26. Evaluate $\iiint\limits_E 3z\,dV$ where E is the region below $x^2+y^2+z^2=1$ and inside $z=\sqrt{x^2+y^2}$.

27. Evaluate $\iiint\limits_E x^2\,dV$ where E is the region above $x^2+y^2+z^2=36$ and inside $z=-\sqrt{3x^2+3y^2}$.

28. Evaluate the following integrals by first converting to an integral in spherical coordinates.

(a) $\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{6x^2+6y^2}}^{\sqrt{7-x^2-y^2}} 18y \;\, dz\,dy\,dx$

(b) $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} \sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$

29. Find the centroid of a homogeneous half-ball of radius R.

30. Find the mass of a ball of radius R whose density σ is proportional to the squared distance from the center.

31. An object occupies the region in the first octant bounded by the cones $\varphi=\pi/4$ and $\varphi=\pi/3$, and the sphere $\rho=\sqrt{6}$, and has density proportional to the distance from the origin. Find the mass.

I.7 Change of Variables

1. Compute the Jacobian of each transformation.

(a) $x=4u-3v^2 \qquad y=u^2-6v.$

(b) $x=u^2v^3 \qquad y=4-2\sqrt{u}.$

(c) $x=\dfrac{v}{u} \qquad y=u^2-4v^2.$

2. If R is the region inside $\dfrac{x^2}{4}+\dfrac{y^2}{36}=1$ determine the region we would get applying the transformation $x=2u, y=6v$ to R.

3. If R is the parallelogram with vertices $(1,0),(4,3),(1,6)$ and $(-2,3)$ determine the region we would get applying the transformation $x=\dfrac{1}{2}(v-u)$, $y=\dfrac{1}{2}(v+u)$ to R.

4. If R is the region bounded by $xy=1, xy=3, y=2$ and $y=6$ determine the region we would get applying the transformation $x=\dfrac{v}{6u}$, $y=2u$ to R.

5. Evaluate $\iint\limits_R xy^3\,dA$ where R is the region bounded by $xy = 1, xy = 3, y = 2$ and $y = 6$ using the transformation $x = \dfrac{v}{6u}$, $y = 2u$.

6. Evaluate $\iint\limits_R 6x - 3y\,dA$ where R is the parallelogram with vertices $(2, 0), (5, 3), (6, 7)$ and $(3, 4)$ using the transformation $x = \dfrac{1}{3}(v - u)$, $y = \dfrac{1}{3}(4v - u)$ to R.

7. Evaluate $\iint\limits_R x + 2y\,dA$ where R is the triangle with vertices $(0, 3), (4, 1)$ and $(2, 6)$ using the transformation $x = \dfrac{1}{2}(u - v)$, $y = \dfrac{1}{4}(3u + v + 12)$ to R.

8. Derive the transformation used in problem 8.

9. Derive a transformation that will convert the triangle with vertices $(1, 0), (6, 0)$ and $(3, 8)$ into a right triangle with the right angle occurring at the origin of the uv system.

10. Evaluate $\iint xy\,dx\,dy$ over the square with corners $(0, 0), (1, 1), (2, 0)$, and $(1, -1)$ in two ways: directly, and using $x = (u + v)/2, y = (u - v)/2$.

11. Evaluate $\iint \sin(9x^2 + 4y^2)\,dA$, over the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.

12. Evaluate $\iiint\limits_E dV$ where E is the solid enclosed by the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$