Chapter 2

Line Integrals

II.1 Line Integrals of Scalar Functions - I

For problems 1–7 evaluate the given line integral. Follow the direction of C as given in the problem statement.

1. Evaluate $\int_C 3x^2 - 2y\,ds$ where C is the line segment from $(3, 6)$ to $(1, -1)$.

2. Evaluate $\int_C 2yx^2 - 4x\,ds$ where C is the lower half of the circle centered at the origin of radius 3 with clockwise rotation.

3. Evaluate $\int_C 6x\,ds$ where C is the portion of $y = x^2$ from $x = -1$ to $x = 2$. The direction of C is in the direction of increasing x.

4. Evaluate $\int_C xy - 4z\,ds$ where C is the line segment from $(1, 1, 0)$ to $(2, 3, -2)$.

5. Evaluate $\int_C x^2y^2\,ds$ where C is the circle centered at the origin of radius 2 centered on the y-axis at $y = 4$. See the sketches below (Figure 2.1) for orientation.

6. Evaluate $\int_C 16y^5\,ds$ where C is the portion of $x = y^4$ from $y = 0$ to $y = 1$ followed by the line segment form $(1, 1)$ to $(1, -2)$ which in turn is followed by the line segment from $(1, -2)$ to $(2, 0)$.

7. Evaluate $\int_C 4y - x\,ds$ where C is the upper portion of the circle centered at

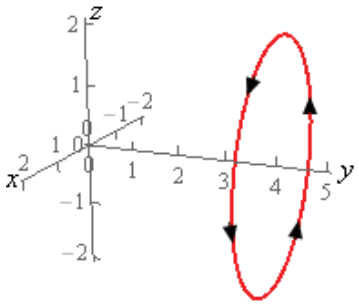

Figure 2.1

the origin of radius 3 from $\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ to $\left(-\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right)$ in the counter clockwise rotation followed by the line segment form $\left(-\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right)$ to $\left(4, -\frac{3}{\sqrt{2}}\right)$ which in turn is followed by the line segment from $\left(4, -\frac{3}{\sqrt{2}}\right)$ to $(4, 4)$.

8. Evaluate $\int_C y^3 - x^2\, ds$ for each of the following curves.

(a) C is the line segment from $(3, 6)$ to $(0, 0)$ followed by the line segment from $(0, 0)$ to $(3, -6)$.

(b) C is the line segment from $(3, 6)$ to $(3, -6)$.

9. Evaluate $\int_C 4x^2\, ds$ for each of the following curves.

(a) C is the portion of the circle centered at the origin of radius 2 in the 1^{st} quadrant rotating in the clockwise direction.

(b) C is the line segment from $(0, 2)$ to $(2, 0)$.

10. Evaluate $\int_C 2x^3\, ds$ for each of the following curves.

(a) C is the portion $y = x^3$ from $x = -1$ to $x = 2$.

(b) C is the portion $y = x^3$ from $x = 2$ to $x = -1$.

II.2 Line Integrals of Scalar Functions - II

For problems 1–5 evaluate the given line integral. Follow the direction of C as given in the problem statement.

1. Evaluate $\int_C \sqrt{1+y}\,dy$ where C is the portion of $y = e^2 x$ from $x = 0$ to $x = 2$.

2. Evaluate $\int_C 2y\,dx + (1-x)\,dy$ where C is portion of $y = 1 - x^3$ from $x = -1$ to $x = 2$.

3. Evaluate $\int_C x^2\,dy - yz\,dz$ where C is the line segment from $(4, -1, 2)$ to $(1, 7, -1)$.

4. Evaluate $\int_C 1 + x^3\,dx$ where C is the right half of the circle of radius 2 with counter clockwise rotation followed by the line segment from $(0, 2)$ to $(-3, -4)$.

5. Evaluate $\int_C 2x^2\,dy - xy\,dx$ where C is the line segment from $(1, -5)$ to $(-2, -3)$ followed by the portion of $y = 1 - x^2$ from $x = -2$ to $x = 2$ which in turn is followed by the line segment from $(2, -3)$ to $(4, -3)$.

6. Evaluate $\int_C (x-y)\,dx - yx^2\,dy$ for each of the following curves.

(a) C is the portion of the circle of radius 6 in the $1^{st}, 2^{nd}$ and 3^{rd} quadrant with clockwise rotation.

(b) C is the line segment from $(0, -6)$ to $(6, 0)$.

7. Evaluate $\int_C x^3\,dy - (y+1)\,dx$ for each of the following curves.

(a) C is the line segment from $(1, 7)$ to $(-2, 4)$.

(b) C is the line segment from $(-2, 4)$ to $(1, 7)$.

II.3 Line Integrals of Vector Fields

1. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = y^2\,\vec{i} + (3x - 6y)\,\vec{j}$ and C is the line segment from $(3, 7)$ to $(0, 12)$.

2. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = (x+y)\,\vec{i} + (1-x)\,\vec{j}$ and C is the portion of $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ that is in the 4^{th} quadrant with the counter clockwise rotation.

3. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = y^2\,\vec{i} + \left(x^2-4\right)\vec{j}$ and C is the portion of $y = (x-1)^2$ from $x = 0$ to $x = 3$.

4. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y,z) = \mathbf{e}^{2x}\,\vec{i} + z\,(y+1)\,\vec{j} + z^3\,\vec{k}$ and C is given by $\vec{r}(t) = t^3\,\vec{i} + (1-3t)\,\vec{j} + \mathbf{e}^t\,\vec{k}$ for $0 \le t \le 2$.

5. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = 3y\,\vec{i} + \left(x^2-y\right)\vec{j}$ and C is the upper half of the circle centered at the origin of radius 1 with counter clockwise rotation and the portion of $y = x^2 - 1$ from $x = -1$ to $x = 1$.

6. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = xy\,\vec{i} + (1+3y)\,\vec{j}$ and C is the line segment from $(0,-4)$ to $(-2,-4)$ followed by portion of $y = -x^2$ from $x = -2$ to $x = 2$ which is in turn followed by the line segment from $(2,-4)$ to $(5,1)$.

7. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = (6x-2y)\,\vec{i} + x^2\vec{j}$ for each of the following curves.

(a) C is the line segment from $(6,-3)$ to $(0,0)$ followed by the line segment from $(0,0)$ to $(6,3)$.

(b) C is the line segment from $(6,-3)$ to $(6,3)$.

8. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = 3\,\vec{i} + (xy-2x)\,\vec{j}$ for each of the following curves.

(a) C is the upper half of the circle centered at the origin of radius 4 with counter clockwise rotation.

(b) C is the upper half of the circle centered at the origin of radius 4 with clockwise rotation.

9. An object moves from $(1,1,1)$ to $(2,4,8)$ along the path $\vec{r}(t) = \langle t, t^2, t^3\rangle$, subject to the force $\vec{F} = \langle \sin x, \sin y, \sin z\rangle$. Find the work done.