Chapter 2

Force Vectors

Chapter Objectives

- To show how to add forces and resolve them into components using the Parallelogram Law.
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction.
- To introduce the dot product in order to determine the angle between two vectors or the projection of one vector onto another.

2-1 Scalars and Vectors

All physical quantities in engineering mechanics are measured using either scalars or vectors.

Scalar. A scalar is any positive or negative physical quantity that can be completely specified by its magnitude. Examples of scalar quantities include length, mass, and time.

Vector. A vector is any physical quantity that requires both a magnitude and a direction for its complete description. Examples of vectors encountered in statics are force, position, and moment. A vector is shown graphically by an arrow. The length of the arrow represents the magnitude of the vector, and the angle θ between the vector and a fixed axis defines the direction of its line of action. The head or tip of the arrow indicates the sense of direction of the vector, Fig. 2–1.

In print, vector quantities are represented by boldface letters such as **A,** and the magnitude of a vector is italicized, *A*. For handwritten work, it is often convenient to denote a vector quantity by simply drawing an arrow above it $\vec{A}$.

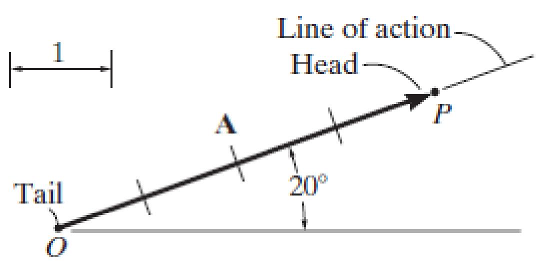


Fig. 2–1

2-2 Vector Operations

Multiplication and Division of a Vector by a Scalar. If a vector is multiplied by a positive scalar, its magnitude is increased by that amount. Multiplying by a negative scalar will also change the directional sense of the vector. Graphic examples of these operations are shown in Fig. 2–2.

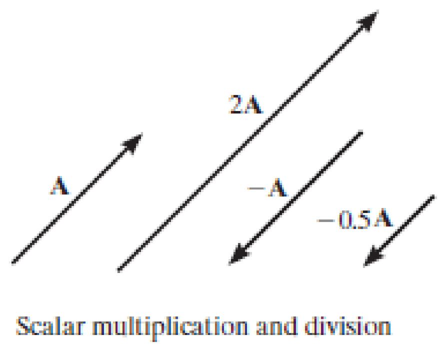


Fig. 2–2

Vector Addition. All vector quantities obey the *parallelogram law of addition.* To illustrate, the two "component" *vectors* **A** and **B** in Fig. 2–3*a* are added to form a "resultant" *vector* **R** = **A** + **B** using the following procedure:

• First join the tails of the components at a point to make them concurrent, Fig. 2–3 b.

• From the head of **B,** draw a line parallel to **A.** Draw another line from the head of **A** that is parallel to **B.** These two lines intersect at point *P* to form the adjacent sides of a parallelogram.

• The diagonal of this parallelogram that extends to *P* forms **R,** which then represents the resultant vector **R** = **A** + **B,** Fig. 2–3 *c.*

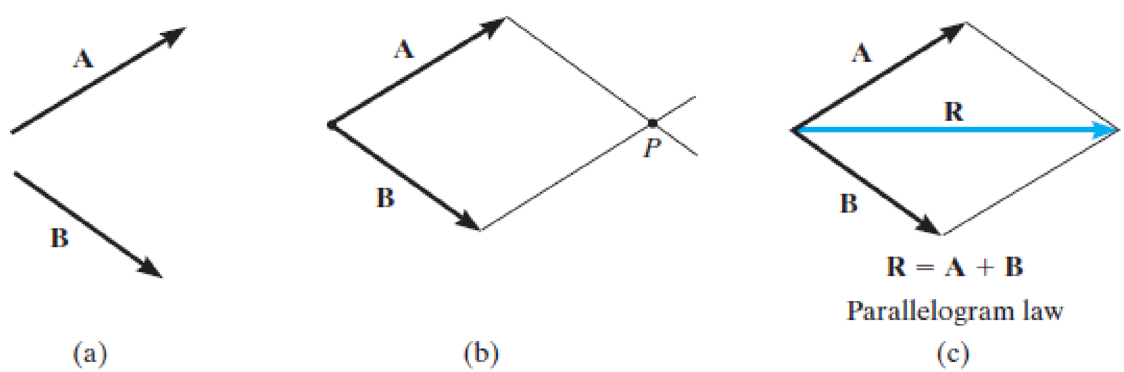
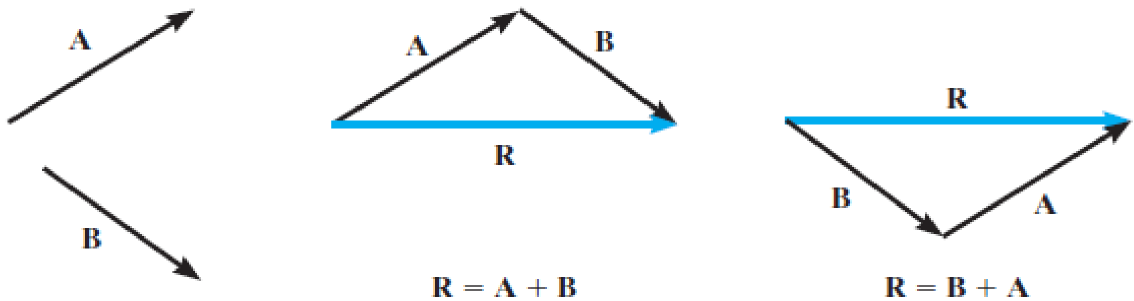


Fig. 2–3

We can also add B to A , Fig. 2–4 a , using the triangle rule , which is a special case of the parallelogram law, whereby vector B is added to vector A in a “head-to-tail” fashion, i.e., by connecting the head of A to the tail of B , Fig. 2–4 b . The resultant R extends from the tail of A to the head of B. In a similar manner, R can also be obtained by adding A to B, Fig. 2–4 c. By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either order, i.e.

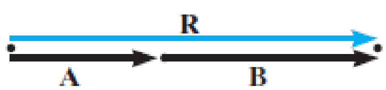
R = A + B = B + A.



R = A + B Triangle rule (b) | R = B + A Triangle rule (c) | (a)

Fig. 2–4

As a special case, if the two vectors **A** and **B** are *collinear,* i.e., both have the same line of action, the parallelogram law reduces to an *algebraic* or *scalar addition* $R = A + B$, as shown in Fig. 2–5.



$R = A + B$

Addition of collinear vectors

Fig. 2–5

Vector Subtraction. The resultant of the *difference* between two vectors **A** and **B** of the same type may be expressed as

$$\boldsymbol{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \qquad (2\text{-}1)$$

This vector sum is shown graphically in Fig. 2–6. Subtraction is therefore defined as a special case of addition, so the rules of vector addition also apply to vector subtraction.

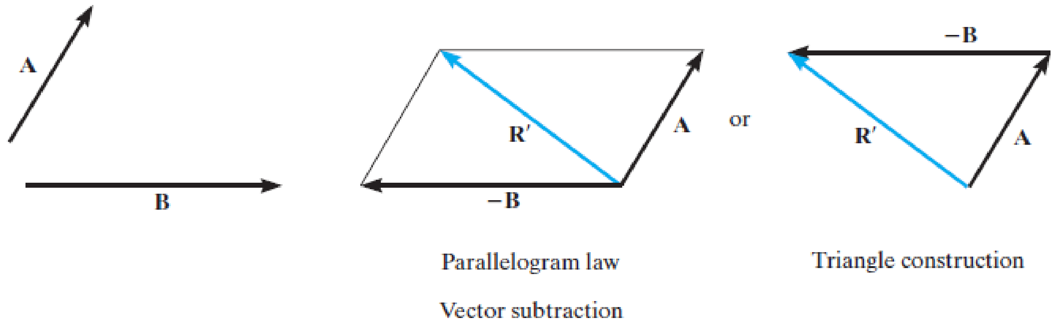


Fig. 2–6

2.3 Vector Addition of Forces

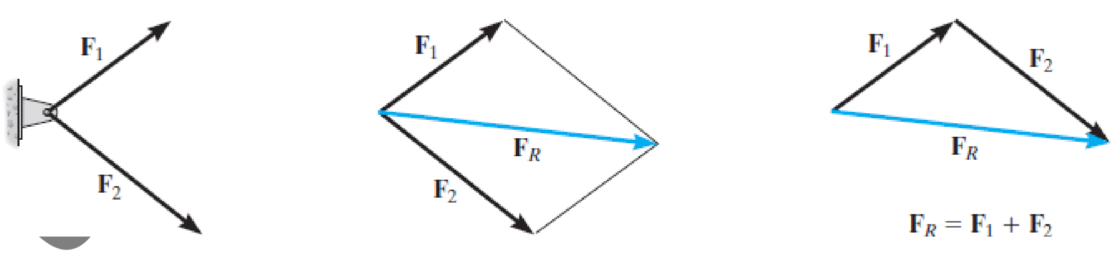
Experimental evidence has shown that a force is a vector quantity since it has a specified magnitude, direction, and sense and it adds according to the parallelogram law. Two common problems in statics involve either finding the resultant force, knowing its components, or resolving a known force into two components.

Finding a Resultant Force.

The two component forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the pin in Fig. 2–7*a* can be added together to form the resultant force, as shown in Fig. 2–7*b*.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 \qquad (2\text{-}2)$$

From this construction, or using the triangle rule, Fig. 2–7*c*, we can apply the law of cosines or the law of sines to the triangle in order to obtain the magnitude of the resultant force and its direction.



(a) **(b)** **(c)**

Fig. 2-7

Finding the Components of a Force.

Sometimes it is necessary to resolve a force into two *components* in order to study its pushing effect in two specific directions. For example, in Fig. 2–8*a*, **F** is to be resolved into two components along the two members, defined by the u and v axes. In order to determine the magnitude of each component, a parallelogram is constructed first, by drawing lines starting from the tip of **F**, one line parallel to u, and the other line parallel to v. These lines then intersect with the v and u axes, forming a parallelogram. The force components $\mathbf{F}_u$ and $\mathbf{F}_v$ are then established by simply joining the tail of **F** to the intersection points on the u and v axes, Fig. 2–8*b*. This parallelogram can then be reduced to a triangle, which represents the triangle rule, Fig. 2–8*c*. From this, the law of sines can then be applied to determine the unknown magnitudes of the components.

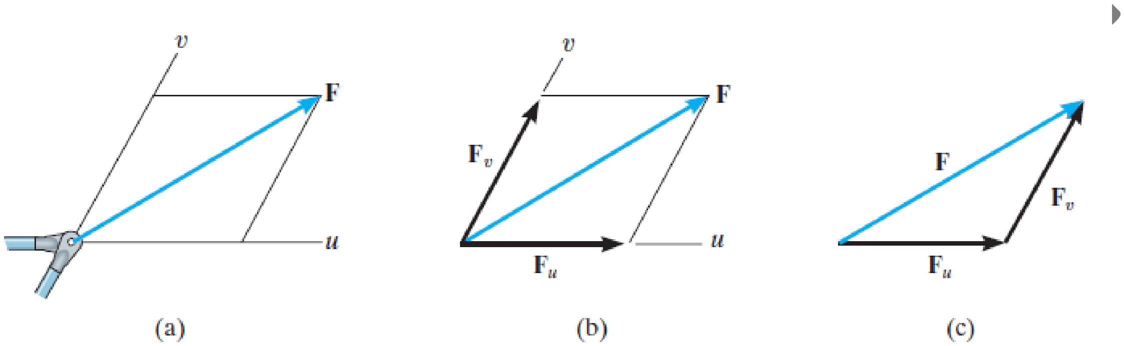


Fig. 2-8

Addition of Several Forces.

If more than two forces are to be added, successive applications of the parallelogram law can be carried out in order to obtain the resultant force. For example, if three forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ act at a point *O*, Fig. 2–9, the resultant of any two of the forces is found, say, $\mathbf{F}_1 + \mathbf{F}_2$ —and then this resultant is added to the third force, yielding the resultant of all three forces; i.e.,

$$\mathbf{F}_R = (F_1 + \mathbf{F}_2) + \mathbf{F}_3 \quad (2\text{-}3)$$

Using the parallelogram law to add more than two forces, as shown here, often requires extensive geometric and trigonometric calculation to determine the numerical values for the magnitude and direction of the resultant. Instead, problems of this type are easily solved by using the “rectangular component method.

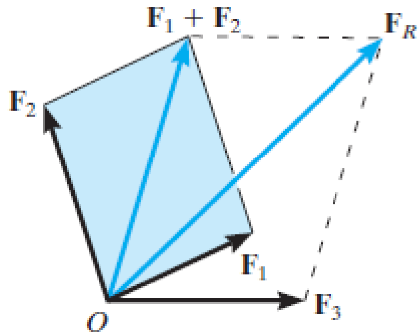


Fig. 2-9

Procedure for Analysis

Problems that involve the addition of two forces can be solved as follows:

Parallelogram Law.

• Two "component" forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 2–10a add according to the parallelogram law, yielding a *resultant* force $\mathbf{F}_R$ that forms the diagonal of the parallelogram.

• If a force **F** is to be resolved into *components* along two axes *u* and v, Fig. 2–10b, then start at the head of force **F** and construct lines parallel to the axes, thereby forming the parallelogram. The sides of the parallelogram represent the components, $\mathbf{F}_u$ and $\mathbf{F}_v$.

• Label all the known and unknown force magnitudes and the angles on the sketch and identify the two unknowns as the magnitude and direction of $\mathbf{F}_R$, or the magnitudes of its components.

Trigonometry.

• Redraw a half portion of the parallelogram to illustrate the triangular head-to-tail addition of the components.

• From this triangle, the magnitude of the resultant force can be determined using the law of cosines, and its direction is determined from the law of sines. The magnitudes of two force components are determined from the law of sines. The formulas are given in Fig. 2–10*c*.

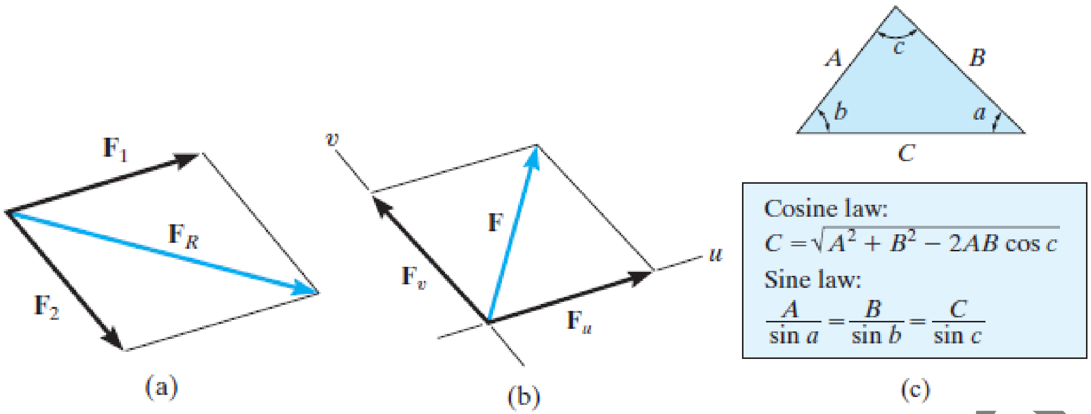


Fig. 2–10

EXAMPLE 2.1

The screw eye in Fig. 2–11*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.

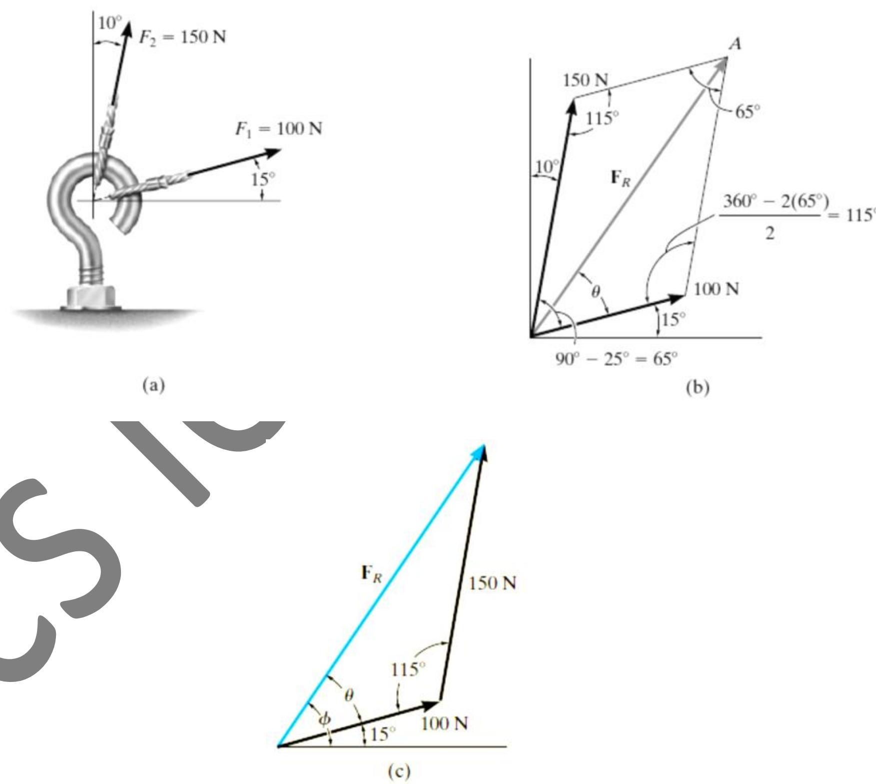


Fig. 2–11

SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of F_1 that is parallel to F_2, and another line from the head of F_2 that is parallel to F_1. The resultant force F_R

extends to where these lines intersect at point *A*, Fig. 2–11 *b*. The two unknowns are the magnitude of F_R and the angle u (theta).

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115^\circ}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N}$$

Applying the law of sines to determine Θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115^\circ} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115^\circ)$$
$$\theta = 39.8^\circ$$

Thus, the direction ø (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8^\circ + 15.0^\circ = 54.8^\circ$$

NOTE: The results seem reasonable, since Fig. 2–11 *b* shows $\mathbf{F}_R$ to have a magnitude larger than its components and a direction that is between them

EXAMPLE 2.2

Resolve the horizontal 600-lb force in Fig. 2–12*a* into components acting along the *u* and v axes and determine the magnitudes of these components.

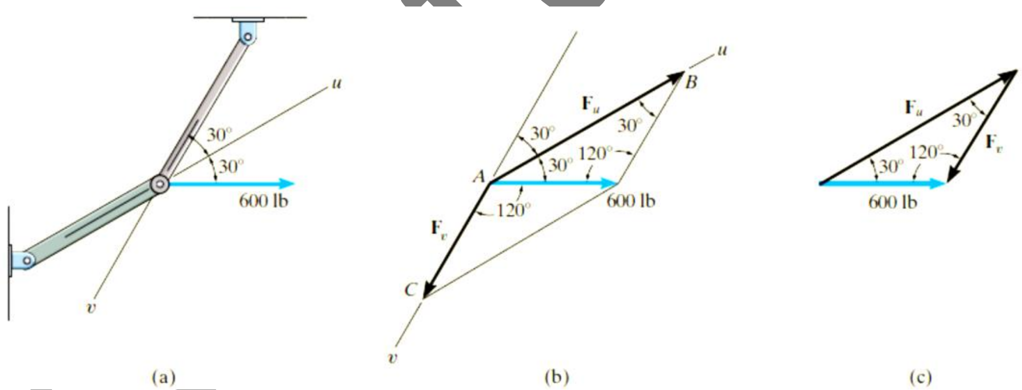


Fig. 2–12

SOLUTION

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the v axis until it intersects the *u* axis at point *B*, Fig. 2–12*b*. The arrow from *A* to *B* represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the *u* axis intersects the v axis at point *C*, which gives $\mathbf{F}_v$. The vector addition using the triangle rule is shown in Fig. 2–12*c*. The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,

$$\frac{F_u}{\sin 120^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039 \text{ lb}$$

$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_v = 600 \text{ lb}$$

EXAMPLE 2.3

Determine the magnitude of the component force **F** in Fig. 2–13*a* and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive *y* axis.

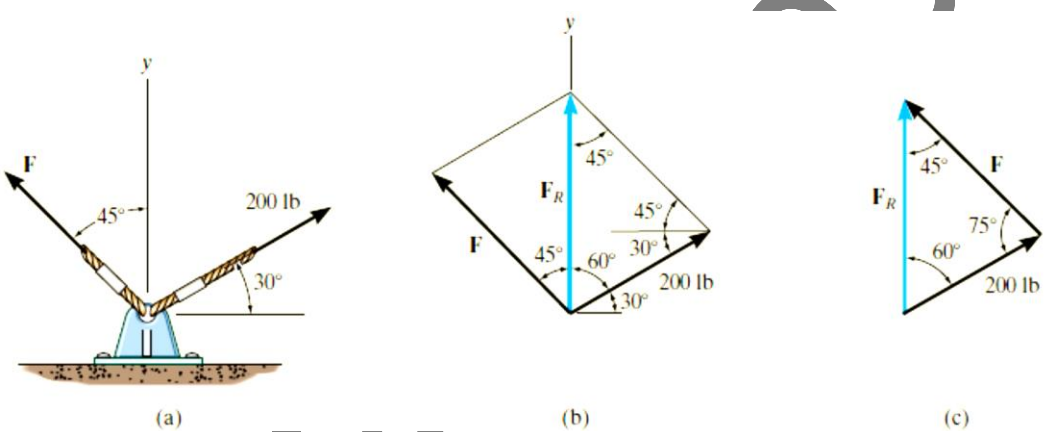


Fig. 2–13

SOLUTION

The parallelogram law of addition is shown in Fig. 2–13*b*, and the triangle rule is shown in Fig. 2–13*c*. The magnitudes of F_R and F are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F = 245 \text{ lb}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F_R = 273 \text{ lb}$$