Systems of Linear Equations

1 - Determine which equations are linear equations in the variables x, y, and z. If any equation is not linear, explain why not.

(a) $x - \pi y + \sqrt[3]{5z} = 0$

(b) $x^{-1} + 7y + z = \sin\left(\frac{\pi}{9}\right)$

(c) $\cos x - 4y + z = \sqrt{3}$

2 - Find a linear equation that has the same solution set as the given equation (possibly with some restrictions on the variables).

(a) $2x + y = 7 - 3y$

(b) $\frac{1}{x} + \frac{1}{y} = \frac{4}{xy}$

3 - Find the solution set of each equation.

(a) $3x - 6y = 0$

(b) $x + 2y + 3z = 4$

4 - Draw graphs corresponding to the given linear systems. Determine geometrically whether each system has a unique solution, infinitely many solutions, or no solution. Then solve each system algebraically to confirm your answer.

(a) $x + y = 0$
$2x + y = 3$

(b) $3x - 6y = 3$
$-x + 2y = 1$

5 - Solve the given system by back substitution.

(a) $x - 2y = 1$
$y = 3$

(b) $x - y + z = 0$
$2y - z = 1$
$3z = -1$

6 - Solve the given system.

$$\begin{aligned} x \quad &= 2 \\ 2x + y \quad &= -3 \\ -3x - 4y + z &= -10 \end{aligned}$$

7 - Find the augmented matrices of the linear systems.

(a) $x - y = 0$
$2x + y = 3$

(b) $2x_1 + 3x_2 - x_3 = 1$
$x_1 + x_3 = 0$
$-x_1 + 2x_2 - 2x_3 = 0$

8 - Find a system of linear equations that has the given matrix as its augmented matrix.

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 2 & -1 & 1 & 1 \end{array}\right]$$

9 - (a) Find a system of two linear equations in the variables x and y whose solution set is given by the parametric equations $x = t$ and $y = 3 - 2t$.

(b) Find another parametric solution to the system in part (a) in which the parameter is s and $y = s$.

10 - The system of equations are nonlinear. Find substitutions (changes of variables) that convert each system into a linear system and use this linear system to help solve the given system.

$$\frac{2}{x} + \frac{3}{y} = 0$$

$$\frac{3}{x} + \frac{4}{y} = 1$$

11 - Determine whether the given matrix is in row echelon form. If it is, state whether it is also in reduced row echelon form.

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 0 & 1 & 0 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

12 - Use elementary row operations to reduce the given matrices to:
(a) row echelon form and (b) reduced row echelon form.

$$\text{Matrix (1)} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{Matrix (2)} = \begin{bmatrix} 3 & -2 & -1 \\ 2 & -1 & -1 \\ 4 & -3 & -1 \end{bmatrix}$$

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13 - In general, what is the elementary row operation that" undoes" each of the three elementary row operations.

$$R_i \leftrightarrow R_j,\ kR_i, \text{ and } R_i + kR_j?$$

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14 - Show that the given matrices are row equivalent and find a sequence of elementary row operations that will convert A into B.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$$

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15 - What is the net effect of performing the following sequence of elementary row operations on a matrix (with at least two rows)?

$$R_2 + R_1,\ R_1 - R_2,\ R_2 + R_1,\ -R_1$$

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16 - Students frequently perform the following type of calculation to introduce a zero into a matrix :

$$\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \xrightarrow{3R_2 - 2R_1} \begin{bmatrix} 3 & 1 \\ 0 & 10 \end{bmatrix}$$

However, $3R_2 - 2R_1$ is not an elementary row operation. Why not? Show how to achieve the same result using elementary row operations.

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17 - Solve the given system of equations using either Gaussian or Gauss-Jordan elimination.

(a) $x_1 + 2x_2 - 3x_3 = 9$ (b) $w + x + 2y + z = 1$

$$2x_1 - x_2 + x_3 = 0$$
$$4x_1 - x_2 + x_3 = 4$$

$$w - x - y + z = 0$$
$$x + y = -1$$
$$w + x + z = 2$$

18 - Determine by inspection whether a linear system with the given augmented matrix has a unique solution, infinitely many solutions, or no solution. Justify your answers.

(a) $$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 2 \\ 0 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right]$$

(b) $$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 \\ 7 & 7 & 7 & 7 & 7 & 7 \end{array}\right]$$

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19 - For what value(s) of k, if any, will the systems have (a) no solution, (b) a unique solution, and (c) infinitely many solutions?

(a) $$\begin{aligned} x + ky &= 1 \\ kx + y &= 1 \end{aligned}$$

(b) $$\begin{aligned} x + y + kz &= 1 \\ x + ky + z &= 1 \\ kx + y + z &= -2 \end{aligned}$$

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20 - Find the line of intersection of the given planes.

$$3x + 2y + z = -1 \quad \text{and} \quad 2x - y + 4z = 5$$

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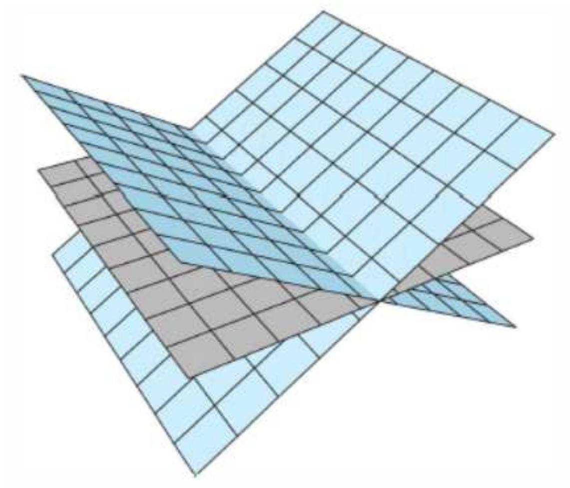
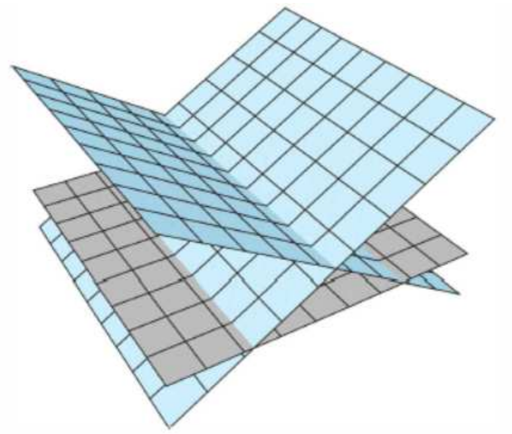
21 - (a) Give an example of three planes that have a common line of intersection



(b) Give an example of three planes that intersect in pairs but have no common point of intersection



22 - Determine whether the lines **x**= **p**+ s**u** and x = **q** + t**v** intersect and, if they do, find their point of intersection.

$$\mathbf{p} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$