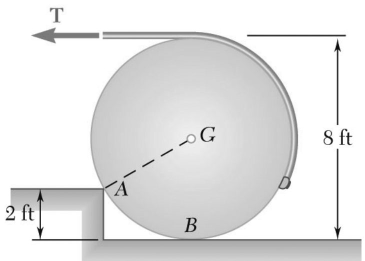
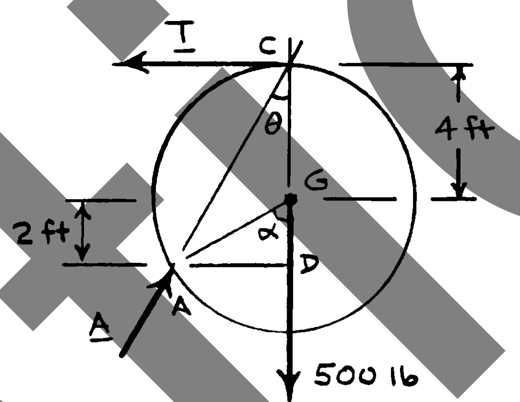


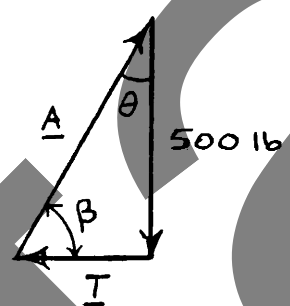
A 500-lb cylindrical tank, 8 ft in diameter, is to be raised over a 2-ft obstruction. A cable is wrapped around the tank and pulled horizontally as shown. Knowing that the corner of the obstruction at *A* is rough, find the required tension in the cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



Force triangle



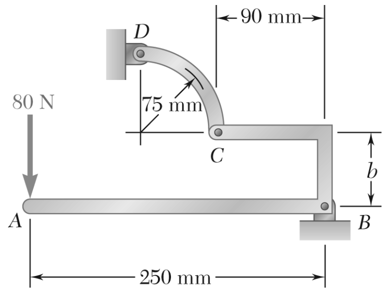
$$\cos\alpha = \frac{GD}{AG} = \frac{2\text{ ft}}{4\text{ ft}} = 0.5 \qquad \alpha = 60°$$

$$\theta = \frac{1}{2}\alpha = 30° \qquad (\beta = 60°)$$

$T = (500\text{ lb})\tan 30° \qquad T = 289\text{ lb}$

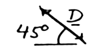
$$A = \frac{500\text{ lb}}{\cos 30°}$$

A = 577 lb ∠ 60.0°



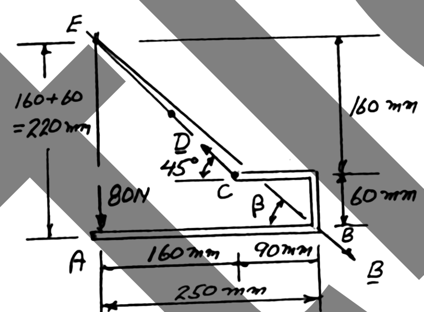
Determine the reactions at B and D when $b = 60$ mm.

SOLUTION

Since CD is a two-force member, the line of action of reaction at D must pass through Points C and D. 

Free-Body Diagram:

(Three-force body)

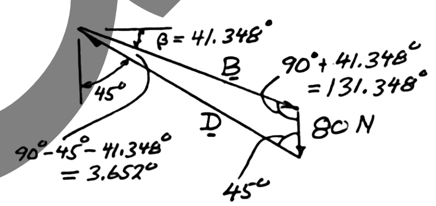


Reaction at B must pass through E, where the reaction at D and the 80-N force intersect.

$$\tan \beta = \frac{220 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 41.348°$$

Force triangle



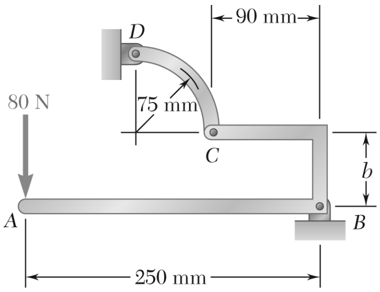
Law of sines:

$$\frac{80 \text{ N}}{\sin 3.652°} = \frac{B}{\sin 45°} = \frac{D}{\sin 131.348°}$$

$$B = 888.0 \text{ N}$$

$$D = 942.8 \text{ N}$$

B = 888 N ↖ 41.3° **D** = 943 N ↘ 45.0° ◀



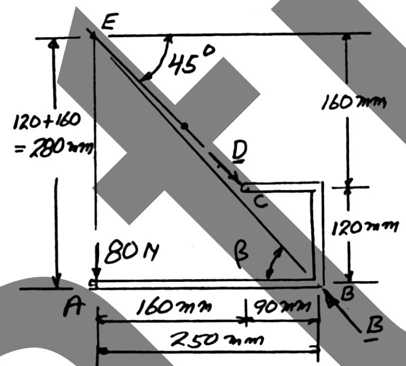
Determine the reactions at B and D when $b = 120$ mm.

SOLUTION

Since CD is a two-force member, line of action of reaction at D must pass through C and D .

Free-Body Diagram:

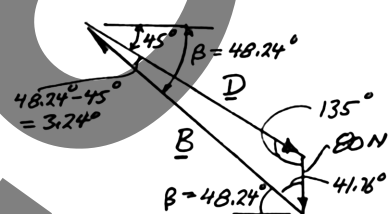
(Three-force body)



Reaction at B must pass through E, where the reaction at D and the 80-N force intersect.

$$\tan \beta = \frac{280 \text{ mm}}{250 \text{ mm}}$$
$$\beta = 48.24°$$

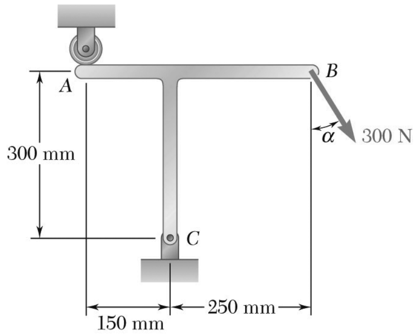
Force triangle



Law of sines:
$$\frac{80 \text{ N}}{\sin 3.24°} = \frac{B}{\sin 135°} = \frac{D}{\sin 41.76°}$$

$$B = 1000.9 \text{ N}$$
$$D = 942.8 \text{ N}$$

$\mathbf{B} = 1001$ N ◣ 48.2° $\mathbf{D} = 943$ N ◥ 45.0° ◀

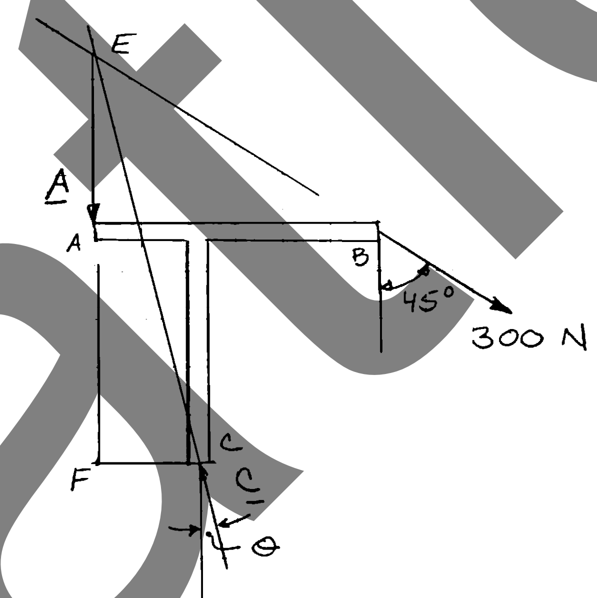


A T-shaped bracket supports a 300-N load as shown. Determine the reactions at A and C when $\alpha = 45°$.

SOLUTION

Free-Body Diagram:

(Three-force body)



The line of action of **C** must pass through E, where **A** and the 300-N force intersect.

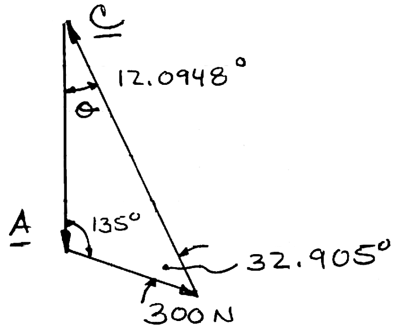
Triangle ABE is isosceles: $EA = AB = 400$ mm

In triangle CEF:

$$\tan\theta = \frac{CF}{EF} = \frac{CF}{EA + AF} = \frac{150 \text{ mm}}{700 \text{ mm}} \qquad \theta = 12.0948°$$

(Continued)

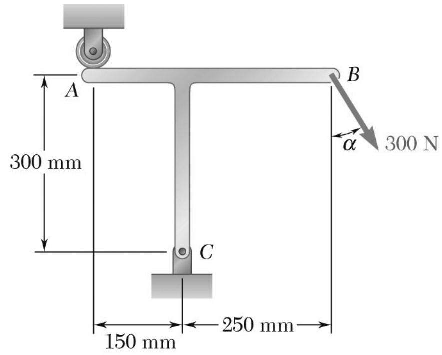
Force Triangle



Law of sines:

$$\frac{A}{\sin 32.905°} = \frac{C}{\sin 135°} = \frac{300\text{ N}}{\sin 12.0948°}$$

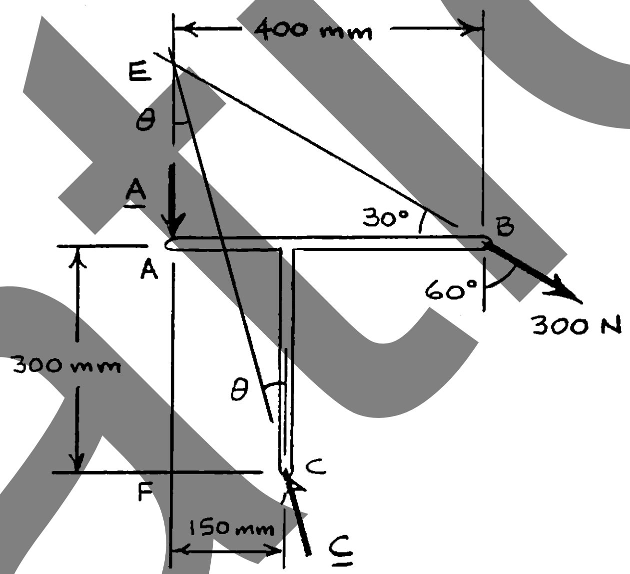
A = 778 N ↓;   **C** = 1012 N ⦫ 77.9° ◀



A T-shaped bracket supports a 300-N load as shown. Determine the reactions at A and C when $\alpha = 60°$.

SOLUTION

Free-Body Diagram:



$$EA = (400 \text{ mm}) \tan 30°$$
$$= 230.94 \text{ mm}$$

In triangle CEF:

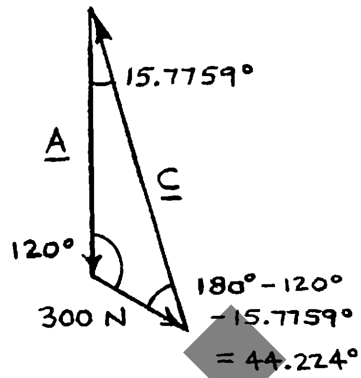
$$\tan\theta = \frac{CF}{EF} = \frac{CF}{EA + AF}$$

$$\tan\theta = \frac{150}{230.94 + 300}$$

$$\theta = 15.7759°$$

(Continued)

Force Triangle



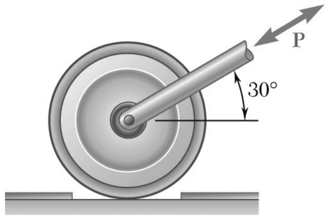
Law of sines:

$$\frac{A}{\sin 44.224^\circ} = \frac{C}{\sin 120^\circ} = \frac{300\text{ N}}{\sin 15.7759^\circ}$$

$$A = 770\text{ N}$$

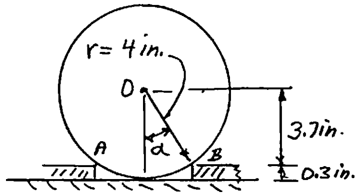
$$C = 956\text{ N}$$

A = 770 N ↓; **C** = 956 N ⦪ 74.2° ◀



A 40-lb roller, of diameter 8 in., which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 0.3 in., determine the force **P** required to move the roller onto the tiles if the roller is (*a*) pushed to the left, (*b*) pulled to the right.

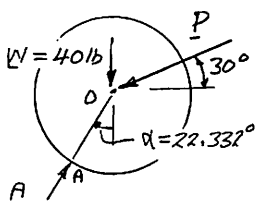
SOLUTION



Geometry: For each case as roller comes into contact with tile,

$$\alpha = \cos^{-1}\frac{3.7\text{ in.}}{4\text{ in.}}$$

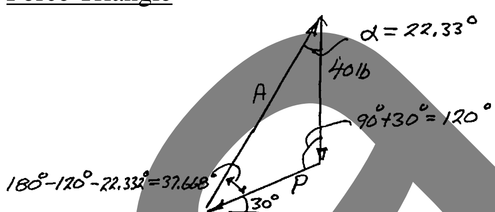
$$\alpha = 22.332°$$



(*a*) Roller pushed to left (three-force body):
Forces must pass through *O*.

Law of sines:
$$\frac{40\text{ lb}}{\sin 37.668°} = \frac{P}{\sin 22.332°};\ P = 24.87\text{ lb}$$

Force Triangle

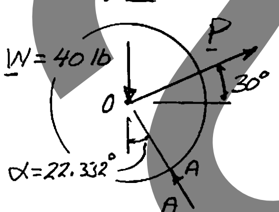


P = 24.9 lb ↗ 30.0° ◀

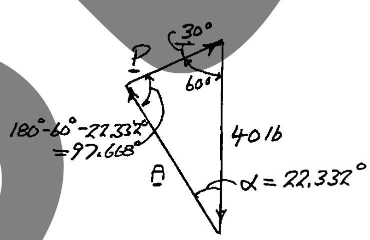
(*b*) Roller pulled to right (three-force body):
Forces must pass through *O*.

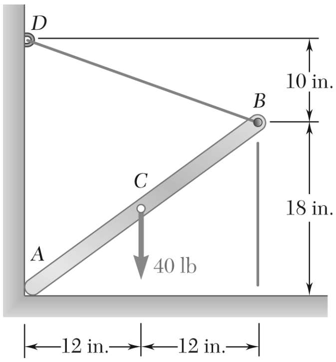
Law of sines:
$$\frac{40\text{ lb}}{\sin 97.668°} = \frac{P}{\sin 22.332°};\ P = 15.3361\text{ lb}$$

P = 15.34 lb ∠ 30.0° ◀



Force Triangle

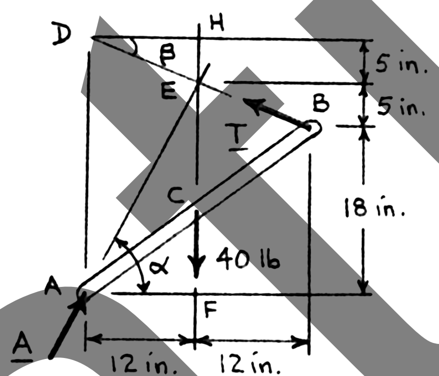




One end of rod AB rests in the corner A and the other end is attached to cord BD. If the rod supports a 40-lb load at its midpoint C, find the reaction at A and the tension in the cord.

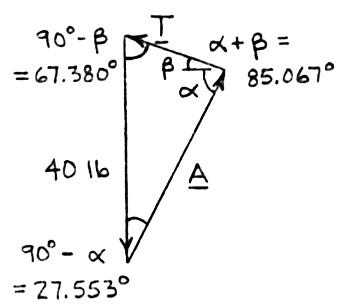
SOLUTION

Free-Body Diagram: (Three-force body)



The line of action of reaction at A must pass through E, where **T** and the 40-lb load intersect.

Force triangle



$$\tan\alpha = \frac{EF}{AF} = \frac{23}{12}$$

$$\alpha = 62.447°$$

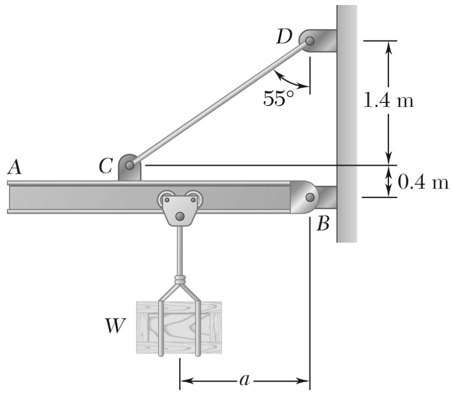
$$\tan\beta = \frac{EH}{DH} = \frac{5}{12}$$

$$\beta = 22.620°$$

$$\frac{A}{\sin 67.380°} = \frac{T}{\sin 27.553°} = \frac{40 \text{ lb}}{\sin 85.067°}$$

$\mathbf{A} = 37.1 \text{ lb} \measuredangle\ 62.4°$ ◀

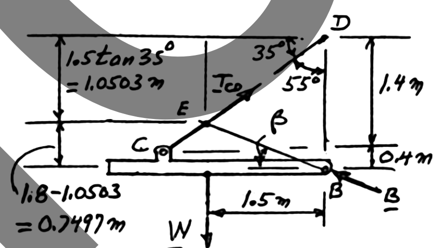
$T = 18.57 \text{ lb}$ ◀



A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION

Three-force body: **W** and $\mathbf{T}_{CD}$ intersect at *E*.



$$\tan \beta = \frac{0.7497 \text{ m}}{1.5 \text{ m}}$$

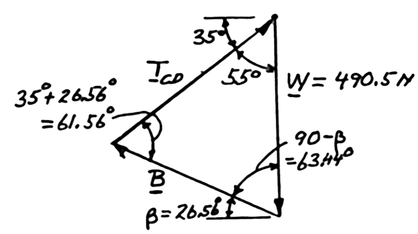
$$\beta = 26.556°$$

Three forces intersect at *E*.

$$W = (50 \text{ kg})\ 9.81 \text{ m/s}^2$$

$$= 490.50 \text{ N}$$

Force triangle



Law of sines:

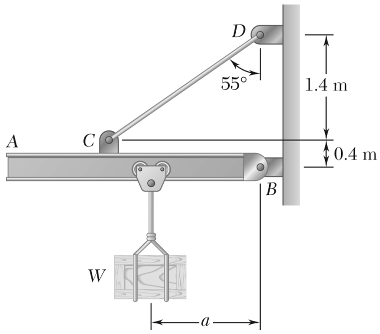
$$\frac{490.50 \text{ N}}{\sin 61.556°} = \frac{T_{CD}}{\sin 63.444°} = \frac{B}{\sin 55°}$$

$$T_{CD} = 498.99 \text{ N}$$

$$B = 456.96 \text{ N}$$

(*a*) $T_{CD} = 499$ N ◀

(*b*) $\mathbf{B} = 457$ N ⦨ 26.6° ◀



Solve Problem 4.73, assuming that $a = 3$ m.

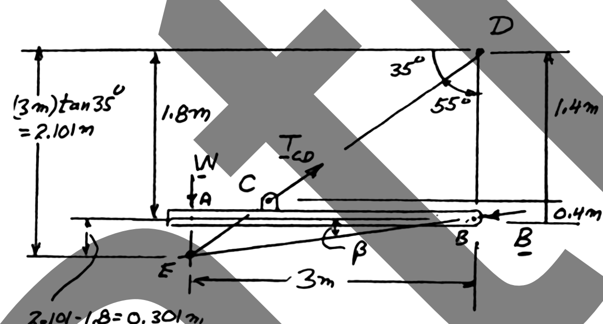
PROBLEM 4.73 A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION

W and $\mathbf{T}_{CD}$ intersect at *E*.

Free-Body Diagram:

Three-Force Body



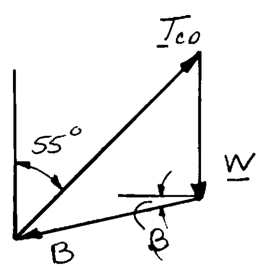
$$\tan\beta = \frac{AE}{AB} = \frac{0.301 \text{ m}}{3 \text{ m}}$$

$$\beta = 5.7295°$$

Three forces intersect at *E*.

$$W = (50 \text{ kg})\, 9.81 \text{ m/s}^2$$
$$= 490.50 \text{ N}$$

Force Triangle



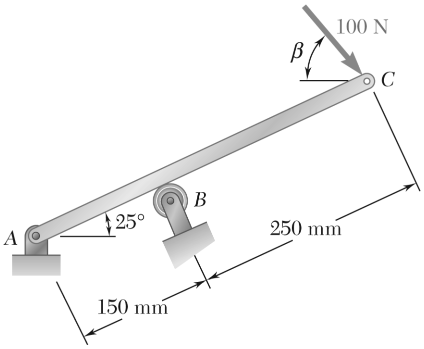
Law of sines:

$$\frac{490.50 \text{ N}}{\sin 29.271°} = \frac{T_{CD}}{\sin 95.730°} = \frac{B}{\sin 55°}$$

$$T_{CD} = 998.18 \text{ N}$$
$$B = 821.76 \text{ N}$$

(*a*) $T_{CD} = 998$ N ◀

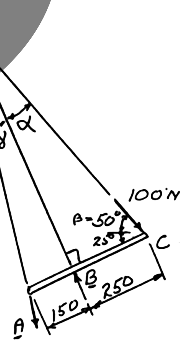
(*b*) $\mathbf{B} = 822$ N ↙ 5.73° ◀



Determine the reactions at A and B when $\beta = 50°$.

SOLUTION

Free-Body Diagram: (Three-force body)



Dimensions in mm

Reaction **A** must pass through Point D where the 100-N force and **B** intersect.

In right $\Delta\, BCD$:

$$\alpha = 90° - 75° = 15°$$

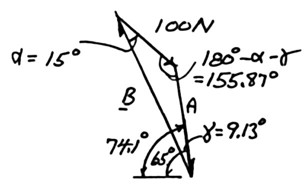
$$BD = 250 \tan 75° = 933.01 \text{ mm}$$

In right $\Delta\, ABD$:

$$\tan\gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}}$$

$$\gamma = 9.1333°$$

Force Triangle

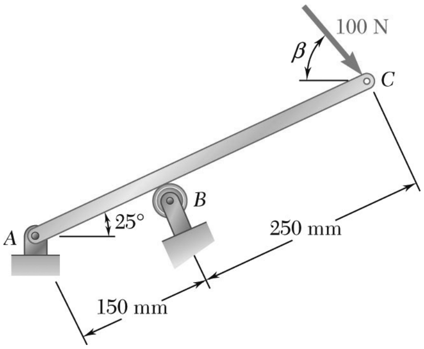


Law of sines:

$$\frac{100 \text{ N}}{\sin 9.1333°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.867°}$$

$$A = 163.1 \text{ N}; \quad B = 257.6 \text{ N}$$

$\mathbf{A} = 163.1$ N ∠ 74.1° $\quad \mathbf{B} = 258$ N ∠ 65.0° ◀

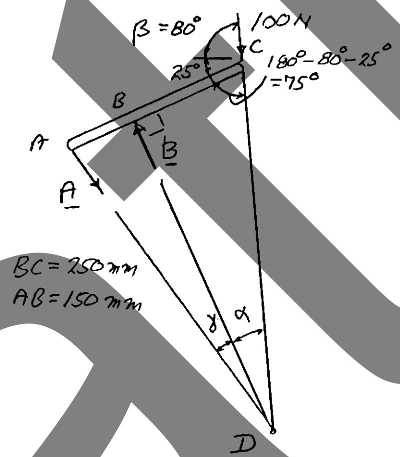


Determine the reactions at A and B when $\beta = 80°$.

SOLUTION

Free-Body Diagram:

(Three-force body)



Reaction **A** must pass through D where the 100-N force and **B** intersect.

In right triangle BCD:

$$\alpha = 90° - 75° = 15°$$

$$BD = BC \tan 75° = 250 \tan 75°$$

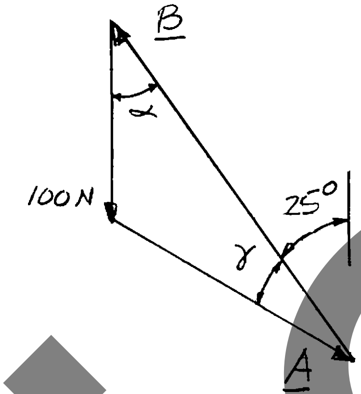
$$BD = 933.01 \text{ mm}$$

In right triangle ABD:

$$\tan \gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}} \qquad \gamma = 9.1333°$$

(Continued)

Force Triangle

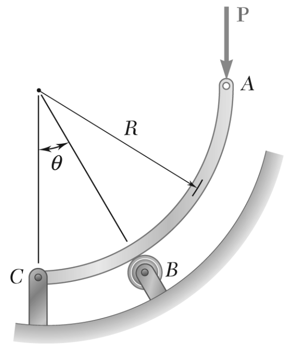


Law of sines:

$$\frac{100\text{ N}}{\sin 9.1333^\circ} = \frac{A}{\sin 15^\circ} = \frac{B}{\sin 155.867^\circ}$$

A = 163.1 N ↘ 55.9° ◀

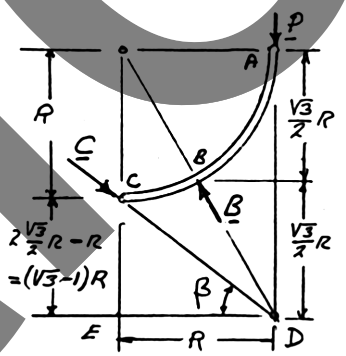
B = 258 N ↖ 65.0° ◀



Knowing that $\theta = 30°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Free-Body Diagram:
(Three-force body)

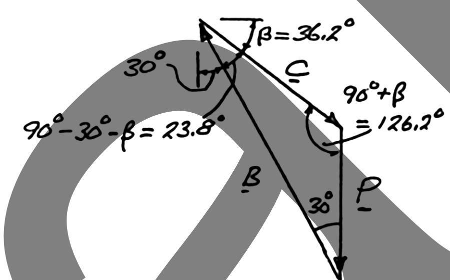


Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

In Δ *CDE*:

$$\tan\beta = \frac{(\sqrt{3}-1)R}{R}$$
$$= \sqrt{3}-1$$
$$\beta = 36.2°$$

Force Triangle

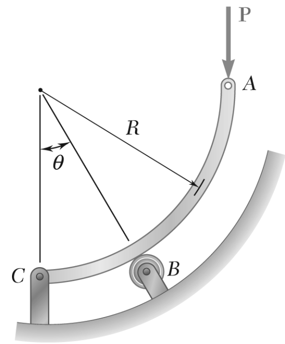


Law of sines:

$$\frac{P}{\sin 23.8°} = \frac{B}{\sin 126.2°} = \frac{C}{\sin 30°}$$
$$B = 2.00P$$
$$C = 1.239P$$

(*a*) $\mathbf{B} = 2P$ ⦬ 60.0° ◀

(*b*) $\mathbf{C} = 1.239P$ ⦫ 36.2° ◀

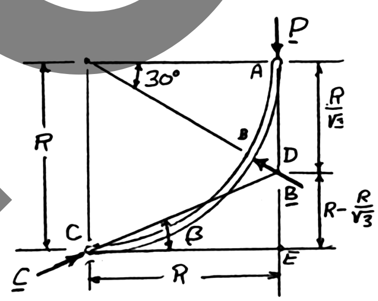


Knowing that $\theta = 60°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

Free-Body Diagram:
(Three-force body)



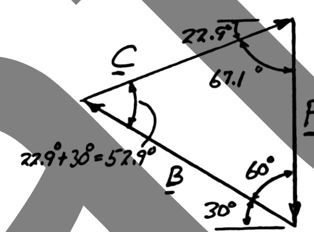
In ΔCDE:

$$\tan\beta = \frac{R - \frac{R}{\sqrt{3}}}{R}$$

$$= 1 - \frac{1}{\sqrt{3}}$$

$$\beta = 22.9°$$

Force Triangle



Law of sines:

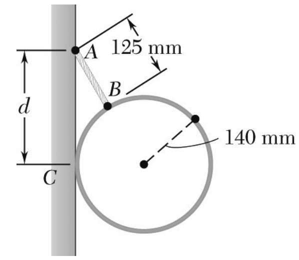
$$\frac{P}{\sin 52.9°} = \frac{B}{\sin 67.1°} = \frac{C}{\sin 60°}$$

$$B = 1.155P$$

$$C = 1.086P$$

(*a*) $\mathbf{B} = 1.155P$ ⦩ $30.0°$ ◀

(*b*) $\mathbf{C} = 1.086P$ ⦨ $22.9°$ ◀

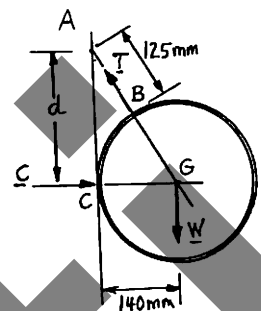


A thin ring of mass 2 kg and radius $r = 140$ mm is held against a frictionless wall by a 125-mm string AB. Determine (*a*) the distance d, (*b*) the tension in the string, (*c*) the reaction at C.

SOLUTION

Free-Body Diagram:

(Three-force body)



The force T exerted at B must pass through the center G of the ring, since **C** and **W** intersect at that point.

Thus, points A, B, and G are in a straight line.

(*a*) From triangle ACG:

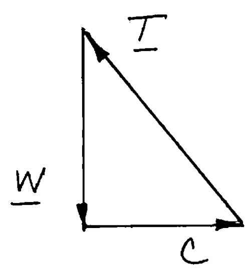
$$d = \sqrt{(AG)^2 - (CG)^2}$$

$$= \sqrt{(265 \text{ mm})^2 - (140 \text{ mm})^2}$$

$$= 225.00 \text{ mm}$$

$d = 225$ mm ◀

Force Triangle



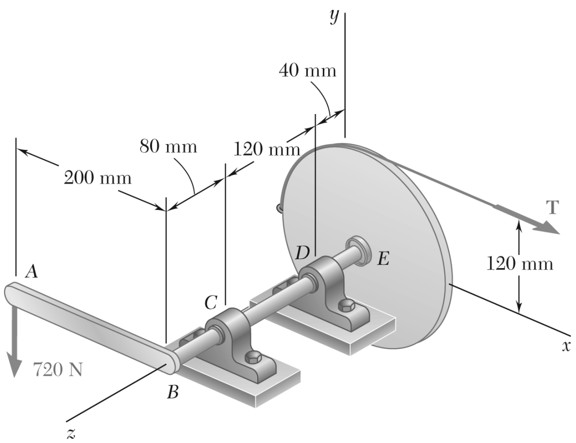
$$W = (2 \text{ kg})(9.81 \text{ m/s}^2) = 19.6200 \text{ N}$$

Law of sines:

$$\frac{T}{265 \text{ mm}} = \frac{C}{140 \text{ mm}} = \frac{19.6200 \text{ N}}{225.00 \text{ mm}}$$

(*b*) $T = 23.1$ N ◀

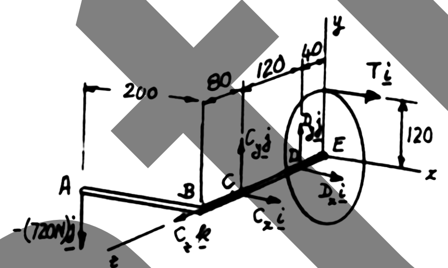
(*c*) $\mathbf{C} = 12.21 \text{ N} \longrightarrow$ ◀



A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six equations of equilibrium. —OK

$$\Sigma \mathbf{M}_C = 0: \quad (-120\mathbf{k}) \times (D_x\mathbf{i} + D_y\mathbf{j}) + (120\mathbf{j} - 160\mathbf{k}) \times T\,\mathbf{i} + (80\mathbf{k} - 200\mathbf{i}) \times (-720\mathbf{j}) = 0$$

$$-120D_x\mathbf{j} + 120D_y\mathbf{i} - 120T\,\mathbf{k} - 160T\mathbf{j} + 57.6\times10^3\,\mathbf{i} + 144\times10^3\,\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors:

$\mathbf{k}$: $\quad -120T + 144\times10^3 = 0 \qquad$ (*a*) $T = 1200$ N ◀

$\mathbf{i}$: $\quad 120D_y + 57.6\times10^3 = 0 \qquad D_y = -480$ N

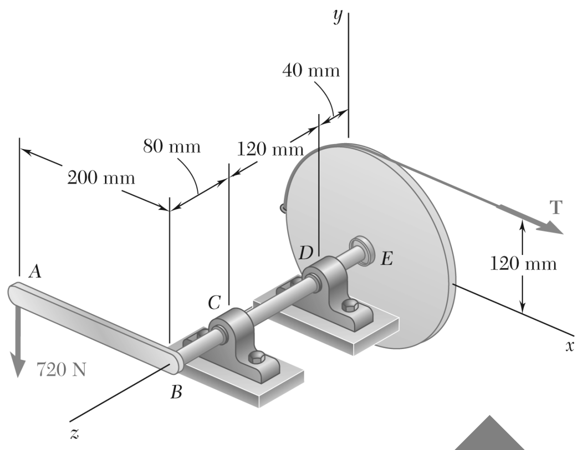
$\mathbf{j}$: $\quad -120D_x - 160(1200\text{ N}) = 0 \qquad D_x = -1600$ N

$\Sigma F_x = 0$: $\quad C_x + D_x + T = 0 \qquad C_x = 1600 - 1200 = 400$ N

$\Sigma F_y = 0$: $\quad C_y + D_y - 720 = 0 \qquad C_y = 480 + 720 = 1200$ N

$\Sigma F_z = 0$: $\quad C_z = 0$

(*b*) $\mathbf{C} = (400\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}; \quad \mathbf{D} = -(1600\text{ N})\mathbf{i} - (480\text{ N})\mathbf{j}$ ◀

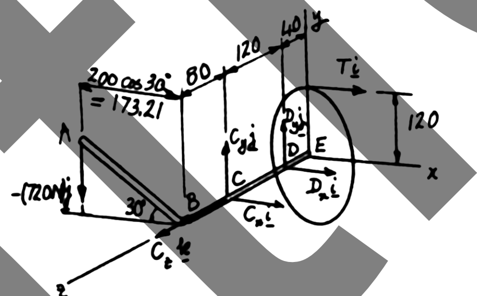


Solve Problem , assuming that the axle has been rotated clockwise in its bearings by 30° and that the 720-N load remains vertical.

A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six equations of equilibrium.

$$\Sigma\mathbf{M}_C = 0: \quad (-120\mathbf{k})\times(D_x\mathbf{i} + D_y\mathbf{j}) + (120\mathbf{j} - 160\mathbf{k})\times T\mathbf{i} + (80\mathbf{k} - 173.21\mathbf{i})\times(-720\mathbf{j}) = 0$$

$$-120D_x\mathbf{j} + 120D_y\mathbf{i} - 120T\,\mathbf{k} - 160T\,\mathbf{j} + 57.6\times10^3\,\mathbf{i} + 124.71\times10^3\,\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors,

$$\mathbf{k}: \quad -120T + 124.71\times10^3 = 0 \qquad T = 1039.2\text{ N} \qquad\qquad T = 1039\text{ N} \blacktriangleleft$$

$$\mathbf{i}: \quad 120D_y + 57.6\times10^3 = 0 \qquad D_y = -480\text{ N}$$

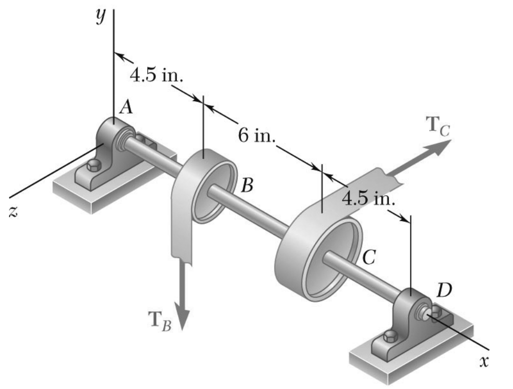
$$\mathbf{j}: \quad -120D_x - 160(1039.2) \qquad D_x = -1385.6\text{ N}$$

$$\Sigma F_x = 0: \quad C_x + D_x + T = 0 \qquad C_x = 1385.6 - 1039.2 = 346.4$$

$$\Sigma F_y = 0: \quad C_y + D_y - 720 = 0 \qquad C_y = 480 + 720 = 1200\text{ N}$$

$$\Sigma F_z = 0: \quad C_z = 0$$

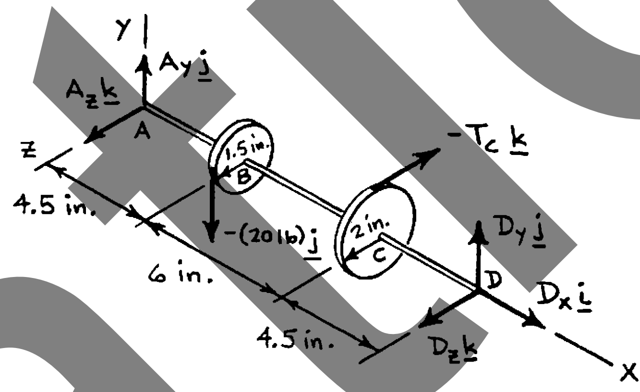
(*b*) $\mathbf{C} = (346\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j} \quad \mathbf{D} = -(1386\text{ N})\mathbf{i} - (480\text{ N})\mathbf{j}$ ◀



Two tape spools are attached to an axle supported by bearings at A and D. The radius of spool B is 1.5 in. and the radius of spool C is 2 in. Knowing that T_B = 20 lb and that the system rotates at a constant rate, determine the reactions at A and D. Assume that the bearing at A does not exert any axial thrust and neglect the weights of the spools and axle.

SOLUTION

Free-Body Diagram:



We have six unknowns and six equations of equilibrium.

$$\Sigma M_A = 0: \quad (4.5\mathbf{i} + 1.5\mathbf{k}) \times (-20\mathbf{j}) + (10.5\mathbf{i} + 2\mathbf{j}) \times (-T_C\mathbf{k}) + (15\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-90\mathbf{k} + 30\mathbf{i} + 10.5T_C\mathbf{j} - 2T_C\mathbf{i} + 15D_y\mathbf{k} - 15D_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$\vec{\mathbf{i}}$: $\quad 30 - 2T_C = 0 \qquad T_C = 15\text{ lb}$

$\vec{\mathbf{j}}$: $\quad 10.5T_C - 15D_z = 0 \quad 10.5(15) - 15D_z = 0 \qquad D_z = 10.5\text{ lb}$

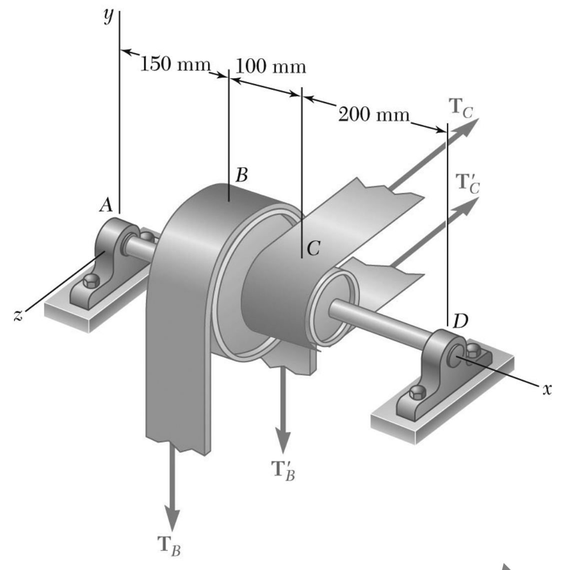
$\vec{\mathbf{k}}$: $\quad -90 + 15D_y = 0 \qquad D_y = 6\text{ lb}$

$\Sigma F_x = 0$: $\quad D_x = 0$

$\Sigma F_y = 0$: $\quad A_y + D_y - 20\text{ lb} = 0 \qquad A_y = 20 - 6 = 14\text{ lb}$

$\Sigma F_z = 0$: $\quad A_z + D_z - 15\text{ lb} = 0 \qquad A_z = 15 - 10.5 = 4.5\text{ lb}$

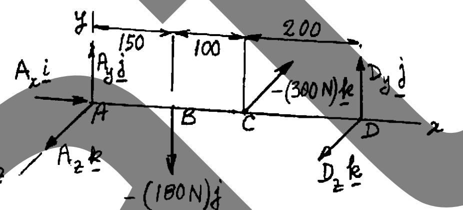
$$\mathbf{A} = (14.00\text{ lb})\mathbf{j} + (4.50\text{ lb})\mathbf{k}; \quad \mathbf{D} = (6.00\text{ lb})\mathbf{j} + (10.50\text{ lb})\mathbf{k} \blacktriangleleft$$



Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at A and D. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt B and 150 N in both portions of belt C, determine the reactions at A and D. Assume that the bearing at D does not exert any axial thrust.

SOLUTION

We replace $\mathbf{T}_B$ and $\mathbf{T}'_B$ by their resultant $(-180\text{ N})\mathbf{j}$ and $\mathbf{T}_C$ and $\mathbf{T}'_C$ by their resultant $(-300\text{ N})\mathbf{k}$.



Dimensions in mm

We have five unknowns and six equations of equilibrium. Axle AD is free to rotate about the x-axis, but equilibrium is maintained $(\Sigma M_x = 0)$.

$$\Sigma \mathbf{M}_A = 0: \quad (150\mathbf{i})\times(-180\mathbf{j}) + (250\mathbf{i})\times(-300\mathbf{k}) + (450\mathbf{i})\times(D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-27\times10^3\mathbf{k} + 75\times10^3\mathbf{j} + 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0$$

Equating coefficients of $\mathbf{j}$ and $\mathbf{k}$ to zero,

$$\mathbf{j}: \quad 75\times10^3 - 450D_z = 0 \qquad D_z = 166.7\text{ N}$$

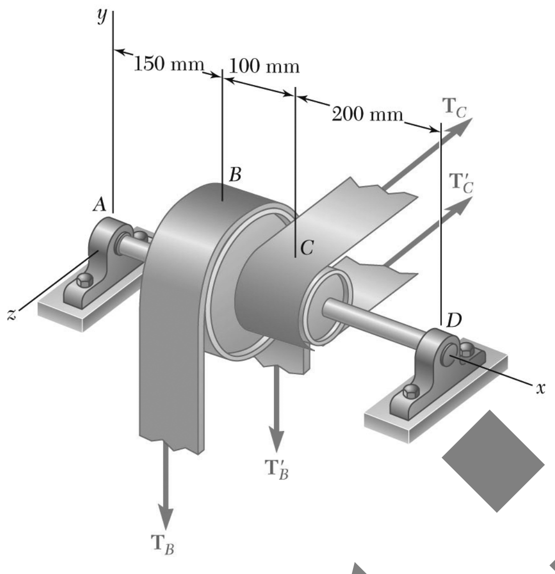
$$\mathbf{k}: \quad -27\times10^3 + 450D_y = 0 \qquad D_y = 60.0\text{ N}$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 180\text{ N} = 0 \qquad A_y = 180 - 60 = 120.0\text{ N}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 300\text{ N} = 0 \qquad A_z = 300 - 166.7 = 133.3\text{ N}$$

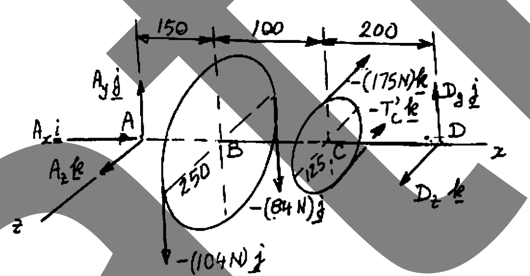
$\mathbf{A} = (120.0\text{ N})\mathbf{j} + (133.3\text{ N})\mathbf{k}; \quad \mathbf{D} = (60.0\text{ N})\mathbf{j} + (166.7\text{ N})\mathbf{k}$ ◀



Solve Problem , assuming that the pulley rotates at a constant rate and that $T_B = 104$ N, $T'_B = 84$ N, $T_C = 175$ N.

PROBLEM Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at *A* and *D*. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt *B* and 150 N in both portions of belt *C*, determine the reactions at *A* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION



Dimensions in mm

We have six unknowns and six equations of equilibrium. —OK

$$\Sigma \mathbf{M}_A = 0: \quad (150\mathbf{i} + 250\mathbf{k}) \times (-104\mathbf{j}) + (150\mathbf{i} - 250\mathbf{k}) \times (-84\mathbf{j})$$
$$+ (250\mathbf{i} + 125\mathbf{j}) \times (-175\mathbf{k}) + (250\mathbf{i} - 125\mathbf{j}) \times (-T_C)$$
$$+ 450\mathbf{i} \times (D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-150(104 + 84)\mathbf{k} + 250(104 - 84)\mathbf{i} + 250(175 + T'_C)\mathbf{j} \;\; -125(175 - T'_C)$$
$$+ 450 D_y\mathbf{k} - 450 D_z\mathbf{j} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: \quad 250(104 - 84) - 125(175 - T'_C) = 0 \qquad 175 = T'_C = 40 \qquad T'_C = 135;$$

$$\mathbf{j}: \quad 250(175 + 135) - 450 D_z = 0 \qquad D_z = 172.2 \text{ N}$$

$$\mathbf{k}: \quad -150(104 + 84) + 450 D_y = 0 \qquad D_y = 62.7 \text{ N}$$

PROBLEM (Continued)

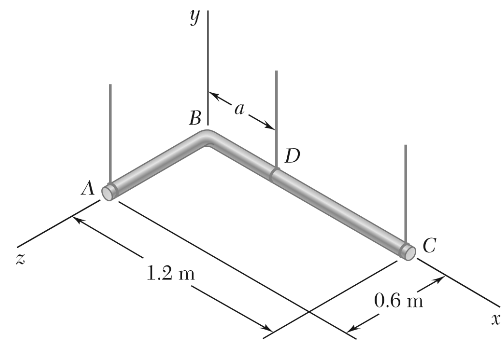
$\Sigma F_x = 0$: $A_x = 0$

$\Sigma F_y = 0$: $A_y - 104 - 84 + 62.7 = 0$ $A_y = 125.3 \text{ N}$

$\Sigma F_z = 0$: $A_z - 175 - 135 + 172.2 = 0$ $A_z = 137.8 \text{ N}$

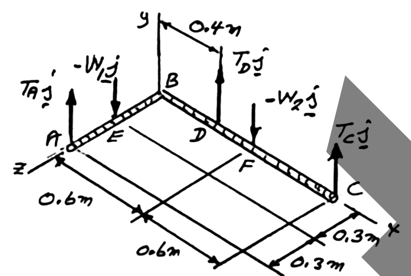
$\mathbf{A} = (125.3 \text{ N})\mathbf{j} + (137.8 \text{ N})\mathbf{k}$; $\mathbf{D} = (62.7 \text{ N})\mathbf{j} + (172.2 \text{ N})\mathbf{k}$ ◀

PROBLEM



Two steel pipes *AB* and *BC*, each having a mass per unit length of 8 kg/m, are welded together at *B* and supported by three wires. Knowing that $a = 0.4$ m, determine the tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$
$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \quad \mathbf{r}_{A/D} \times T_A\mathbf{j} + \mathbf{r}_{E/D} \times (-W_1\mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2\mathbf{j}) + \mathbf{r}_{C/D} \times T_C\mathbf{j} = 0$$

$$(-0.4\mathbf{i} + 0.6\mathbf{k}) \times T_A\mathbf{j} + (-0.4\mathbf{i} + 0.3\mathbf{k}) \times (-W_1\mathbf{j}) + 0.2\mathbf{i} \times (-W_2\mathbf{j}) + 0.8\mathbf{i} \times T_C\mathbf{j} = 0$$

$$-0.4T_A\mathbf{k} - 0.6T_A\mathbf{i} + 0.4W_1\mathbf{k} + 0.3W_1\mathbf{i} - 0.2W_2\mathbf{k} + 0.8T_C\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}0.6m'g = 0.3m'g$$

$$\mathbf{k}: \quad -0.4T_A + 0.4W_1 - 0.2W_2 + 0.8T_C = 0$$
$$-0.4(0.3m'g) + 0.4(0.6m'g) - 0.2(1.2m'g) + 0.8T_C = 0$$

$$T_C = \frac{(0.12 - 0.24 - 0.24)m'g}{0.8} = 0.15m'g$$

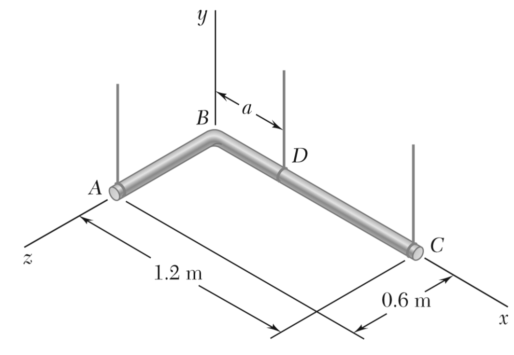
$$\Sigma F_y = 0: \quad T_A + T_C + T_D - W_1 - W_2 = 0$$
$$0.3m'g + 0.15m'g + T_D - 0.6m'g - 1.2m'g = 0$$
$$T_D = 1.35m'g$$

$$m'g = (8 \text{ kg/m})(9.81 \text{m/s}^2) = 78.48 \text{ N/m}$$

$T_A = 0.3m'g = 0.3 \times 78.45$ $\qquad T_A = 23.5$ N ◀

$T_B = 0.15m'g = 0.15 \times 78.45$ $\qquad T_B = 11.77$ N ◀

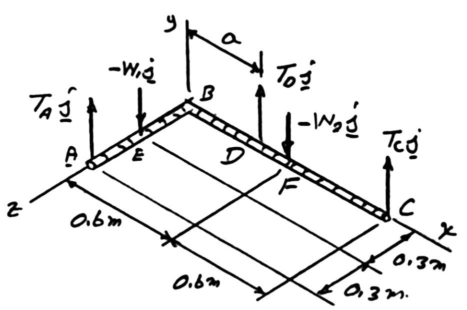
$T_C = 1.35m'g = 1.35 \times 78.45$ $\qquad T_C = 105.9$ N ◀



PROBLEM

For the pipe assembly of Problem 4.97, determine (*a*) the largest permissible value of a if the assembly is not to tip, (*b*) the corresponding tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$
$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \quad \mathbf{r}_{A/D} \times T_A\mathbf{j} + \mathbf{r}_{E/D} \times (-W_1\mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2\mathbf{j}) + \mathbf{r}_{C/D} \times T_C\mathbf{j} = 0$$

$$(-a\mathbf{i} + 0.6\mathbf{k}) \times T_A\mathbf{j} + (-a\mathbf{i} + 0.3\mathbf{k}) \times (-W_1\mathbf{j}) + (0.6 - a)\mathbf{i} \times (-W_2\mathbf{j}) + (1.2 - a)\mathbf{i} \times T_C\mathbf{j} = 0$$

$$-T_A a\mathbf{k} - 0.6T_A\mathbf{i} + W_1 a\mathbf{k} + 0.3W_1\mathbf{i} - W_2(0.6 - a)\mathbf{k} + T_C(1.2 - a)\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}0.6m'g = 0.3m'g$$

$$\mathbf{k}: \quad -T_A a + W_1 a - W_2(0.6 - a) + T_C(1.2 - a) = 0$$

$$-0.3m'ga + 0.6m'ga - 1.2m'g(0.6 - a) + T_C(1.2 - a) = 0$$

$$T_C = \frac{0.3a - 0.6a + 1.2(0.6 - a)}{1.2 - a}$$

For maximum a and no tipping, $T_C = 0$.

(*a*)

$$-0.3a + 1.2(0.6 - a) = 0$$
$$-0.3a + 0.72 - 1.2a = 0$$
$$1.5a = 0.72$$

$a = 0.480$ m ◀

PROBLEM (Continued)

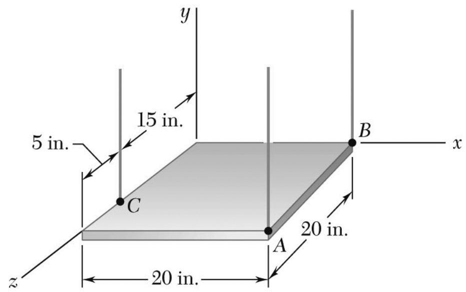
(*b*) Reactions: $m'g = (8\text{ kg/m})\,9.81\text{ m/s}^2 = 78.48\text{ N/m}$

$T_A = 0.3m'g = 0.3 \times 78.48 = 23.544\text{ N}$ $\qquad T_A = 23.5\text{ N}$ ◀

$\Sigma F_y = 0$: $\quad T_A + T_C + T_D - W_1 - W_2 = 0$

$T_A + 0 + T_D - 0.6m'g - 1.2m'g = 0$

$T_D = 1.8m'g - T_A = 1.8 \times 78.48 - 23.544 = 117.72$ $\qquad T_D = 117.7\text{ N}$ ◀

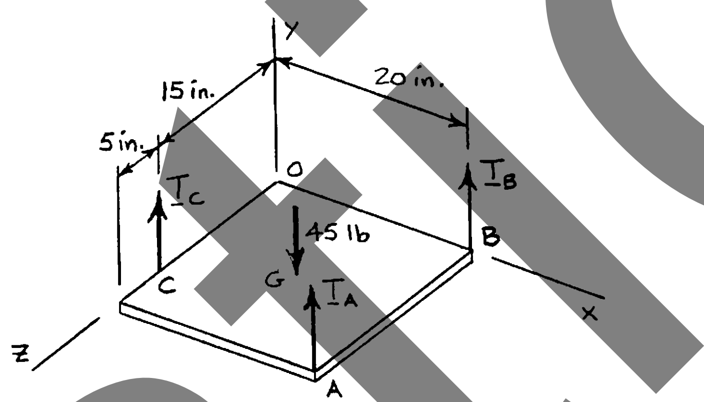


PROBLEM

The 45-lb square plate shown is supported by three vertical wires. Determine the tension in each wire.

SOLUTION

Free-Body Diagram:



$$\Sigma M_B = 0: \quad \mathbf{r}_{C/B} \times T_C\mathbf{j} + \mathbf{r}_{A/B} \times T_A\mathbf{j} + \mathbf{r}_{G/B} \times (-45 \text{ lb})\mathbf{j} = 0$$

$$[-(20 \text{ in.})\mathbf{i} + (15 \text{ in.})\vec{\mathbf{k}}] \times T_C\mathbf{j} + (20 \text{ in.})\mathbf{k} \times T_A\mathbf{j}$$

$$+ [-(10 \text{ in.})\mathbf{i} + (10 \text{ in.})\mathbf{k}] \times [-(45 \text{ lb})\mathbf{j}] = 0$$

$$-20T_C\mathbf{k} - 15T_C\mathbf{i} - 20T_A\mathbf{i} + 450\mathbf{k} + 450\mathbf{i} = 0$$

Equating to zero the coefficients of the unit vectors,

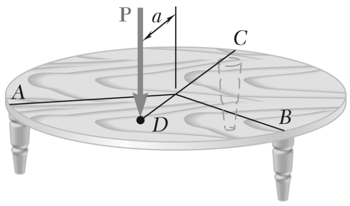
$$\mathbf{k}: \quad -20T_C + 450 = 0 \qquad T_C = 22.5 \text{ lb} \blacktriangleleft$$

$$\mathbf{i}: \quad -15(22.5) - 20T_A + 450 = 0 \qquad T_A = 5.625 \text{ lb} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - 45 \text{ lb} = 0$$

$$5.625 \text{ lb} + T_B + 22.5 \text{ lb} - 45 \text{ lb} = 0 \qquad T_B = 16.875 \text{ lb} \blacktriangleleft$$

$$T_A = 5.63 \text{ lb}; \; T_B = 16.88 \text{ lb}; \; T_C = 22.5 \text{ lb} \blacktriangleleft$$

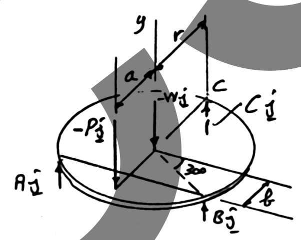


PROBLEM 0

The table shown weighs 30 lb and has a diameter of 4 ft. It is supported by three legs equally spaced around the edge. A vertical load **P** of magnitude 100 lb is applied to the top of the table at D. Determine the maximum value of a if the table is not to tip over. Show, on a sketch, the area of the table over which **P** can act without tipping the table.

SOLUTION

$$r = 2 \text{ ft} \quad b = r\sin 30° = 1 \text{ ft}$$



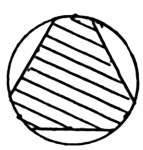
We shall sum moments about AB.

$$(b + r)C + (a - b)P - bW = 0$$

$$(1 + 2)C + (a - 1)100 - (1)30 = 0$$

$$C = \frac{1}{3}[30 - (a - 1)100]$$

If table is not to tip, $C \geq 0$.



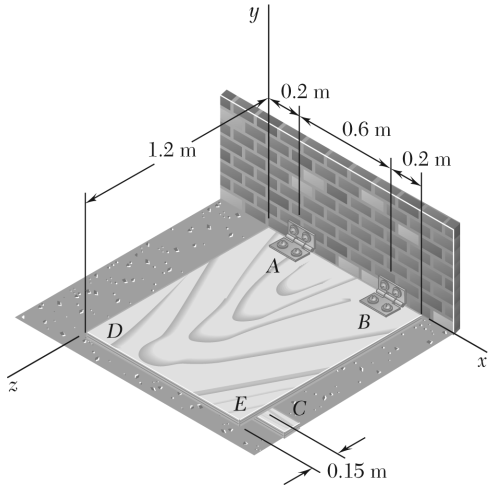
$$[30 - (a - 1)100] \geq 0$$

$$30 \geq (a - 1)100$$

$$a - 1 \leq 0.3 \quad a \leq 1.3 \text{ ft} \quad a = 1.300 \text{ ft}$$

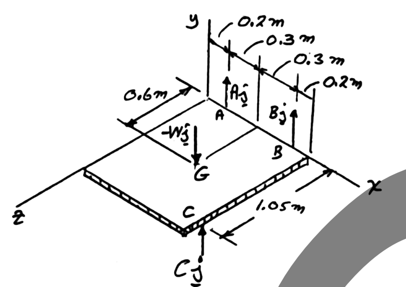
Only $\perp$ distance from P to AB matters. Same condition must be satisfied for each leg. P must be located in shaded area for no tipping.

PROBLEM



An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at *A* and *B* and is maintained in a position slightly above the floor by a small block *C*. Determine the vertical component of the reaction (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.8\mathbf{i} + 1.05\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18\text{ kg})9.81$$
$$W = 176.58\text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(0.6\mathbf{i}) \times B\mathbf{j} + (0.8\mathbf{i} + 1.05\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.8C\mathbf{k} - 1.05C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad 1.05C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.05}\right)176.58\text{ N} = 100.90\text{ N}$$

$$\mathbf{k}: \quad 0.6B + 0.8C - 0.3W = 0$$

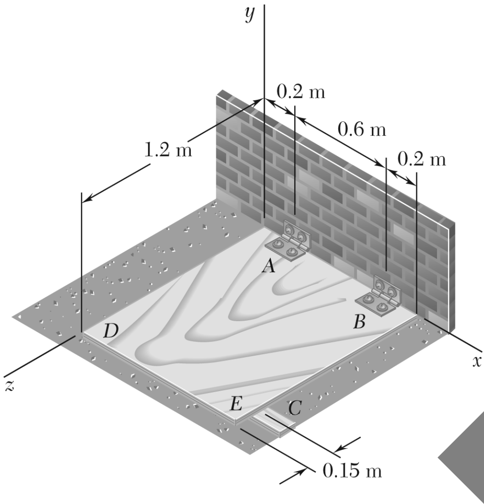
$$0.6B + 0.8(100.90\text{ N}) - 0.3(176.58\text{ N}) = 0 \quad B = -46.24\text{ N}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 46.24\text{ N} + 100.90\text{ N} + 176.58\text{ N} = 0 \quad A = 121.92\text{ N}$$

$$(a) \quad A = 121.9\text{ N} \quad (b) \quad B = -46.2\text{ N} \quad (c) \quad C = 100.9\text{ N} \blacktriangleleft$$

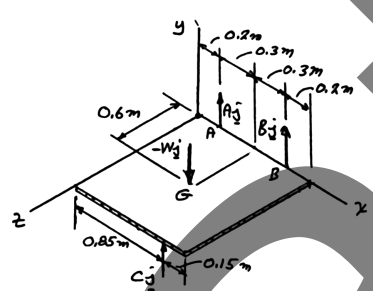
PROBLEM



Solve Problem 4.101, assuming that the small block *C* is moved and placed under edge *DE* at a point 0.15 m from corner *E*.

PROBLEM 4.101 An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at *A* and *B* and is maintained in a position slightly above the floor by a small block *C*. Determine the vertical component of the reaction (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.65\mathbf{i} + 1.2\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18 \text{ kg})\, 9.81 \text{ m/s}^2$$
$$W = 176.58 \text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$0.6\mathbf{i} \times B\mathbf{j} + (0.65\mathbf{i} + 1.2\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.65C\mathbf{k} - 1.2C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -1.2C + 0.6W = 0 \qquad C = \left(\frac{0.6}{1.2}\right) 176.58 \text{ N} = 88.29 \text{ N}$$

$$\mathbf{k}: \quad 0.6B + 0.65C - 0.3W = 0$$

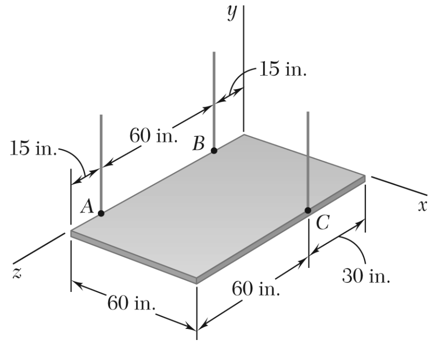
$$0.6B + 0.65(88.29 \text{ N}) - 0.3(176.58 \text{ N}) = 0 \qquad B = -7.36 \text{ N}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 7.36 \text{ N} + 88.29 \text{ N} - 176.58 \text{ N} = 0 \qquad A = 95.648 \text{ N}$$

$$(a) \quad A = 95.6 \text{ N} \quad (b) \quad B = -7.36 \text{ N} \quad (c) \quad C = 88.3 \text{ N} \blacktriangleleft$$

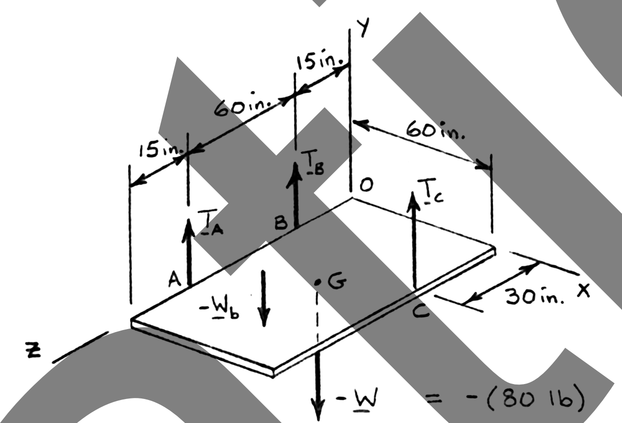
PROBLEM



The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the tension in each wire.

SOLUTION

Free-Body Diagram:



$$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{A/B} \times T_A \mathbf{j} + \mathbf{r}_{C/B} \times T_C \mathbf{j} + \mathbf{r}_{G/B} \times (-80\text{ lb})\mathbf{j} = 0$$

$$(60\text{ in.})\mathbf{k} \times T_A \mathbf{j} + [(60\text{ in.})\mathbf{i} + (15\text{ in.})\mathbf{k}] \times T_C \mathbf{j} + [(30\text{ in.})\mathbf{i} + (30\text{ in.})\mathbf{k}] \times (-80\text{ lb})\mathbf{j} = 0$$

$$-60T_A\mathbf{i} + 60T_C\mathbf{k} - 15T_C\mathbf{i} - 2400\mathbf{k} + 2400\mathbf{i} = 0$$

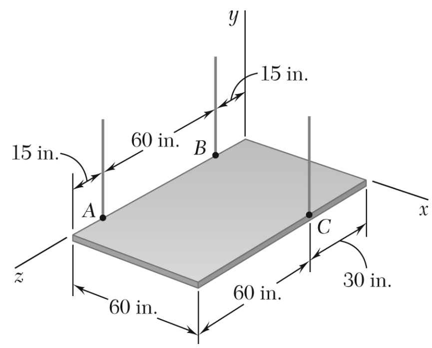
Equating to zero the coefficients of the unit vectors,

$\mathbf{i}$: $\quad 60T_A - 15(40) + 2400 = 0 \qquad T_A = 30.0\text{ lb}$ ◀

$\mathbf{k}$: $\quad 60T_C - 2400 = 0 \qquad T_C = 40.0\text{ lb}$ ◀

$\Sigma F_y = 0$: $\quad T_A + T_B + T_C - 80\text{ lb} = 0$

$$30\text{ lb} + T_B + 40\text{ lb} - 80\text{ lb} = 0 \qquad T_B = 10.00\text{ lb}$$ ◀

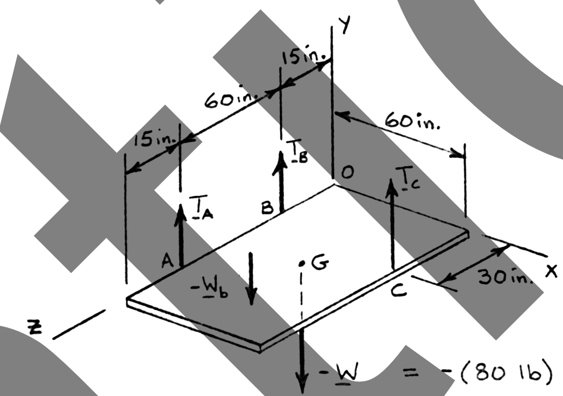


PROBLEM

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the weight and location of the lightest block that should be placed on the plate if the tensions in the three wires are to be equal.

SOLUTION

Free-Body Diagram:



Let $-W_b\mathbf{j}$ be the weight of the block and x and z the block's coordinates.

Since tensions in wires are equal, let

$$T_A = T_B = T_C = T$$

$$\Sigma M_0 = 0: \quad (\mathbf{r}_A \times T\mathbf{j}) + (\mathbf{r}_B \times T\mathbf{j}) + (\mathbf{r}_C \times T\mathbf{j}) + \mathbf{r}_G \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

or $$(75\,\mathbf{k}) \times T\mathbf{j} + (15\,\mathbf{k}) \times T\mathbf{j} + (60\mathbf{i} + 30\mathbf{k}) \times T\mathbf{j} + (30\mathbf{i} + 45\mathbf{k}) \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

or $$-75T\,\mathbf{i} - 15T\,\mathbf{i} + 60T\,\mathbf{k} - 30T\,\mathbf{i} - 30W\,\mathbf{k} + 45W\,\mathbf{i} - W_b \times \mathbf{k} + W_b\, z\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -120T + 45W + W_b z = 0 \qquad (1)$$

$$\mathbf{k}: \quad 60T - 30W - W_b x = 0 \qquad (2)$$

Also, $$\Sigma F_y = 0: \quad 3T - W - W_b = 0 \qquad (3)$$

Eq. (1) + 40 Eq. (3): $$5W + (z - 40)W_b = 0 \qquad (4)$$

Eq. (2) – 20 Eq. (3): $$-10W - (x - 20)W_b = 0 \qquad (5)$$

PROBLEM (Continued)

Solving Eqs. (4) and (5) for W_b/W and recalling that $0 \le x \le 60$ in., $0 \le z \le 90$ in.,

Eq. (4): $$\frac{W_b}{W} = \frac{5}{40-z} \ge \frac{5}{40-0} = 0.125$$

Eq. (5): $$\frac{W_b}{W} = \frac{10}{20-x} \ge \frac{10}{20-0} = 0.5$$

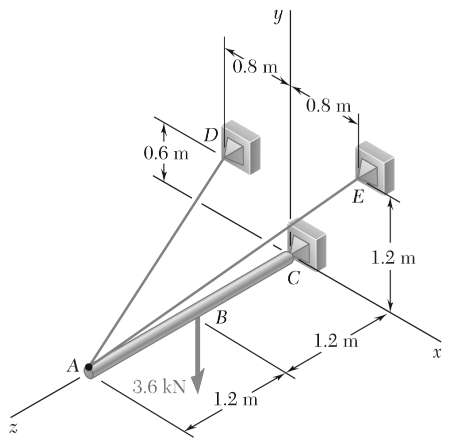
Thus, $(W_b)_{\min} = 0.5W = 0.5(80) = 40$ lb $\qquad (W_b)_{\min} = 40.0$ lb ◀

Making $W_b = 0.5W$ in Eqs. (4) and (5):

$$5W + (z-40)(0.5W) = 0 \qquad z = 30.0 \text{ in.} ◀$$

$$-10W - (x-20)(0.5W) = 0 \qquad x = 0 \text{ in.} ◀$$

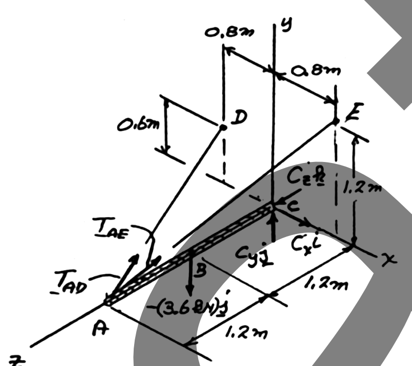
PROBLEM



A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AC} = 0)$.



$$\mathbf{r}_B = 1.2\mathbf{k}$$

$$\mathbf{r}_A = 2.4\mathbf{k}$$

$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$

$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$

$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6 \text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.55385T_{AD} - 1.02857T_{AE} + 4.32 = 0 \qquad (1)$$

$$\mathbf{j}: \quad -0.73846T_{AD} + 0.68671T_{AE} = 0$$

$$T_{AD} = 0.92857T_{AE} \qquad (2)$$

From Eq. (1): $\quad -0.55385(0.92857)T_{AE} - 1.02857T_{AE} + 4.32 = 0$

$$1.54286T_{AE} = 4.32$$

$$T_{AE} = 2.800 \text{ kN} \qquad T_{AE} = 2.80 \text{ kN} ◀$$

PROBLEM (Continued)

From Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600 \text{ kN}$ $\qquad T_{AD} = 2.60 \text{ kN}$ ◀

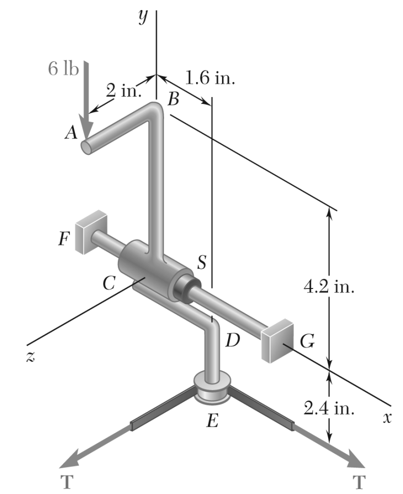
$$\Sigma F_x = 0: \quad C_x - \frac{0.8}{2.6}(2.6 \text{ kN}) + \frac{0.8}{2.8}(2.8 \text{ kN}) = 0 \qquad C_x = 0$$

$$\Sigma F_y = 0: \quad C_y + \frac{0.6}{2.6}(2.6 \text{ kN}) + \frac{1.2}{2.8}(2.8 \text{ kN}) - (3.6 \text{ kN}) = 0 \qquad C_y = 1.800 \text{ kN}$$

$$\Sigma F_z = 0: \quad C_z - \frac{2.4}{2.6}(2.6 \text{ kN}) - \frac{2.4}{2.8}(2.8 \text{ kN}) = 0 \qquad C_z = 4.80 \text{ kN}$$

$\mathbf{C} = (1.800 \text{ kN})\mathbf{j} + (4.80 \text{ kN})\mathbf{k}$ ◀

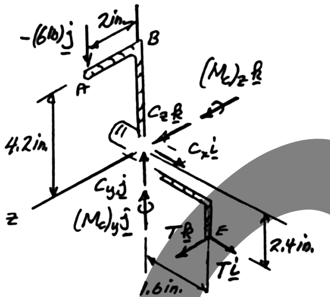
PROBLEM



The assembly shown is used to control the tension T in a tape that passes around a frictionless spool at E. Collar C is welded to rods ABC and CDE. It can rotate about shaft FG but its motion along the shaft is prevented by a washer S. For the loading shown, determine (*a*) the tension T in the tape, (*b*) the reaction at C.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{A/C} = 4.2\mathbf{j} + 2\mathbf{k}$$
$$\mathbf{r}_{E/C} = 1.6\mathbf{i} - 2.4\mathbf{j}$$

$$\Sigma M_C = 0: \quad \mathbf{r}_{A/C} \times (-6\mathbf{j}) + \mathbf{r}_{E/C} \times T(\mathbf{i}+\mathbf{k}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

$$(4.2\,\mathbf{j} + 2\mathbf{k}) \times (-6\mathbf{j}) + (1.6\mathbf{i} - 2.4\mathbf{j}) \times T(\mathbf{i}+\mathbf{k}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

Coefficient of **i**: $\quad 12 - 2.4T = 0 \qquad T = 5.00 \text{ lb}$ ◀

Coefficient of **j**: $\quad -1.6(5 \text{ lb}) + (M_C)_y = 0 \quad (M_C)_y = 8 \text{ lb}\cdot\text{in.}$

Coefficient of **k**: $\quad 2.4(5 \text{ lb}) + (M_C)_z = 0 \quad (M_C)_z = -12 \text{ lb}\cdot\text{in.}$

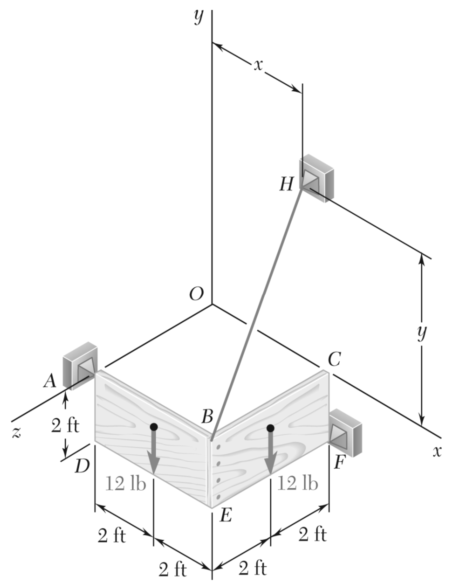
$$\mathbf{M}_C = (8.00 \text{ lb}\cdot\text{in.})\mathbf{j} - (12.00 \text{ lb}\cdot\text{in.})\mathbf{k} \;◀$$

$$\Sigma F = 0: \quad C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} - (6 \text{ lb})\mathbf{j} + (5 \text{ lb})\mathbf{i} + (5 \text{ lb})\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero.

$$C_x = -5 \text{ lb} \quad C_y = 6 \text{ lb} \quad C_z = -5 \text{ lb} \;◀$$

$$\mathbf{C} = -(5.00 \text{ lb})\mathbf{i} + (6.00 \text{ lb})\mathbf{j} - (5.00 \text{ lb})\mathbf{k} \;◀$$



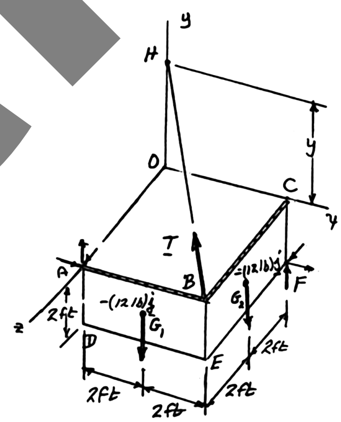
PROBLEM

Solve Problem 4.136, subject to the restriction that H must lie on the y-axis.

PROBLEM 4.136 Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (a) the location of H in the xy plane if the tension in the wire is to be minimum, (b) the corresponding minimum tension.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$$

$$\boldsymbol{\lambda}_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{G/A}\times(-12\mathbf{j}) + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{G_2/A}\times(-12\mathbf{j})) + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times T) = 0$$

$$\begin{vmatrix} 2 & -1 & 2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix}\frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix}\frac{1}{3} + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) = 0$$

$$(2\times 2\times 12)\frac{1}{3} + (-2\times 2\times 12 + 2\times 4\times 12)\frac{1}{3} + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) = 0$$

$$\boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) = -32 \qquad (1)$$

$$\overrightarrow{BH} = -4\mathbf{i} + y\mathbf{j} - 4\mathbf{k} \qquad BH = (32 + y^2)^{1/2}$$

$$\mathbf{T} = T\frac{\overrightarrow{BH}}{BH} = T\frac{-4\mathbf{i} + y\mathbf{j} - 4\mathbf{k}}{(32 + y^2)^{1/2}}$$

PROBLEM (Continued)

From Eq. (1):

$$\boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times T)=\begin{vmatrix}2 & -1 & -2\\ 4 & 0 & 0\\ -4 & y & -4\end{vmatrix}\frac{T}{3(32+y^2)^{1/2}}=-32$$

$$(-16-8y)T=-3\times 32(32+y^2)^{1/2} \qquad T=96\frac{(32+y^2)^{1/2}}{8y+16} \qquad (2)$$

$$\frac{dT}{dy}=0\!:\quad 96\frac{(8y+16)\frac{1}{2}(32+y^2)^{-1/2}(2y)+(32+y^2)^{1/2}(8)}{(8y+16)^2}$$

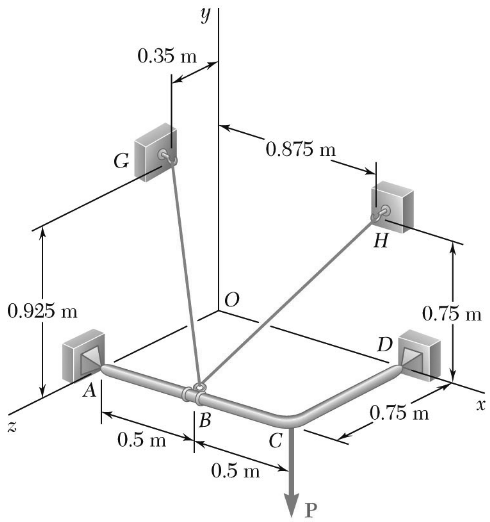
Numerator = 0: $(8y+16)y=(32+y^2)8$

$$8y^2+16y=32\times 8+8y^2$$

$x=0$ ft; $y=16.00$ ft ◀

From Eq. (2): $T=96\dfrac{(32+16^2)^{1/2}}{8\times 16+16}=11.3137$ lb

$T_{\min}=11.31$ lb ◀

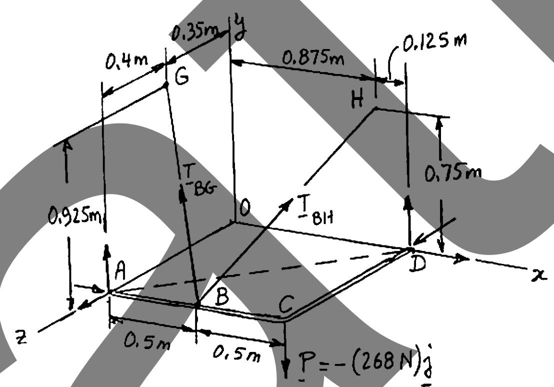


PROBLEM

The frame *ACD* is supported by ball-and-socket joints at *A* and *D* and by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the frame supports at Point *C* a load of magnitude $P = 268$ N, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\boldsymbol{\lambda}_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{(1\text{ m})\mathbf{i} - (0.75\text{ m})\mathbf{k}}{1.25\text{ m}}$$

$$\boldsymbol{\lambda}_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$\mathbf{T}_{BG} = T_{BG}\frac{\overrightarrow{BG}}{BG}$$

$$= T_{BG}\frac{-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k}}{1.125}$$

$$T_{BH} = T_{BH}\frac{\overrightarrow{BH}}{BH}$$

$$= T_{BH}\frac{0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k}}{1.125}$$

PROBLEM (Continued)

$$\mathbf{r}_{B/A} = (0.5\text{ m})\mathbf{i}; \quad \mathbf{r}_{C/A} = (1\text{ m})\mathbf{i}; \quad \mathbf{P} = -(268\text{ N})\mathbf{j}$$

To eliminate the reactions at A and D, we shall write

$$\Sigma\mathbf{M}_{AD} = 0: \quad \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}_{BG}) + \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}_{BH}) + \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{C/A}\times\mathbf{P}) = 0 \qquad (1)$$

Substituting for terms in Eq. (1) and using determinants,

$$\begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ -0.5 & 0.925 & -0.4 \end{vmatrix}\frac{T_{BG}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ 0.375 & 0.75 & -0.75 \end{vmatrix}\frac{T_{BH}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1 & 0 & 0 \\ 0 & -268 & 0 \end{vmatrix} = 0$$

Multiplying all terms by (–1.125),

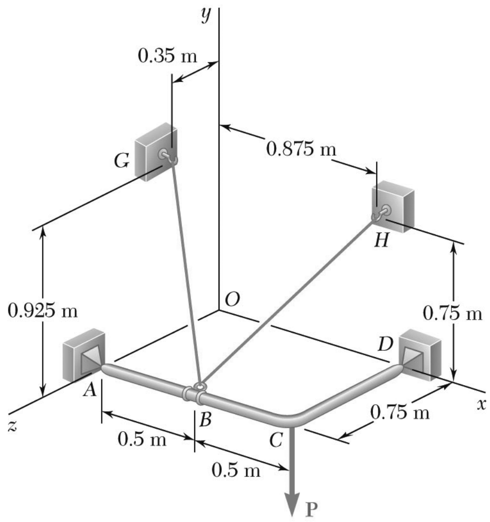
$$0.27750T_{BG} + 0.22500T_{BH} = 180.900 \qquad (2)$$

For this problem, $T_{BG} = T_{BH} = T$

$$(0.27750 + 0.22500)T = 180.900$$

$T = 360\text{ N}$ ◀

PROBLEM

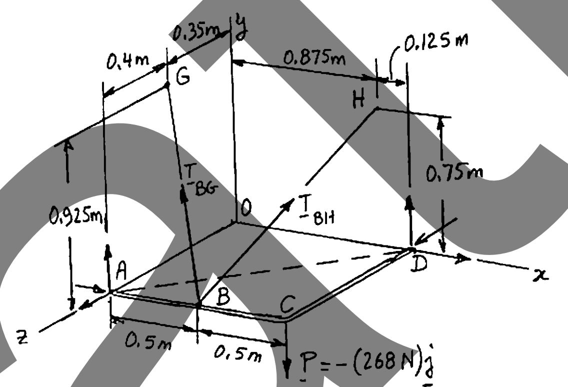


Solve Prob. assuming that cable *GBH* is replaced by a cable *GB* attached at *G* and *B*.

PROBLEM The frame *ACD* is supported by ball-and-socket joints at *A* and *D* and by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the frame supports at Point *C* a load of magnitude *P* = 268 N, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\lambda_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{(1\text{ m})\mathbf{i} - (0.75\text{ m})\mathbf{k}}{1.25\text{ m}}$$

$$\lambda_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$T_{BG} = T_{BG}\frac{\overrightarrow{BG}}{BG}$$

$$= T_{BG}\frac{-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k}}{1.125}$$

$$T_{BH} = T_{BH}\frac{\overrightarrow{BH}}{BH}$$

$$= T_{BH}\frac{0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k}}{1.125}$$

PROBLEM (Continued)

$$\mathbf{r}_{B/A} = (0.5\text{ m})\mathbf{i}; \quad \mathbf{r}_{C/A} = (1\text{ m})\mathbf{i}; \quad \mathbf{P} = -(268\text{ N})\mathbf{j}$$

To eliminate the reactions at A and D, we shall write

$$\Sigma \mathbf{M}_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BG}) + \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH}) + \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{C/A} \times \mathbf{P}) = 0 \qquad (1)$$

Substituting for terms in Eq. (1) and using determinants,

$$\begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ -0.5 & 0.925 & -0.4 \end{vmatrix} \frac{T_{BG}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ 0.375 & 0.75 & -0.75 \end{vmatrix} \frac{T_{BH}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1 & 0 & 0 \\ 0 & -268 & 0 \end{vmatrix} = 0$$

Multiplying all terms by (–1.125),

$$0.27750T_{BG} + 0.22500T_{BH} = 180.900 \qquad (2)$$

For this problem, $T_{BH} = 0$.

Thus, Eq. (2) reduces to

$$0.27750T_{BG} = 180.900 \qquad T_{BG} = 652\text{ N} \blacktriangleleft$$