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Mechanics lectures

Lecture (5)

2-8 Couple

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.

Consider the action of two equal and opposite forces **F** and **-F** a distance *d* apart, as shown in Fig. 2-30*a*. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as *O* in their plane is the couple **M**. This couple has a magnitude

$$M = F(a + d) - Fa$$

OR

$$M = Fd$$

Its direction is counterclockwise when viewed from above for the case illustrated. Note especially that the magnitude of the couple is independent of the distance *a* which locates the forces with respect to the moment center *O*. It follows that the moment of a couple has the same value for all moment centers.

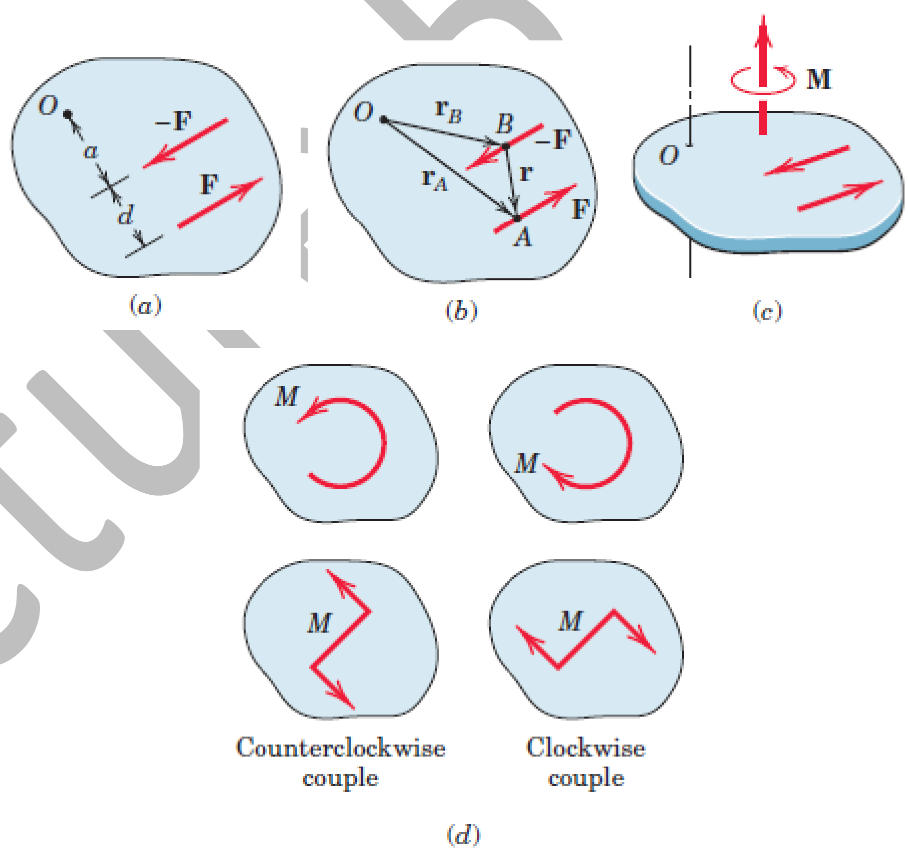

Fig. 2-30

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Vector Algebra Method

We may also express the moment of a couple by using vector algebra. With the cross-product notation of Eq. 2-16, the combined moment about point *O* of the forces forming the couple of Fig. 2-30*b* is

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

where $\mathbf{r}_A$ and $\mathbf{r}_B$ are position vectors which run from point O to arbitrary points A and B on the lines of action of $\mathbf{F}$ and $-\mathbf{F}$, respectively. Because $\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$, we can express $\mathbf{M}$ as

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Here again, the moment expression contains no reference to the moment center *O* and, therefore, is the same for all moment centers. Thus, we may represent **M** by a free vector, as show in Fig. 2-30*c*, where the direction of **M** is normal to the plane of the couple and the sense of **M** is established by the right-hand rule.

Because the couple vector **M** is always perpendicular to the plane of the forces which constitute the couple, in two-dimensional analysis we can represent the sense of a couple vector as clockwise or counterclockwise by one of the conventions shown in Fig. 2-30*d*. Later, when we deal with couple vectors in three-dimensional problems, we will make full use of vector notation to represent them, and the mathematics will automatically account for their sense.

Equivalent Couples

Changing the values of *F* and *d* does not change a given couple as long as the product *Fd* remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane. Figure 2-31 shows four different configurations of the same couple **M**. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.

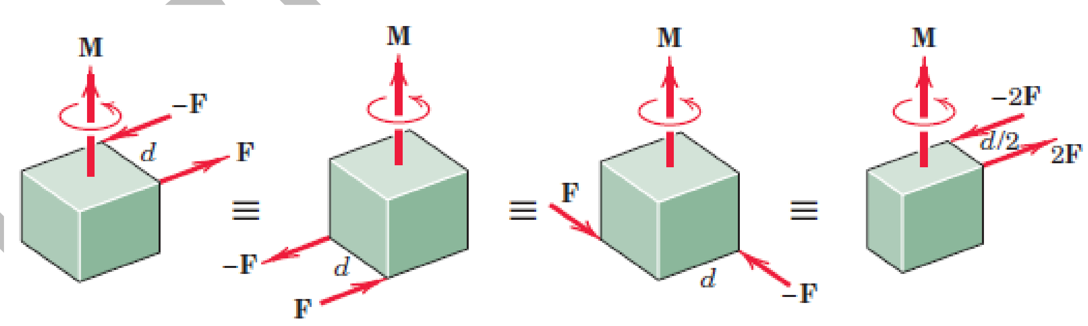

Fig. 2-31

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Force–Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in Fig. 2-32, where the given force **F** acting at point A is replaced by an equal force **F** at some point B and the counterclockwise couple $M = Fd$.

The transfer is seen in the middle figure, where the equal and opposite forces **F** and **-F** are added at point B without introducing any net external effects on the body. We now see that the original force at A and the equal and opposite one at B constitute the couple $M = Fd$, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at A by the same force acting at a different point B and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig. 2-32 is referred to as a *force–couple system*.

By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force. Replacement of a force by an equivalent force–couple system, and the reverse procedure, have many applications in mechanics and should be mastered.

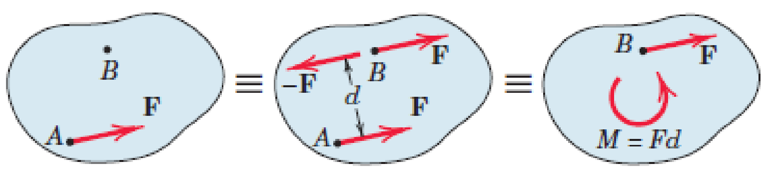

Fig. 2-32

EXAMPLE 2-10

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and **-P**, each of which has a magnitude of 400 N. Determine the proper angle θ.

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Solution. The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$[M = Fd]$ $\qquad$ $M = 100(0.1) = 10\ \text{N}\cdot\text{m}$

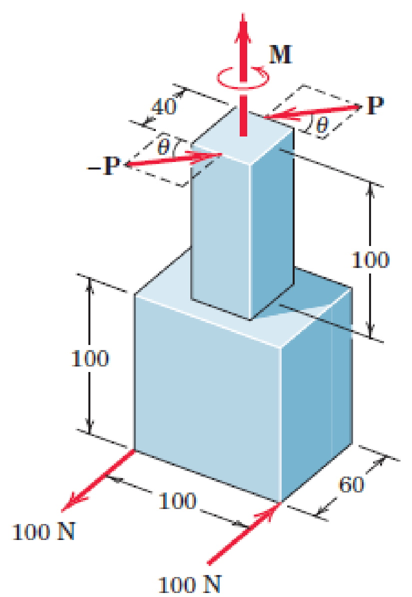
The forces **P** and −**P** produce a counterclockwise couple

$$M = 400(0.040)\cos\theta$$

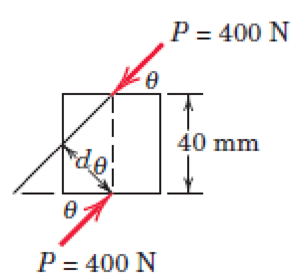
Equating the two expressions gives

$$10 = (400)(0.040)\cos\theta$$

$$\theta = \cos^{-1}\frac{10}{16} = 51.3° \qquad \textit{Ans.}$$



Dimensions in millimeters



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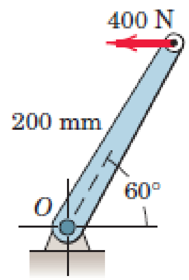
EXAMPLE 2-11

Replace the horizontal 400-N force acting on the lever by an equivalent system consisting of a force at *O* and a couple.



Solution. We apply two equal and opposite 400-N forces at *O* and identify the counterclockwise couple

$[M = Fd]$ $\quad M = 400(0.200 \sin 60°) = 69.3\ \text{N}\cdot\text{m}$ *Ans.*

Thus, the original force is equivalent to the 400-N force at *O* and the 69.3-N·m couple as shown in the third of the three equivalent figures.