

Course Title: Mathematics 2	Course Code: MPH 102	Spec.: Preparatory Year
Examiner: Dr. Moataz El-Zekey	M_A	Total Marks: 30
Date: 7 / 4 / 2019	Mid-term Exam (2019)	Time: 1.5 Hours

Answer the following multiple choice questions by circling the correct response.

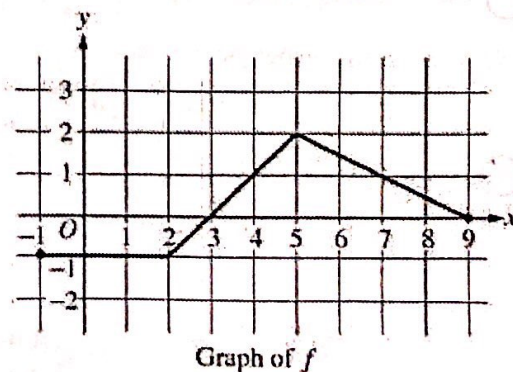
- The improper integral $\int_{-\infty}^{\infty} x \, dx$ is
 - convergent since the area to the left of $x = 0$ cancels with the area to the right of $x = 0$.
 - convergent since it equals $\lim_{t \rightarrow -\infty} \int_t^0 x \, dx + \lim_{s \rightarrow \infty} \int_0^s x \, dx = -\infty + \infty = 0$.
 - divergent by comparison to $\int_{-\infty}^{\infty} x e^{-x} \, dx$.
 - ☒ divergent since both integrals $\int_0^{\infty} x \, dx$ and $\int_{-\infty}^0 x \, dx$ are divergent.
- Let f be a differentiable function such that

$$\int f(x) \sin x \, dx = -f(x) \cos x + \int 4x^3 \cos x \, dx$$
 which of the following could be $f(x)$?
 - $\cos x$
 - $\sin x$
 - ☒ x^4
 - $-x^4$
 - $4x^3$
- $\int_0^{\pi} \sin^8 x \, dx =$ (Hint: Don't try to evaluate the integral; instead eliminate the three wrong answers)
 - π
 - $\frac{3\pi}{90} - 1$
 - ☒ $\frac{35\pi}{128}$
 - $\frac{\pi}{2}$
- Let f be a function such that $\int_6^{12} f(2x) \, dx = 10$, which of the following must be true?
 - $\int_{12}^{24} f(x) \, dx = 5$
 - ☒ $\int_{12}^{24} f(x) \, dx = 20$
 - $\int_6^{12} f(x) \, dx = 5$
 - $\int_6^{12} f(x) \, dx = 20$
- An antiderivative of $y = e^{x+e^x}$ is
 - $\frac{e^{x+e^x}}{1+e^x}$
 - $(1+e^x)e^{x+e^x}$
 - e^{x+e^x}
 - e^{1+e^x}
 - ☒ e^{e^x}
- Assume $f(x)$ is a function with continuous derivatives, and $f(7) = 10$; $f(2) = 5$; and $f'(7) = 3$. What is

$$\int_2^7 (2x-4)f''(x) \, dx$$
 - 25
 - 10
 - ☒ 20
 - 0
 - Can't be determined without knowing $f'(2)$.
- If $f(x) = \int_2^{\sin x} \sqrt{1+t^2} \, dt$, then $\frac{df}{dx} =$
 - $(\cos x)\sqrt{1+\sin x}$
 - $\sqrt{1+\sin^2 x}$
 - ☒ $(\cos x)\sqrt{1+\sin^2 x}$
 - $(\cos x)(1+\sin^2 x)^{3/2}$
- The graph of the function f is shown below. What is the value of the following integral?

$$\int_{-1}^9 (3f(x) + 2) \, dx =$$

- 7.5
- 9.5
- ☒ 27.5
- 47
- 48.5



9. For what values of p does the integral $\int_1^{\infty} 1/x^p dx$ converge?

A. $p < 1$

B. $p \geq 1$

☒ C. $p > 1$

D. None of these

10. Which of the following is not an improper?

A. $\int_0^1 \ln x dx$

☒ B. $\int_{-1}^1 \frac{dx}{x^2+1}$

C. $\int_2^5 \frac{dx}{\sqrt{x^2-4}}$

D. None of these

11. $\int \frac{1}{x^2+x} dx =$

A. $\ln|x^2+1| + C$

B. $\ln\left|\frac{x+1}{x}\right| + C$

☒ C. $\ln\left|\frac{x}{x+1}\right| + C$

D. None of these.

12. The substitution of $x = \sin \theta$ in the following integral results in

$$\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}} dx =$$

A. $\int_0^{1/2} \frac{\sin^2 \theta}{\cos \theta} d\theta$

B. $\int_0^{1/2} \sin^2 \theta d\theta$

☒ C. $\int_0^{\pi/6} \sin^2 \theta d\theta$

D. $\int_0^{\pi/3} \sin^2 \theta d\theta$

13. Using a trigonometric substitution, the following integral

$$\int \frac{x^2}{\sqrt{x^2+25}} dx =$$

☒ A. $25 \int \tan^2 \theta \sec \theta d\theta$

B. $5 \int \tan^2 \theta \sec \theta d\theta$

C. $25 \int (\tan^2 \theta / \sec \theta) d\theta$

D. $5 \int (\tan^2 \theta / \sec \theta) d\theta$

14. What is the form of the complete partial fractions decomposition of this function:

$$\frac{2x+1}{(x^2+4)^2(x-1)^2(x-3)}$$

A. $\frac{A}{x^2+4} + \frac{B}{(x^2+4)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{F}{x-3}$

☒ B. $\frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2} + \frac{E}{x-1} + \frac{F}{(x-1)^2} + \frac{G}{x-3}$

C. $\frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2} + \frac{Ex+F}{(x-1)^2} + \frac{G}{x-3}$

D. $\frac{Ax+B}{(x^2+4)^2} + \frac{Cx+D}{(x-1)^2} + \frac{F}{x-3}$

15. $\int \tan^{-1}(2x) dx =$

A. $x \tan^{-1}(2x) + C$

B. $x \tan^{-1}(2x) + \frac{1}{4} \ln(1+4x^2) + C$

☒ C. $x \tan^{-1}(2x) - \frac{1}{4} \ln(1+4x^2) + C$

D. None of these

16. What can be said about the integrals

(i) $\int_0^1 \frac{1}{x^2} dx$

(ii) $\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$

☒ A. (i) diverges and (ii) converges

B. Both (i) and (ii) converge

C. (i) converges and (ii) diverges

D. Both (i) and (ii) diverge.

GOOD LUCK!

Course Title: Mathematics 2	Course Code: MPH 102	Spec.: Preparatory Year
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Answer the following multiple choice questions by circling the correct response.

1. What is the form of the complete partial fractions decomposition of this function:

$$\frac{2x + 1}{(x^2 + 4)^2(x - 1)^2(x - 3)}$$

A. $\frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2} + \frac{E}{x - 1} + \frac{F}{(x - 1)^2} + \frac{G}{x - 3}$

B. $\frac{A}{x^2 + 4} + \frac{B}{(x^2 + 4)^2} + \frac{C}{x - 1} + \frac{D}{(x - 1)^2} + \frac{F}{x - 3}$

C. $\frac{Ax + B}{(x^2 + 4)^2} + \frac{Cx + D}{(x - 1)^2} + \frac{F}{x - 3}$

D. $\frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2} + \frac{Ex + F}{(x - 1)^2} + \frac{G}{x - 3}$

2. $\int \frac{1}{x^2 + x} dx =$

A. $\ln \left| \frac{x+1}{x} \right| + C$

B. $\ln \left| \frac{x}{x+1} \right| + C$

C. $\ln |x^2 + 1| + C$

D. None of these.

3. The substitution of $x = \sin \theta$ in the following integral results in

$$\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}} dx =$$

A. $\int_0^{\pi/3} \sin^2 \theta d\theta$

B. $\int_0^{\pi/6} \sin^2 \theta d\theta$

C. $\int_0^{1/2} \sin^2 \theta d\theta$

D. $\int_0^{1/2} \frac{\sin^2 \theta}{\cos \theta} d\theta$

4. $\int_0^{\pi} \sin^8 x dx =$ (Hint: Don't try to evaluate the integral; instead eliminate the three wrong answers)

A. $\frac{\pi}{2}$

B. $\frac{35\pi}{128}$

C. $\frac{3\pi}{90} - 1$

D. π

5. Let f be a function such that $\int_6^{12} f(2x) dx = 10$, which of the following must be true?

A. $\int_{12}^{24} f(x) dx = 20$

B. $\int_{12}^{24} f(x) dx = 5$

C. $\int_6^{12} f(x) dx = 5$

D. $\int_6^{12} f(x) dx = 20$

6. An antiderivative of $y = e^{x+e^x}$ is

A. $\frac{e^{x+e^x}}{1+e^x}$

B. e^{1+e^x}

C. e^{e^x}

D. $(1 + e^x)e^{x+e^x}$

E. e^{x+e^x}

7. Assume $f(x)$ is a function with continuous derivatives, and $f(7) = 10$; $f(2) = 5$; and $f'(7) = 3$. What is

$$\int_2^7 (2x - 4)f''(x) dx$$

A. 10

B. 20

C. 25

D. 0

E. Can't be determined without knowing $f'(2)$.

8. If $f(x) = \int_2^{\sin x} \sqrt{1+t^2} dt$, then $\frac{df}{dx} =$

A. $\sqrt{1 + \sin^2 x}$

B. $(\cos x)\sqrt{1 + \sin^2 x}$

C. $(\cos x)\sqrt{1 + \sin x}$

D. $(\cos x)(1 + \sin^2 x)^{3/2}$

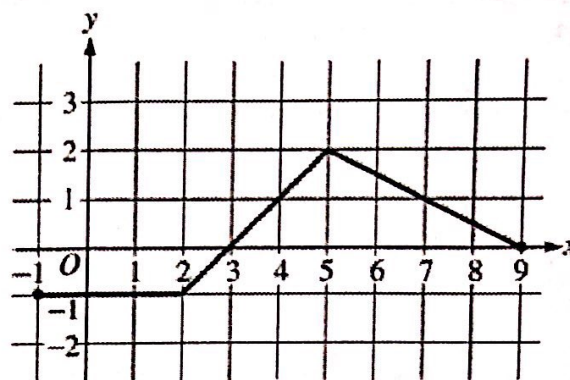
9. For what values of p does the integral $\int_1^\infty 1/x^p dx$ converge?

- ☒ A. $p > 1$ B. $p \geq 1$ C. $p < 1$ D. None of these

10. The graph of the function f is shown below. What is the value of the following integral?

$$\int_{-1}^9 (3f(x) + 2) dx =$$

- A. 9.5
B. 7.5
C. 47
D. 48.5
☒ E. 27.5



Graph of f

11. The improper integral $\int_{-\infty}^\infty x dx$ is

- A. convergent since it equals $\lim_{t \rightarrow -\infty} \int_t^0 x dx + \lim_{s \rightarrow \infty} \int_0^s x dx = -\infty + \infty = 0$.
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13. Let f be a differentiable function such that

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which of the following could be $f(x)$?

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14. What can be said about the integrals

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- A. (i) converges and (ii) diverges
☒ C. (i) diverges and (ii) converges
B. Both (i) and (ii) diverge
D. Both (i) and (ii) converge.

15. Using a trigonometric substitution, the following integral

$$\int \frac{x^2}{\sqrt{x^2+25}} =$$

- A. $25 \int (\tan^2 \theta / \sec \theta) d\theta$ B. $5 \int (\tan^2 \theta / \sec \theta) d\theta$
☒ C. $25 \int \tan^2 \theta \sec \theta d\theta$ D. $5 \int \tan^2 \theta \sec \theta d\theta$

16. $\int \tan^{-1}(2x) dx =$

- A. $x \tan^{-1}(2x) + C$ ☒ B. $x \tan^{-1}(2x) - \frac{1}{4} \ln(1 + 4x^2) + C$
C. $x \tan^{-1}(2x) + \frac{1}{4} \ln(1 + 4x^2) + C$ D. None of these

GOOD LUCK!