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THERMODYNAMICS OF GEOTHERMAL POWER PRODUCTION - REVIEW

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ABSTRACT

The following text is covering the basic principles of thermodynamics to be treated in the lectures "Thermodynamics of geothermal power production cycles and cycle selection" and "Exergy, efficiency and efficiency improvements by combining cycles".

1. ENERGY AND POWER, HEAT AND WORK

The production of electricity from a geothermal source is about producing work from heat. Electricity production from heat will never be successful unless appropriate respect is paid to the second law of thermodynamics.

Energy is utilized in two forms, as heat and as work. Work moves bodies, changes their form, but heat changes temperature (changes the molecular random kinetic energy). Work is thus the ordered energy, whereas heat is the random "unorganized" energy. Heat and work are totally different products for a power station, but these two energy forms cannot be produced independent of each other. Independent production of heat and work is in a way similar to have cattle producing three hind legs per animal when required.

It is as well appropriate to discuss the relation between power and energy right here in the introduction to this chapter. A power station is built to be able to supply certain maximum power. The source heat supply and the design of the power plant internals are based on this maximum power. On the other hand the income of the power station will be depending on the energy sold, on the integral of produced power with respect to time.

Geothermal installations have normally zero energy cost. The inflow into the well is not charged for. The only cost is the investment cost in equipment and installations to get the fluid to the surface, and to process it appropriately in the power plant in order to obtain the product, be it heat for a direct use application or an electricity producing power plant.

As a consequence of this, a geothermal power plant (Figure 1) is a typical base load plant, the bulk portion of the cost is there regardless of how much power the plant is producing. Duration curves and utilization time will be discussed later in this chapter.

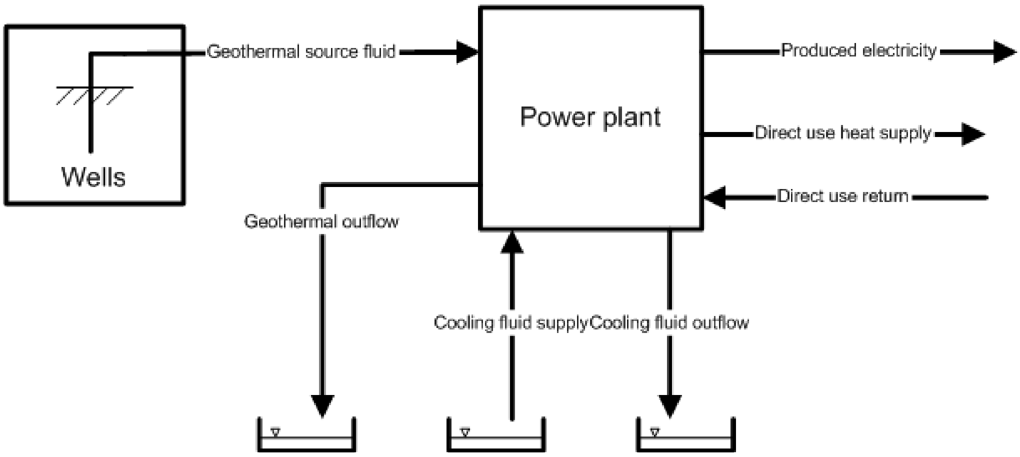

FIGURE 1: Schematic of a geothermal power plant

2. CONVERSION OF HEAT TO WORK

Work can always be changed into heat. Even during the Stone Age, work was used to light fire by friction, by rubbing wood sticks to a hard surface. The same applies today, the electric heater is converting work into heat with 100% efficiency.

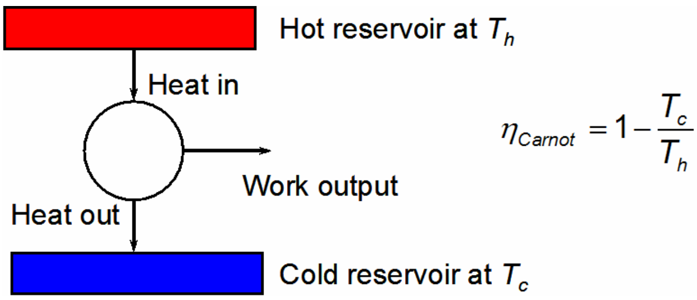

FIGURE 2: The Carnot engine

Conversion of heat into work is difficult and is limited by the laws of thermodynamics. A part of the heat used has always to be rejected to the surroundings, so there is always an upper limit of the possible work production from a given heat stream.

Textbooks use the Reversible Heat Engine (RHE, Carnot engine) as a reference (Figure 2). RHE is the best engine for producing work from heat, assuming that the engine is operating between two infinitely large heat reservoirs. The reference to the Carnot engine has to be taken with caution, as the real heat reservoirs are usually not infinitely large, and the heat supply or rejection will happen at a variable temperature.

2.1 Exergy

The second law of thermodynamics demands that a part of the heat input to any heat engine is rejected to the environment. The portion of the input heat, which can be converted into work, is called *exergy* (availability, convertible energy**)**. The unconvertible portion is called *anergy*. Thus the exergy of any system or flow stream is equal to the maximum work (or electricity) which can be produced from the source. The thermodynamic definition of exergy for a flow stream is:

$$x = h - h_0 - T_0\left(s - s_0\right) \qquad (1)$$

The zero index refers to the environmental conditions for the subject conversion. The local environment for the power plant defines the available cold heat reservoir, and all the anergy rejected tot the environment will finally be at the environmental conditions.

The exergy of a flow stream is thus the maximum theoretical work which can be produced if the stream is subjected to a process bringing it down to the environmental conditions.
If the stream is a liquid with constant heat capacity, the above equation can be written as:

$$x = c_{liquid}\left[\left(T - T_0\right) - T_0 \ln\left(\frac{T}{T_0}\right)\right] \tag{2}$$

Economics of power production are conveniently analyzed by using exergy. A power plant has the main purpose of converting heat into work, and therefore the relevant physical variable for cost and economic performance calculation is the exergy rather than the total energy or the heat flow.

2.2 Efficiency definitions

Efficiency is the ratio of input to output, a performance measure for the process. There are many possibilities of defining input and output, but the most standard definition of efficiency is the power plant thermal efficiency.

The following schematic (Figure 3) shows the energy streams for a binary power plant.

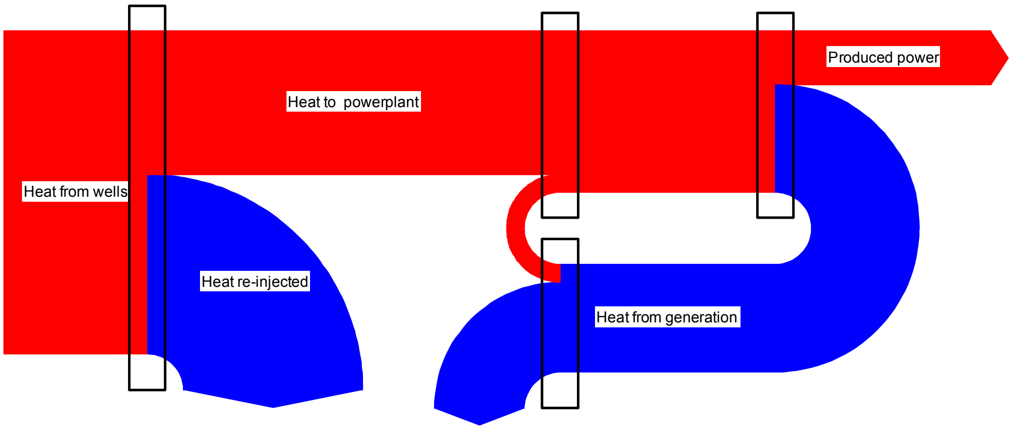

FIGURE 3: Energy streams in a binary power plant

The thermal efficiency is seen as the ratio of produced power to the heat transferred to the power plant. The effectiveness is the ratio of the heat transferred to the power plant to the heat available from the wells. It is obvious that the total power plant efficiency will be the multiple of power plant efficiency and effectiveness.

2.3 Power plant thermal efficiency

The power plant thermal efficiency is the ratio between power produced and the heat flow to the power plant. The power plant thermal efficiency is traditionally defined as:

$$\eta_{th} = \frac{\dot{W}}{\dot{Q}_{in}} \tag{3}$$

The heat input is then the heat input to the power plant, and takes no notice of how much heat is available from the wells. This can be very misleading. The wells make up a great portion of the power plant cost, and the economics of the power plant will be decided by the utilization of the well investment. The Carnot efficiency is as well misleading, it is based on the assumption that the thermal reservoirs are infinitely large, no cooling will occur in the hot reservoir by heat removal, and no heating will be in the cold reservoir by heat addition. Therefore the only relevant performance measure will have to be based on the exergetic efficiency, and due to the importance of the well investment, the effectiveness as well.

Calculating efficiency based on the first law for a cogeneration power plant is in no way easy, because the plant has two products, heat and work. The first law does not provide any equivalence between heat and work, or the value of these products. A cogeneration plant will only be analyzed properly by exergetic analysis.

If the power plant effectiveness is high, the geothermal fluid return temperature is low, and the average temperature of the heat input to the power plant is low. This will lead to lower efficiency, but larger power plant. If the well flow is given, then high effectiveness will lead to a plant with higher power, but lower first law efficiency.

2.4 Example

Assume that an ideal power with 100% isentropic efficiency plant has a source of 120°C and 150 kg/s flow. The cooling water is assumed to enter the power plant at 10°C and leave the plant at 20°C. Let's assume that the first ideal power plant (Figures 4 and 5) is able to cool the geothermal fluid down to 80°C.

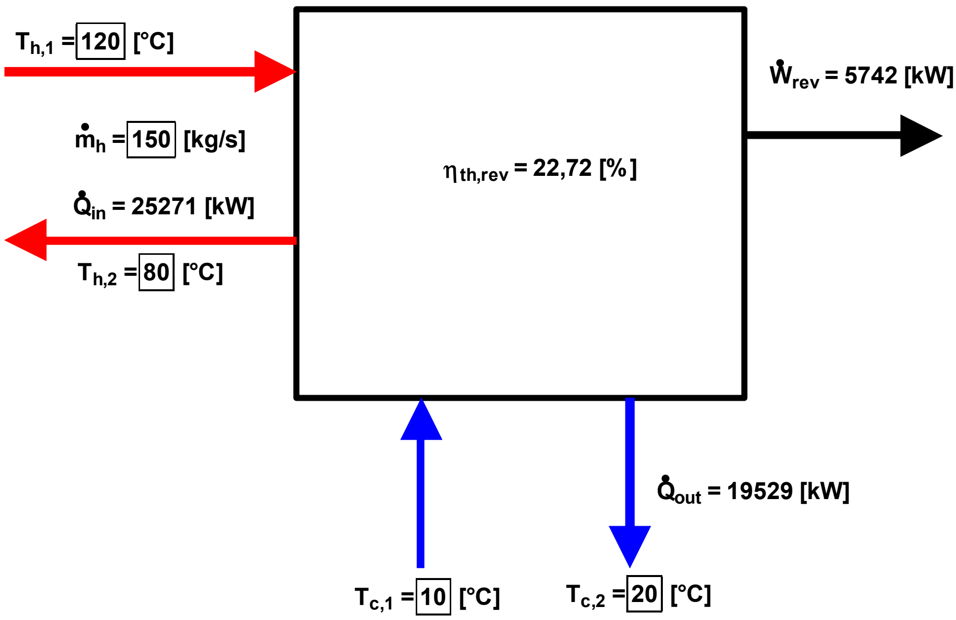

FIGURE 4: A low effectiveness ideal power plant

The obtained power is 5.7 MW, efficiency is 22.7%. What will happen if the effectiveness is doubled, and the geothermal return water temperature is brought down to 40°C?

The efficiency falls down to 18.1%, but the output power is increased to 9.1 MW. It is very obvious that the power plant with lower efficiency, but higher effectiveness is more powerful and will be more economic, at least if the technical design limitations do not hurt too badly.

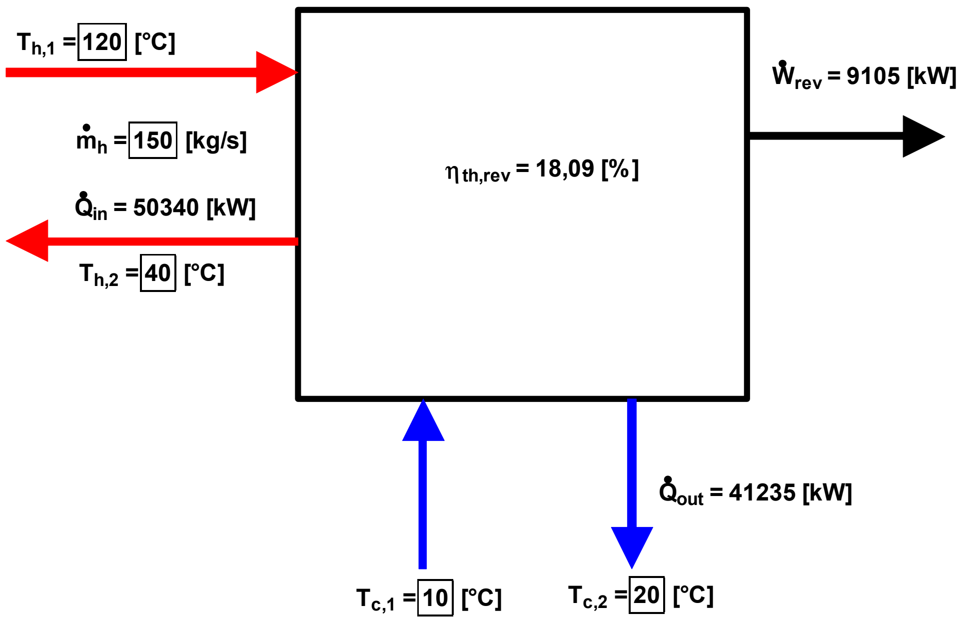

FIGURE 5: A high effectiveness ideal power plant

The general relation between output power and efficiency for this example are given in the diagram in Figure 6.

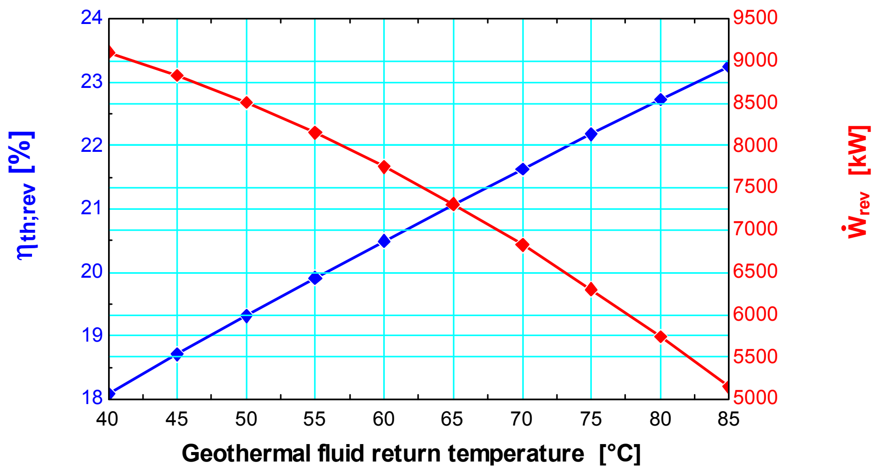

FIGURE 6: Net power and efficiency as a function of re-injection temperature

2.5 Effectiveness

The power plant effectiveness is the ratio between the available energy to the energy input to the power plant. The available energy is found by assuming that the geothermal fluid can be cooled down to the environmental conditions. Effectiveness will be the deciding factor for the possible power plant size, rather than the quality of the power plant.

2.6 Second law efficiency and effectiveness

Exergy is the portion of the energy which can theoretically be converted into work. It is logical to base performance criteria for production of electricity on exergy rather than heat or energy, because then the performance calculation will take into account what can be done, and not incorporate any "perpetuum mobile" in the calculations.

The second law approach makes as well easy to treat cogeneration. Then the exergy stream in the sold heat is treated in the same way as the produced electrical power, having the same exergy unitary cost.

3. ANALYSIS

Efficiency is the ratio of benefit to cost. In order to be able to define efficiency, the inputs (cost) and outputs have to be defined. In a low temperature heat conversion process, two cases regarding the stream $\dot{m}_s$ are possible, depending on if the heat contained in that stream can be sold to a heat consuming process.

The conversion efficiency is a measure of how much of the available heat is converted into work. It has to be kept in mind that only a part of the heat can be converted into work due to the limitations imposed by the second law of thermodynamics. Exergy, the potential of any system to produce work, is the correct property to consider, when the conversion efficiency is analyzed. Exergy is dependent of the properties of the source as well as the properties of the environment, where the environmental temperature and pressure are the main properties.

The temperature of the entering cooling fluid is taken to be the environmental temperature, the lowest temperature which can be obtained, as well as defining the thermal sink temperature for the Carnot engine efficiency. The environmental pressure is logically the ambient atmospheric pressure.

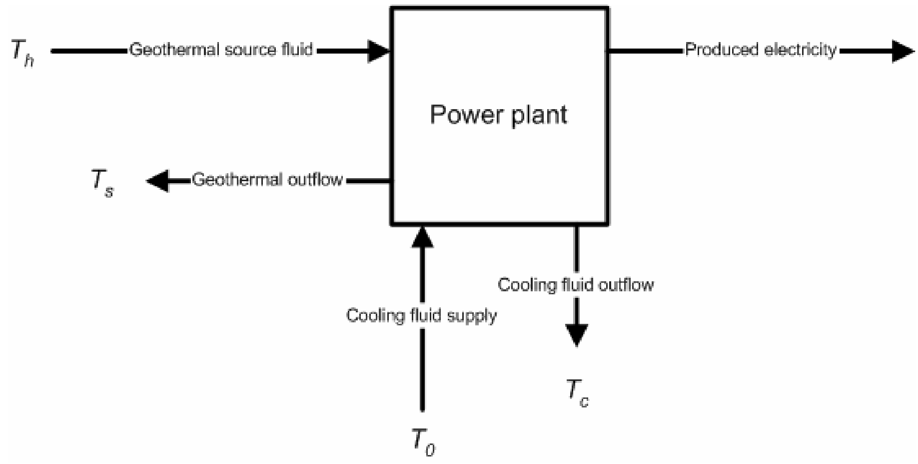

FIGURE 7: Electrical power plant schematic

This process can be seen as a non-conserving heat exchange process between the source stream and the cooling fluid stream. Figure 7 is a block diagram of a power plant converting heat into electricity.

The variables related to the conversion are as follows:

c_h = Source fluid heat capacity;
m_h = Flow rate of source fluid;
T_h = Source fluid inlet temperature;
T_s = Source fluid outlet temperature;
c_c = Cooling fluid heat capacity;
m_c = Cooling fluid flow rate;
T_c = Cooling fluid outlet temperature; and
T_0 = Cooling fluid inlet temperature (environmental temperature).

In the following this system will be analyzed in order to gain a better understanding of the conversion of low temperature heat into electricity. It is assumed that the geothermal source fluid is liquid water with constant heat capacity.

The streams in and out_of the system have four flow properties: mass, heat capacity, enthalpy and exergy. The mass conservation is obvious, no mixing of the source and cooling streams is assumed. The heat capacity is important for the characteristics of the heat conversion, and will be treated here as a heat capacity flow, the product of fluid heat capacity and flow rate. The product of the enthalpy relative to the environmental temperature and the flow rate defines the heat flow in and out of the system. The exergy will give information on the work producing potential of the system, and is calculated in the same way as the enthalpy. Reference textbooks such as Cengel (2002) give basic information on exergy and its definition, but here the analysis is as well based on Kotas (1985) and Szargut (1988). Thórólfsson (2002), Valdimarsson (2002) and Dorj (2005) apply these methods on specific geothermal applications.

The heat ($\dot{Q}$) and exergy ($\dot{X}$) flows are given by:

$$\dot{Q}_h = c_h \dot{m}_h \left(T_h - T_0\right) \tag{4}$$

$$\dot{Q}_s = c_h \dot{m}_h \left(T_s - T_0\right) \tag{5}$$

$$\dot{Q}_c = c_c \dot{m}_c \left(T_c - T_0\right) \tag{6}$$

$$\dot{X}_h = c_h \dot{m}_h \left[\left(T_h - T_0\right) - T_0 \ln\left(\frac{T_h}{T_0}\right) \right] = \dot{Q}_h - c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_0}\right) \tag{7}$$

$$\dot{X}_s = c_h \dot{m}_h \left[\left(T_s - T_0\right) - T_0 \ln\left(\frac{T_s}{T_0}\right) \right] = \dot{Q}_s - c_h \dot{m}_h T_0 \ln\left(\frac{T_s}{T_0}\right) \tag{8}$$

$$\dot{X}_c = c_c \dot{m}_c \left[\left(T_c - T_0\right) - T_0 \ln\left(\frac{T_c}{T_0}\right) \right] = \dot{Q}_c - c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right) \tag{9}$$

The energy (1. law) and exergy (2. law) balances are:

$$\dot{Q}_h - \dot{Q}_s - \dot{Q}_c = \dot{W} \tag{10}$$

$$\dot{X}_h - \dot{X}_s - \dot{X}_c = \dot{W}_{rev} \quad \text{or}$$

$$\dot{Q}_h - \dot{Q}_s - \dot{Q}_c - c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) + c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right) = \dot{W}_{rev} \tag{11}$$

The energy balance is valid for all processes, ideal and real. The exergy balance gives only information on the reversible work, or the largest amount of work that can be obtained from the power plant.

If the power plant is ideal, then:

$$\dot{W}_{rev} = \dot{W} \quad or \quad -c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) + c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right) = 0 \tag{12}$$

Then the heat capacity flow ratio for a reversible power plant is:

$$C_{rev} = \left.\frac{c_c \dot{m}_c}{c_h \dot{m}_h}\right|_{rev} = \frac{\ln\left(\frac{T_h}{T_s}\right)}{\ln\left(\frac{T_c}{T_0}\right)} \tag{13}$$

Assume that electricity is the only output of the power plant. The heat contained in the stream $\dot{m}_s$ is rejected to the surroundings.

$$\begin{aligned} \text{Product:} \quad & \dot{W} \\ \text{Input:} \quad & \dot{Q}_h \\ \text{Rejected:} \quad & \dot{Q}_s \textit{ and } \dot{Q}_c \end{aligned}$$

First law efficiency:

$$\eta_{I,E} = \frac{\dot{W}}{\dot{Q}_h} = \frac{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c}{\dot{Q}_h} = \frac{(T_h - T_s) - C(T_c - T_0)}{T_h - T_0} \tag{14}$$

First law maximum efficiency:

$$\begin{aligned} \eta_{I,\max,E} &= \frac{\dot{W}_{rev}}{\dot{Q}_h} = \frac{\dot{X}_h - \dot{X}_s - \dot{X}_c}{\dot{Q}_h} = 1 - \frac{\dot{Q}_s + \dot{Q}_c + c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) - c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right)}{\dot{Q}_h} \\ &= \frac{(T_h - T_s) - T_0 \ln\left(\frac{T_h}{T_s}\right)}{(T_h - T_0)} - C \frac{(T_c - T_0) - T_0 \ln\left(\frac{T_c}{T_0}\right)}{(T_h - T_0)} \\ &= \frac{\ln\left(\frac{T_c}{T_0}\right)(T_h - T_s) - \ln\left(\frac{T_h}{T_s}\right)(T_c - T_0)}{\ln\left(\frac{T_c}{T_0}\right)(T_h - T_0)} \end{aligned} \tag{15}$$

Second law efficiency:

$$\begin{aligned} \eta_{II,E} &= \frac{\dot{W}}{\dot{W}_{rev}} = \frac{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c}{\dot{X}_h - \dot{X}_s - \dot{X}_c} = \frac{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c}{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c - c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) + c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right)} \\ &= \frac{(T_h - T_s) - C(T_c - T_0)}{(T_h - T_s) - T_0 \ln\left(\frac{T_h}{T_s}\right) - C\left((T_c - T_0) - T_0 \ln\left(\frac{T_c}{T_0}\right)\right)} \end{aligned} \tag{16}$$

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