric procedures described in this chapter. We have summarized each study's research problem and the researchers' rationale(s) for choosing a nonparametric approach. We encourage you to obtain these studies if you are interested in their results.

Boser and Poppen (1978) sought to determine which verbal responses by teacher held the greatest potential for improving student–teacher relationships. The seven verbal responses were feelings, thoughts, motives, behaviors, encounter/encouragement, confrontation, and sharing. They used a Wilcoxon signed rank test to examine 101 9th-grader responses because the student participants rank ordered their responses.

Vaughn et al. (1999) investigated kindergarten teachers' perceptions of practices identified to improve outcomes for children with disabilities transitioning from prekindergarten to kindergarten. The researchers compared the paired ratings of teachers' desirability to employ the identified practices with feasibility using a Wilcoxon signed rank test. This nonparametric procedure was considered the most appropriate because the study's measure was a Likert-type scale (1 = *low*, 5 = *high*).

Rinderknecht and Smith (2004) used a 7-month nutrition intervention to improve the dietary self-efficacy of Native American children (5–10 years) and adolescents (11–18 years). Wilcoxon signed rank tests were used to determine whether fat and sugar intake changed significantly between pre- and postintervention among adolescents. The researchers chose nonparametric tests for their data that were not normally distributed.

Seiver and Hatfield (2002) asked environmental health professionals about their willingness to dine in certain restaurants based on the method and history of health code evaluations. A paired-sample sign test was used to determine which health code evaluation method and history that participants preferred. The researchers chose a nonparametric test since they administered questionnaires with rank ordered scales (0 = *never*, 10 = *always*).

3.8 SUMMARY

Two samples that are paired, or related, may be compared using a nonparametric procedure called the Wilcoxon signed rank test or the sign test. The parametric equivalent to this test is known as the Student's *t*-test, *t*-test for matched pairs, or *t*-test for dependent samples.

In this chapter, we described how to perform and interpret a Wilcoxon signed rank test and a sign test, using both small samples and large samples. We also explained how to perform the procedure for both tests using SPSS. Finally, we offered varied examples of these nonparametric statistics from the literature. The next chapter will involve comparing two samples that are not related.

3.9 PRACTICE QUESTIONS

1. A teacher wished to determine if providing a bilingual dictionary to students with limited English proficiency improves math test scores. A small class of students ($n = 10$) was selected. Students were given two math tests. Each test covered the same type of math content; however, students were provided a bilingual dictionary on the second test. The data in Table 3.10 represent the students' performance on each math test.

TABLE 3.10

Student	Math test without a bilingual dictionary	Math test with a bilingual dictionary
1	30	39
2	56	46
3	48	37
4	47	44
5	43	32
6	45	39
7	36	41
8	44	40
9	44	38
10	40	46

Use a one-tailed Wilcoxon signed rank test and a one-tailed sign test to determine which testing condition resulted in higher scores. Use $\alpha = 0.05$. Report your findings.

2. A research study was done to investigate the influence of being alone at night on the human male heart rate. Ten men were sent into a wooded area, one at a time, at night, for 20 min. They had a heart monitor to record their pulse rate. The second night, the same men were sent into a similar wooded area accompanied by a companion. Their pulse rate was recorded again. The researcher wanted to see if having a companion would change their pulse rate. The median rates are reported in Table 3.11.

   Use a two-tailed Wilcoxon signed rank test and a two-tailed sign test to determine which condition produced a higher pulse rate. Use $\alpha = 0.05$. Report your findings.

TABLE 3.11

Participant	Median rate alone	Median rate with companion
A	88	72
B	77	74
C	91	80
D	70	77
E	80	71
F	85	83
G	90	80
H	82	91
I	93	86
J	75	69

3. A researcher conducts a pilot study to compare two treatments to help obese female teenagers lose weight. She tests each individual in two different treatment conditions. The data in Table 3.12 provide the number of pounds that each participant lost.

TABLE 3.12

	Pounds lost	
Participant	Treatment 1	Treatment 2
1	10	18
2	20	12
3	15	16
4	9	7
5	18	21
6	11	17
7	6	13
8	12	14

Use a two-tailed Wilcoxon signed rank test and a two-tailed sign test to determine which treatment resulted in greater weight loss. Use $\alpha = 0.05$. Report your findings.

4. Twenty participants in an exercise program were measured on the number of sit-ups they could do before other physical exercise (first count) and the number they could do after they had done at least 45 min of other physical exercise (second count). Table 3.13 shows the results for 20 participants obtained during two separate physical exercise sessions. Determine the ES for a calculated z-score.

TABLE 3.13

Participant	First count	Second count
1	18	28
2	19	18
3	20	28
4	29	20
5	15	30
6	22	25
7	21	28
8	30	18
9	22	27
10	11	30
11	20	24
12	21	27
13	21	10
14	20	40
15	18	20
16	27	14
17	24	29
18	13	30
19	10	24
20	10	36

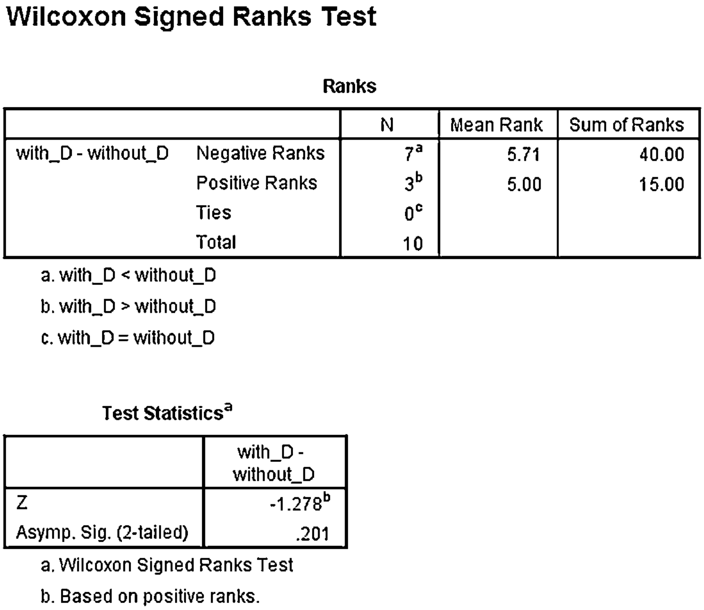
5. A school is trying to get more students to participate in activities that will make learning more desirable. Table 3.14 shows the number of activities that each of the 10 students in one class participated in last year before a new activity program was implemented and this year after it was implemented. Construct a 95% median confidence interval based on the Wilcoxon signed rank test to determine whether the new activity program had a significant positive effect on the student participation.

TABLE 3.14

Participants	Last year	This year
1	18	20
2	22	28
3	10	18
4	25	23
5	16	20
6	14	21
7	21	17
8	13	18
9	28	22
10	12	21

3.10 SOLUTIONS TO PRACTICE QUESTIONS

1. The results from the analysis are displayed in SPSS Outputs 3.3 and 3.4. Both tests report the two-tailed significance, but the question asked for the one-tailed significance. Therefore, divide the two-tailed significance by 2 to find the one-tailed significance.



Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
with_D - without_D	Negative Ranks	7[a]	5.71	40.00
	Positive Ranks	3[b]	5.00	15.00
	Ties	0[c]		
	Total	10		

a. with_D < without_D

b. with_D > without_D

c. with_D = without_D

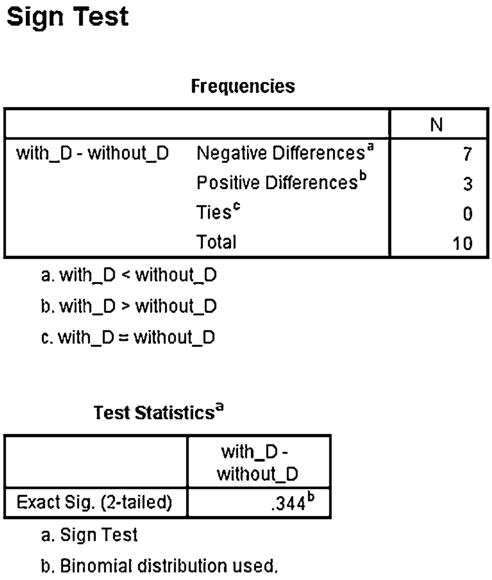
Test Statistics[a]

	with_D - without_D
Z	-1.278[b]
Asymp. Sig. (2-tailed)	.201

a. Wilcoxon Signed Ranks Test

b. Based on positive ranks.

SPSS OUTPUT 3.3



Sign Test

Frequencies

		N
with_D - without_D	Negative Differences[a]	7
	Positive Differences[b]	3
	Ties[c]	0
	Total	10

a. with_D < without_D

b. with_D > without_D

c. with_D = without_D

Test Statistics[a]

	with_D - without_D
Exact Sig. (2-tailed)	.344[b]

a. Sign Test

b. Binomial distribution used.

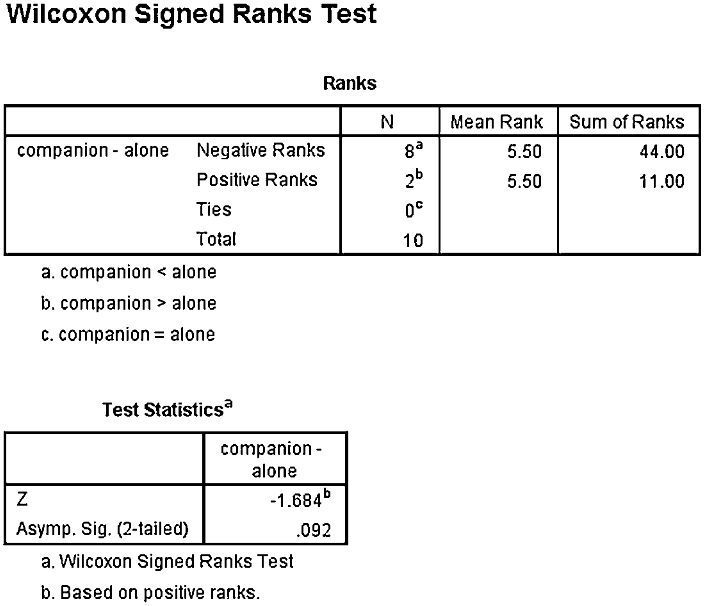
SPSS OUTPUT 3.4

The results from the Wilcoxon signed rank test reported a one-tailed significance of $p = 0.201/2 = 0.101$. The test results ($T = 15.0$, $n = 10$, $p > 0.05$) indicated that the two testing conditions were not significantly different.

The results from the sign test reported a one-tailed significance of $p = 0.344/2 = 0.172$. These test results ($p > 0.05$) also indicated that the two testing conditions were not significantly different.

Therefore, based on this study, the use of bilingual dictionaries on a math test did not significantly improve scores among limited English proficient students.

2. The results from the analysis are displayed in SPSS Outputs 3.5 and 3.6.



Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
companion - alone	Negative Ranks	8[a]	5.50	44.00
	Positive Ranks	2[b]	5.50	11.00
	Ties	0[c]		
	Total	10		

a. companion < alone

b. companion > alone

c. companion = alone

Test Statistics[a]

	companion - alone
Z	-1.684[b]
Asymp. Sig. (2-tailed)	.092

a. Wilcoxon Signed Ranks Test

b. Based on positive ranks.

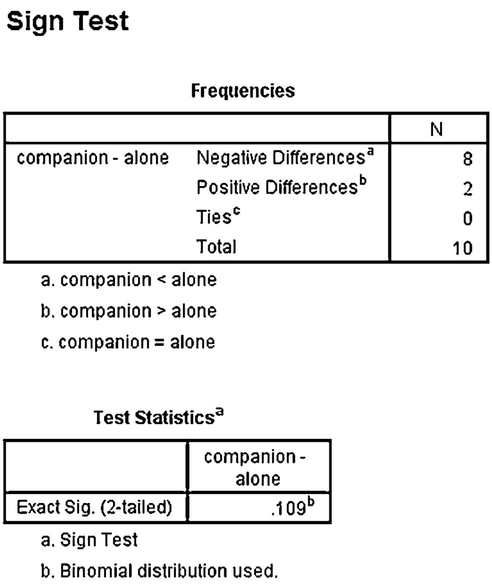
SPSS OUTPUT 3.5

The results from the Wilcoxon signed rank test reported a two-tailed significance of $p = 0.092$. The test results ($T = 11.0$, $n = 10$, $p > 0.05$) indicated that the two conditions were not significantly different.

The results from the sign test reported a two-tailed significance of $p = 0.109$. These test results ($p > 0.05$) also indicated that the two testing conditions were not significantly different.

Therefore, based on this study, the presence of a companion in the woods at night did not significantly influence the males' pulse rates.

3. The results from the analysis are displayed in SPSS Outputs 3.7 and 3.8. The results from the Wilcoxon signed rank test ($T = 10.0$, $n = 8$, $p > 0.05$) indicated that the two treatments were not significantly different.



Sign Test

Frequencies

		N
companion - alone	Negative Differences[a]	8
	Positive Differences[b]	2
	Ties[c]	0
	Total	10

a. companion < alone

b. companion > alone

c. companion = alone

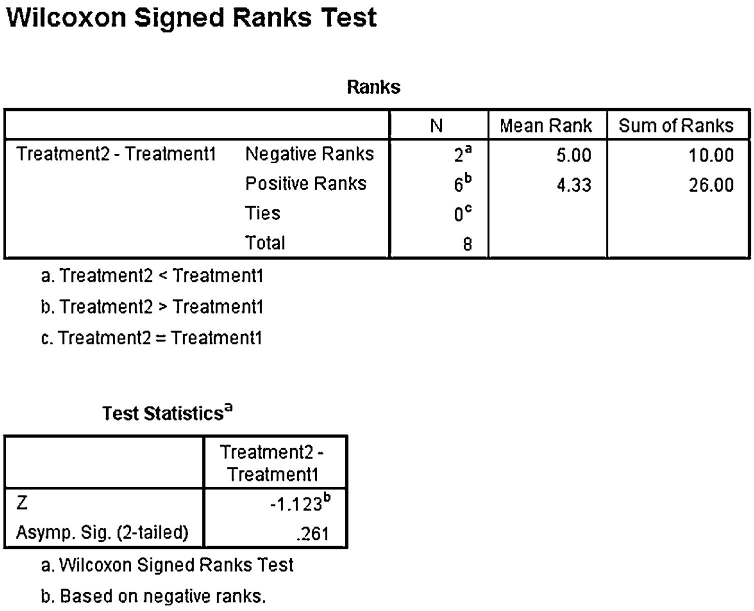
Test Statistics[a]

	companion - alone
Exact Sig. (2-tailed)	.109[b]

a. Sign Test

b. Binomial distribution used.

SPSS OUTPUT 3.6



Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
Treatment2 - Treatment1	Negative Ranks	2[a]	5.00	10.00
	Positive Ranks	6[b]	4.33	26.00
	Ties	0[c]		
	Total	8		

a. Treatment2 < Treatment1

b. Treatment2 > Treatment1

c. Treatment2 = Treatment1

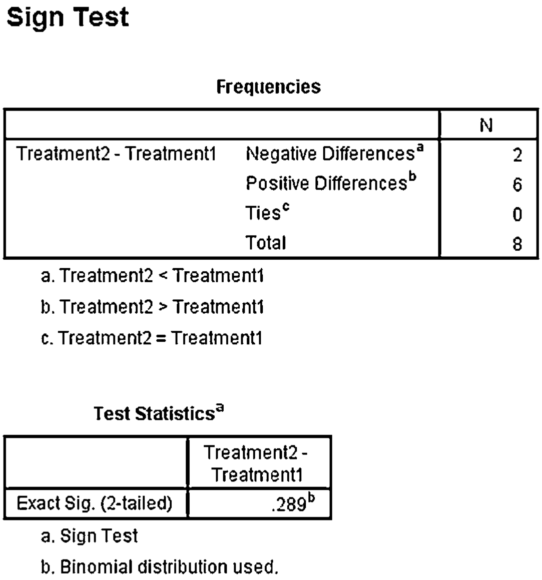
Test Statistics[a]

	Treatment2 - Treatment1
Z	-1.123[b]
Asymp. Sig. (2-tailed)	.261

a. Wilcoxon Signed Ranks Test

b. Based on negative ranks.

SPSS OUTPUT 3.7



Sign Test

Frequencies

		N
Treatment2 - Treatment1	Negative Differences[a]	2
	Positive Differences[b]	6
	Ties[c]	0
	Total	8

a. Treatment2 < Treatment1

b. Treatment2 > Treatment1

c. Treatment2 = Treatment1

Test Statistics[a]

	Treatment2 - Treatment1
Exact Sig. (2-tailed)	.289[b]

a. Sign Test

b. Binomial distribution used.

SPSS OUTPUT 3.8

The results from the sign test ($p > 0.05$) also indicated that the two testing conditions were not significantly different.

Therefore, based on this study, neither treatment program resulted in a significantly higher weight loss among obese female teenagers.

4. The results from the analysis are as follows:

$$T = 50$$

$$x_r = 105 \text{ and } s_r = 26.79$$

$$z^* = -2.05$$

$$ES = 0.46$$

This is a reasonably high ES which indicates a strong measure of association.

5. For our example, $n = 10$ and $p = 0.05/2$. Thus, $T = 8$ and $K = 9$. The ninth value from the bottom is -1.0 and the ninth value from the top is 7.0. Based on these findings, it is estimated with 95% confidence that the difference in students' number of activities before and after the new program lies between -1.0 and 7.0.

CHAPTER 4

COMPARING TWO UNRELATED SAMPLES: THE MANN–WHITNEY U*-TEST AND THE KOLMOGOROV–SMIRNOV TWO-SAMPLE TEST*

4.1 OBJECTIVES

In this chapter, you will learn the following items:

- How to perform the Mann–Whitney *U*-test.
- How to construct a median confidence interval based on the difference between two independent samples.
- How to perform the Kolmogorov–Smirnov two-sample test.
- How to perform the Mann–Whitney *U*-test and the Kolmogorov–Smirnov two-sample test using SPSS®.

4.2 INTRODUCTION

Suppose a teacher wants to know if his first-period's early class time has been reducing student performance. To test his idea, he compares the final exam scores of students in his first-period class with those in his fourth-period class. In this example, each score from one class period is independent, or unrelated, to the other class period.

The Mann–Whitney *U*-test and the Kolmogorov–Smirnov two-sample test are nonparametric statistical procedures for comparing two samples that are independent, or not related. The parametric equivalent to these tests is the *t*-test for independent samples.

In this chapter, we will describe how to perform and interpret a Mann–Whitney *U*-test and a Kolmogorov–Smirnov two-sample test. We will demonstrate both small samples and large samples for each test. We will also explain how to perform the procedure using SPSS. Finally, we offer varied examples of these nonparametric statistics from the literature.

4.3 COMPUTING THE MANN–WHITNEY *U*-TEST STATISTIC

The Mann–Whitney *U*-test is used to compare two unrelated, or independent, samples. The two samples are combined and rank ordered together. The strategy is to determine if the values from the two samples are randomly mixed in the rank ordering or if they are clustered at opposite ends when combined. A random rank ordered would mean that the two samples are not different, while a cluster of one sample's values would indicate a difference between them. In Figure 4.1, two sample comparisons illustrate this concept.

The scores in Comparison 1 are rank ordered in clusters at opposite ends. This suggests that treatment X might be higher than treatment O.	COMPARISON 1 X X X O X X X X O O O O 1 2 3 4 5 6 7 8 9 10 11 12
The scores in Comparison 2 are spread along the entire distribution. This suggests that there is no clear difference between treatments.	COMPARISON 2 X O O X X O X O X O X X 1 2 3 4 5 6 7 8 9 10 11 12

FIGURE 4.1

Use Formula 4.1 to determine a Mann–Whitney *U*-test statistic for each of the two samples. The smaller of the two *U* statistics is the obtained value:

$$U_i = n_1 n_2 + \frac{n_i(n_i + 1)}{2} - \sum R_i \tag{4.1}$$

where U_i is the test statistic for the sample of interest, n_i is the number of values from the sample of interest, n_1 is the number of values from the first sample, n_2 is the number of values from the second sample, and ΣR_i is the sum of the ranks from the sample of interest.

After the U statistic is computed, it must be examined for significance. We may use a table of critical values (see Table B.4 in Appendix B). However, if the numbers of values in each sample, n_i, exceeds those available from the table, then a large sample approximation may be performed. For large samples, compute a z-score and use a table with the normal distribution (see Table B.1 in Appendix B) to obtain a critical region of z-scores. Formula 4.2, Formula 4.3, and Formula 4.4 are used to find the z-score of a Mann–Whitney U-test for large samples:

$$\bar{x}_U = \frac{n_1 n_2}{2} \tag{4.2}$$

where $\bar{x}_U$ is the mean, n_1 is the number of values from the first sample, and n_2 is the number of values from the second sample;

$$s_U = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} \tag{4.3}$$

where s_U is the standard deviation;

$$z^* = \frac{U_i - \bar{x}_U}{s_U} \tag{4.4}$$

where z^* is the z-score for a normal approximation of the data and U_i is the U statistic from the sample of interest.

At this point, the analysis is limited to identifying the presence or absence of a significant difference between the groups and does not describe the strength of the treatment. We can consider the effect size (ES) to determine the degree of association between the groups. We use Formula 4.5 to calculate the ES:

$$ES = \frac{|z|}{\sqrt{n}} \tag{4.5}$$

where $|z|$ is the absolute value of the z-score and n is the total number of observations.

The ES ranges from 0 to 1. Cohen (1988) defined the conventions for ES as *small* = 0.10, *medium* = 0.30, and *large* = 0.50. (Correlation coefficient and ES are both measures of association. See Chapter 7 concerning correlation for more information on Cohen's assignment of ES's relative strength.)

4.3.1 Sample Mann–Whitney *U*-Test (Small Data Samples)

The following data were collected from a study comparing two methods being used to teach reading recovery in the 4th grade. Method 1 was a pull-out program in which the children were taken out of the classroom for 30 min a day, 4 days a week. Method 2 was a small group program in which children were taught in groups of four or five for 45 min a day in the classroom, 4 days a week. The students were tested using a reading comprehension test after 4 weeks of the program. The test results are shown in Table 4.1.

TABLE 4.1

Method 1	Method 2
48	14
40	18
39	20
50	10
41	12
38	102
53	17

4.3.1.1 State the Null and Research Hypotheses

The null hypothesis states that there is no tendency of the ranks of one method to be systematically higher or lower than the other. The hypothesis is stated in terms of comparison of distributions, not means. The research hypothesis states that the ranks of one method are systematically higher or lower than the other. Our research hypothesis is a two-tailed, nondirectional hypothesis because it indicates a difference, but in no particular direction.

The null hypothesis is

> H_O: There is no tendency for ranks of one method to be significantly higher (or lower) than the other.

The research hypothesis is

> H_A: The ranks of one method are systematically higher (or lower) than the other.

4.3.1.2 Set the Level of Risk (or the Level of Significance) Associated with the Null Hypothesis

The level of risk, also called an alpha (α), is frequently set at 0.05. We will use $\alpha = 0.05$ in our example. In other words, there is a 95% chance that any observed statistical difference will be real and not due to chance.

4.3.1.3 Choose the Appropriate Test Statistic

The data are obtained from two independent, or unrelated, samples of 4th-grade children being taught reading. Both the small sample sizes and an existing outlier in the second sample violate our assumptions of normality. Since we are comparing two unrelated, or independent, samples, we will use the Mann–Whitney U-test.

4.3.1.4 Compute the Test Statistic

First, combine and rank both data samples together (see Table 4.2).

Next, compute the sum of ranks for each method. Method 1 is ΣR_1 and method 2 is ΣR_2. Using Table 4.2,

$$\sum R_1 = 7+8+9+10+11+12+13$$

$$\sum R_1 = 70$$

TABLE 4.2

Ordered scores

Rank	Score	Sample
1	10	Method 2
2	12	Method 2
3	14	Method 2
4	17	Method 2
5	18	Method 2
6	20	Method 2
7	38	Method 1
8	39	Method 1
9	40	Method 1
10	41	Method 1
11	48	Method 1
12	50	Method 1
13	53	Method 1
14	102	Method 2

and

$$\sum R_2 = 1+2+3+4+5+6+14$$
$$\sum R_2 = 35$$

Now, compute the U-value for each sample. For sample 1,

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - \sum R_1 = 7(7) + \frac{7(7+1)}{2} - 70 = 49 + 28 - 70$$
$$U_1 = 7$$

and for sample 2,

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - \sum R_2 = 7(7) + \frac{7(7+1)}{2} - 35 = 49 + 28 - 35$$
$$U_2 = 42$$

The Mann–Whitney U-test statistic is the smaller of U_1 and U_2. Therefore, $U = 7$.

4.3.1.5 Determine the Value Needed for Rejection of the Null Hypothesis Using the Appropriate Table of Critical Values for the Particular Statistic

Since the sample sizes are small ($n < 20$), we use Table B.4 in Appendix B, which lists the critical values for the Mann–Whitney U. The critical values are found on the table at the point for $n_1 = 7$ and $n_2 = 7$. We set $\alpha = 0.05$. The critical value for the Mann–Whitney U is 8. A calculated value that is less than or equal to 8 will lead us to reject our null hypothesis.

4.3.1.6 Compare the Obtained Value with the Critical Value The critical value for rejecting the null hypothesis is 8 and the obtained value is $U = 7$. If the critical value equals or exceeds the obtained value, we must reject the null hypothesis. If instead, the critical value is less than the obtained value, we must not reject the null hypothesis. Since the critical value exceeds the obtained value, we must reject the null hypothesis.

4.3.1.7 Interpret the Results We rejected the null hypothesis, suggesting that a real difference exists between the two methods. In addition, since the sum of the ranks for method 1 (ΣR_1) was larger than method 2 (ΣR_2), we see that method 1 had significantly higher scores.

4.3.1.8 Reporting the Results The reporting of results for the Mann–Whitney U-test should include such information as the sample sizes for each group, the U statistic, the p-value's relation to α, and the sums of ranks for each group.

For this example, two methods were used to provide students with reading instruction. Method 1 involved a pull-out program and method 2 involved a small group program. Using the ranked reading comprehension test scores, the results indicated a significant difference between the two methods ($U = 7$, $n_1 = 7$, $n_2 = 7$, $p < 0.05$). The sum of ranks for method 1 ($\Sigma R_1 = 70$) was larger than the sum of ranks for method 2 ($\Sigma R_2 = 35$). Therefore, we can state that the data support the pull-out program as a more effective reading program for teaching comprehension to 4th-grade children at this school.

4.3.2 Confidence Interval for the Difference between Two Location Parameters

The American Psychological Association (2001) has suggested that researchers report the *confidence interval* for research data. A confidence interval is an inference to a population in terms of an estimation of sampling error. More specifically, it provides a range of values that fall within the population with a level of confidence of $100(1 - \alpha)\%$.

A median confidence interval can be constructed based on the difference between two independent samples. It consists of possible values of differences for which we do not reject the null hypothesis at a defined significance level of α.

The test depends on the following assumptions:

1. Data consist of two independent random samples: $X_1, X_2, \ldots, X_n$ from one population and $Y_1, Y_2, \ldots, Y_n$ from the second population.
2. The distribution functions of the two populations are identical except for possible location parameters.

To perform the analysis, set up a table that identifies all possible differences for each possible sample pair such that $D_{ij} = X_i - Y_j$ for (X_i, Y_j). Placing the values for X from smallest to largest across the top and the values for Y from smallest to largest down the side will eliminate the need to order the values of D_{ij} later.

TABLE 4.3

	X_i						
Y_j	38	39	40	41	48	50	53
10	28	29	30	31	38	40	43
12	26	27	28	29	36	38	41
14	24	25	26	27	34	36	39
17	21	22	23	24	31	33	36
18	20	21	22	23	30	32	35
20	18	19	20	21	28	30	33
102	−64	−63	−62	−61	−54	−52	−49

The sample procedure to be presented later is based on the data from Table 4.2 (small data sample Mann–Whitney *U*-test) near the beginning of this chapter.

The values from Table 4.2 are arranged in Table 4.3 so that the method 1 (X) scores are placed in order across the top and the method 2 (Y) scores are placed in order down the side. Then, the n_1n_2 differences are calculated by subtracting each Y value from each X value. The differences are shown in Table 4.3. Notice that the values of D_{ij} are ordered in the table from highest to lowest starting at the top right and ending at the bottom left.

We use Table B.4 in Appendix B to find the lower limit of the confidence interval, L, and the upper limit U. For a two-tailed test, L is the $w_{\alpha/2}$th smallest difference and U is the $w_{\alpha/2}$th largest difference that correspond to $\alpha/2$ for n_1 and n_2 for a confidence interval of $(1 - \alpha)$.

For our example, $n_1 = 7$ and $n_2 = 7$. For $\alpha/2 = 0.05/2 = 0.025$, Table B.4 returns $w_{\alpha/2} = 9$. This means that the ninth values from the top and bottom mark the limits of the 95% confidence interval on both ends. Therefore, $L = 19$ and $U = 36$. Based on these results, we are 95% certain that the difference in population median is between 18 and 36.

4.3.3 Sample Mann–Whitney *U*-Test (Large Data Samples)

The previous comparison of teaching methods for reading recovery was repeated with 5th-grade students. The 5th-grade used the same two methods. Method 1 was a pull-out program in which the children were taken out of the classroom for 30 min a day, 4 days a week. Method 2 was a small group program in which children were taught in groups of four or five for 45 min a day in the classroom, 4 days a week. The students were tested using the same reading comprehension test after 4 weeks of the program. The test results are shown in Table 4.4.

4.3.3.1 State the Null and Research Hypotheses The null hypothesis states that there is no tendency of the ranks of one method to be systematically higher or lower than the other. The hypothesis is stated in terms of comparison of distributions, not means. The research hypothesis states that the ranks of one method are

TABLE 4.4

Method 1	Method 2
48	14
40	18
39	20
50	10
41	12
38	102
71	21
30	19
15	100
33	23
47	16
51	82
60	13
59	25
58	24
42	97
11	28
46	9
36	34
27	52
93	70
72	22
57	26
45	8
53	17

systematically higher or lower than the other. Our research hypothesis is a two-tailed, nondirectional hypothesis because it indicates a difference, but in no particular direction.

The null hypothesis is

> H_O: There is no tendency for ranks of one method to be significantly higher (or lower) than the other.

The research hypothesis is

> H_A: The ranks of one method are systematically higher (or lower) than the other.

4.3.3.2 Set the Level of Risk (or the Level of Significance) Associated with the Null Hypothesis The level of risk, also called an alpha (α), is frequently set at 0.05. We will use $\alpha = 0.05$ in our example. In other words, there is a 95% chance that any observed statistical difference will be real and not due to chance.

4.3.3.3 Choose the Appropriate Test Statistic The data are obtained from two independent, or unrelated, samples of 5th-grade children being taught reading. Since we are comparing two unrelated, or independent, samples, we will use the Mann−Whitney *U*-test.

4.3.3.4 Compute the Test Statistic First, combine and rank both data samples together (see Table 4.5). Next, compute the sum of ranks for each method. Method 1 is ΣR_1 and method 2 is ΣR_2. Using Table 4.5,

TABLE 4.5

Ordered scores

Rank	Score	Sample
1	8	Method 2
2	9	Method 2
3	10	Method 2
4	11	Method 1
5	12	Method 2
6	13	Method 2
7	14	Method 2
8	15	Method 1
9	16	Method 2
10	17	Method 2
11	18	Method 2
12	19	Method 2
13	20	Method 2
14	21	Method 2
15	22	Method 2
16	23	Method 2
17	24	Method 2
18	25	Method 2
19	26	Method 2
20	27	Method 1
21	28	Method 2
22	30	Method 1
23	33	Method 1
24	34	Method 2
25	36	Method 1
26	38	Method 1
27	39	Method 1
28	40	Method 1
29	41	Method 1
30	42	Method 1
31	45	Method 1

(*Continued*)

TABLE 4.5 (*Continued*)

Ordered scores

Rank	Score	Sample
32	46	Method 1
33	47	Method 1
34	48	Method 1
35	50	Method 1
36	51	Method 1
37	52	Method 2
38	53	Method 1
39	57	Method 1
40	58	Method 1
41	59	Method 1
42	60	Method 1
43	70	Method 2
44	71	Method 1
45	72	Method 1
46	82	Method 2
47	93	Method 1
48	97	Method 2
49	100	Method 2
50	102	Method 2

$$\sum R_1 = 779$$

and

$$\sum R_2 = 496$$

Now, compute the U-value for each sample. For sample 1,

$$\begin{aligned} U_1 &= n_1 n_2 + \frac{n_1(n_1+1)}{2} - \sum R_1 \\ &= 25(25) + \frac{25(25+1)}{2} - 779 = 625 + 325 - 779 \\ U_1 &= 171 \end{aligned}$$

and for sample 2,

$$\begin{aligned} U_2 &= n_1 n_2 + \frac{n_2(n_2+1)}{2} - \sum R_2 \\ &= 25(25) + \frac{25(25+1)}{2} - 496 = 625 + 325 - 496 \\ U_2 &= 454 \end{aligned}$$

The Mann–Whitney U-test statistic is the smaller of U_1 and U_2. Therefore, $U = 171$.

Since our sample sizes are large, we will approximate them to a normal distribution. Therefore, we will find a z-score for our data using a normal approximation. We must find the mean $\overline{x}_U$ and the standard deviation s_U for the data:

$$\overline{x}_U = \frac{n_1 n_2}{2} = \frac{(25)(25)}{2}$$
$$\overline{x}_U = 312.5$$

and

$$s_U = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} = \sqrt{\frac{(25)(25)(25+25+1)}{12}} = \sqrt{\frac{31{,}875}{12}}$$
$$s_U = 51.54$$

Next, we use the mean, standard deviation, and the U-test statistic to calculate a z-score. Remember, we are testing the hypothesis that there is no difference in the ranks of the scores for two different methods of reading instruction for 5th-grade students:

$$z^* = \frac{U_i - \overline{x}_U}{s_U} = \frac{171 - 312.5}{51.54}$$
$$z^* = -2.75$$

4.3.3.5 Determine the Value Needed for Rejection of the Null Hypothesis Using the Appropriate Table of Critical Values for the Particular Statistic

Table B.1 in Appendix B is used to establish the critical region of z-scores. For a two-tailed test with $\alpha = 0.05$, we must not reject the null hypothesis if $-1.96 \leq z^* \leq 1.96$.

4.3.3.6 Compare the Obtained Value with the Critical Value

We find that z^* is not within the critical region of the distribution, $-2.75 < -1.96$. Therefore, we reject the null hypothesis. This suggests a difference between method 1 and method 2.

4.3.3.7 Interpret the Results

We rejected the null hypothesis, suggesting that a real difference exists between the two methods. In addition, since the sum of the ranks for method 1 (ΣR_1) was larger than method 2 (ΣR_2), we see that method 1 had significantly higher scores.

At this point, the analysis is limited to identifying the presence or absence of a significant difference between the groups. In other words, the statistical test's level of significance does not describe the strength of the treatment. The American Psychological Association (2001), however, has called for a measure of the strength called the *effect size*.

We can consider the ES for this large sample test to determine the degree of association between the groups. We can use Formula 4.5 to calculate the ES. For the example, $z = -2.75$ and $n = 50$: