I.4 Double Integrals In Polar Coordinates

1. Evaluate $\iint\limits_D y^2 + 3x\, dA$ where D is the region in the 3rd quadrant between $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

2. Evaluate $\iint\limits_D \sqrt{1 + 4x^2 + 4y^2}\, dA$ where D is the bottom half of $x^2 + y^2 = 16$.

3. Evaluate $\iint\limits_D 4xy - 7\, dA$ where D is the portion of $x^2 + y^2 = 2$ in the 1st quadrant.

4. Use a double integral to determine the area of the region that is inside $r = 4 + 2\sin\theta$ and outside $r = 3 - \sin\theta$.

5. Evaluate the following integral by first converting to an integral in polar coordinates.

(a) $\displaystyle\int_0^3 \int_{-\sqrt{9-x^2}}^0 \mathbf{e}^{x^2+y^2}\, dy\, dx$

(b) $\displaystyle\int_{-2}^0 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2\, dx\, dy$

6. Use a double integral to determine the volume of the solid that is inside the cylinder $x^2 + y^2 = 16$, below $z = 2x^2 + 2y^2$ and above the xy-plane.

7. Use a double integral to determine the volume of the solid that is bounded by $z = 8 - x^2 - y^2$ and $z = 3x^2 + 3y^2 - 4$.

8. Find the area of the portion of the cone $x^2 + y^2 = 3z^2$ lying above the xy-plane and inside the cylinder $x^2 + y^2 = 4y$.

9. Use a double integral to derive the area of a circle of radius a.

10. Use a double integral to derive the area of the region between circles of radius a and b with $\alpha \leq \theta \leq \beta$.

11. Consider the integral $\iint\limits_D \dfrac{1}{\sqrt{x^2 + y^2}}\, dA$, where D is the unit disk centered at the origin.

(a) Why might this integral be considered improper?

(b) Calculate the value of the integral of the same function $1/\sqrt{x^2 + y^2}$ over the annulus with outer radius 1 and inner radius δ.

(c) Obtain a value for the integral on the whole disk by letting δ approach 0.

(d) For which values λ can we replace the denominator with $(x^2 + y^2)^\lambda$ in the original integral and still get a finite value for the improper integral?

I.5 Triple Integrals

1. Evaluate $\displaystyle\int_2^3 \int_{-1}^4 \int_1^0 4x^2y - z^3 \, dz \, dy \, dx$.

2. Evaluate $\displaystyle\int_0^1 \int_0^{z^2} \int_0^3 y \cos\left(z^5\right) \, dx \, dy \, dz$

3. Evaluate $\displaystyle\iiint\limits_E 6z^2 \, dV$ where E is the region below $4x + y + 2z = 10$ in the first octant.

4. Evaluate $\displaystyle\iiint\limits_E 3 - 4x \, dV$ where E is the region below $z = 4 - xy$ and above the region in the xy-plane defined by $0 \le x \le 2$, $0 \le y \le 1$.

5. Evaluate $\displaystyle\iiint\limits_E 12y - 8x \, dV$ where E is the region behind $y = 10 - 2z$ and in front of the region in the xz-plane bounded by $z = 2x$, $z = 5$ and $x = 0$.

6. Evaluate $\displaystyle\iiint\limits_E yz \, dV$ where E is the region bounded by $x = 2y^2 + 2z^2 - 5$ and the plane $x = 1$.

7. Evaluate $\displaystyle\iiint\limits_E 15z \, dV$ where E is the region between $2x + y + z = 4$ and $4x + 4y + 2z = 20$ that is in front of the region in the yz-plane bounded by $z = 2y^2$ and $z = \sqrt{4y}$.

8. Use a triple integral to determine the volume of the region below $z = 4 - xy$ and above the region in the xy-plane defined by $0 \le x \le 2$ $0 \le y \le 1$.

9. Use a triple integral to determine the volume of the region that is below $z = 8 - x^2 - y^2$ above $z = -\sqrt{4x^2 + 4y^2}$ and inside $x^2 + y^2 = 4$.

10. Express the triple integral $\displaystyle\iiint\limits_E dxdydz$ in terms of iterated integrals in six different ways. The region E lies in the first octant and is bounded by the cylinder $x^2 + z^2 = 4$ and the plane $y = 3$. Find the value of the integral.