Advanced Electric Circuits
ELE213

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Lecture - 11

Course Content

Chapter (5)

Harmonics

5.1 Introduction

5.2 Basics of Harmonic Theory

5.3 Linear and Nonlinear Loads

5.4 Effects of Harmonics on Distribution Systems

5.5 Harmonic Analyses

5.6 The General Equation for a Complex Waveform

5.7 Harmonic Synthesis

5.8 RMS Value, Mean Value and the Form Factor of a Complex Wave

5.9 Power Associated with Complex Waveforms

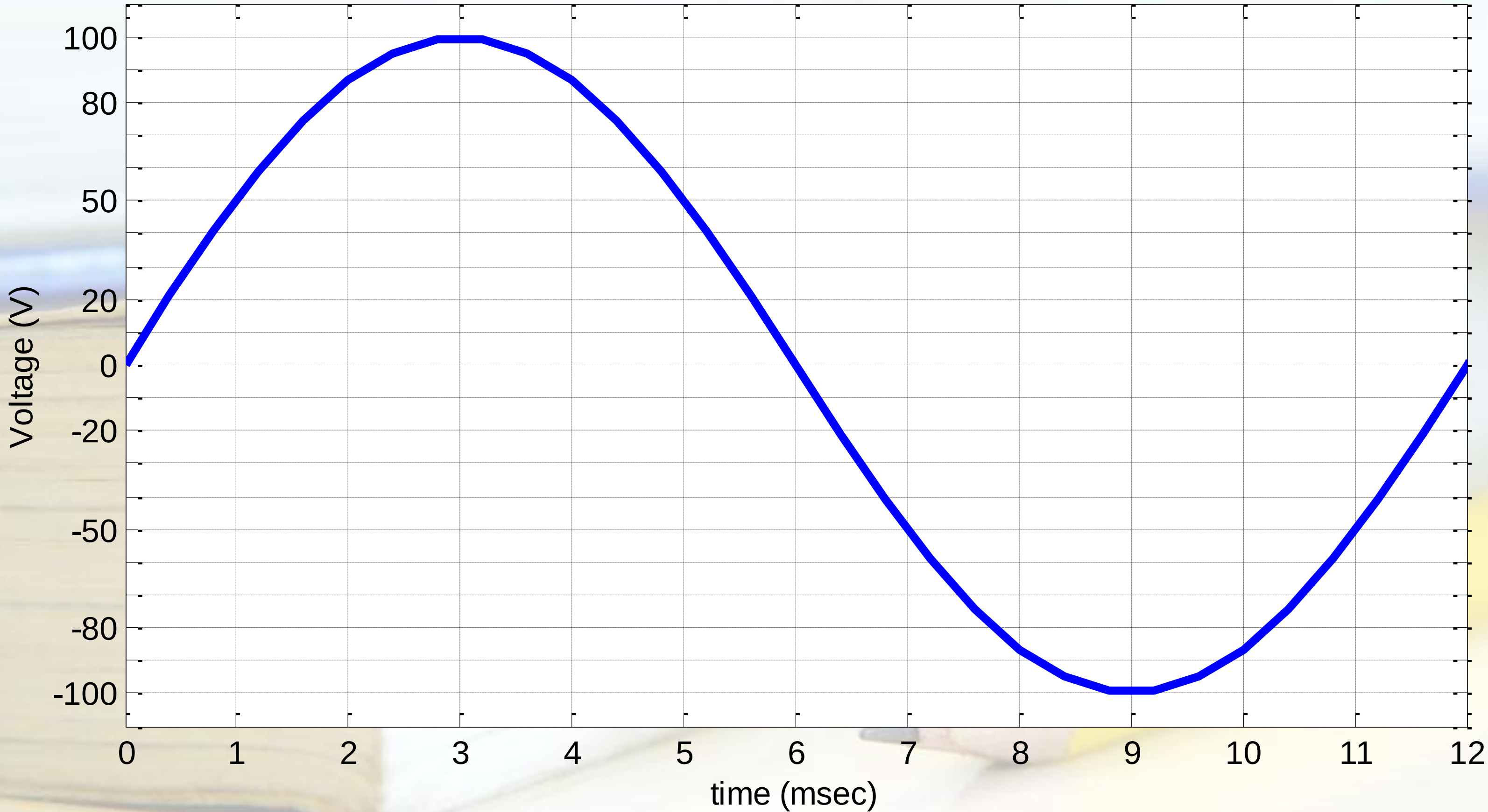
5.10 Power Factor

5.11 Harmonics in Single Phase Circuits

Introduction

Introduction

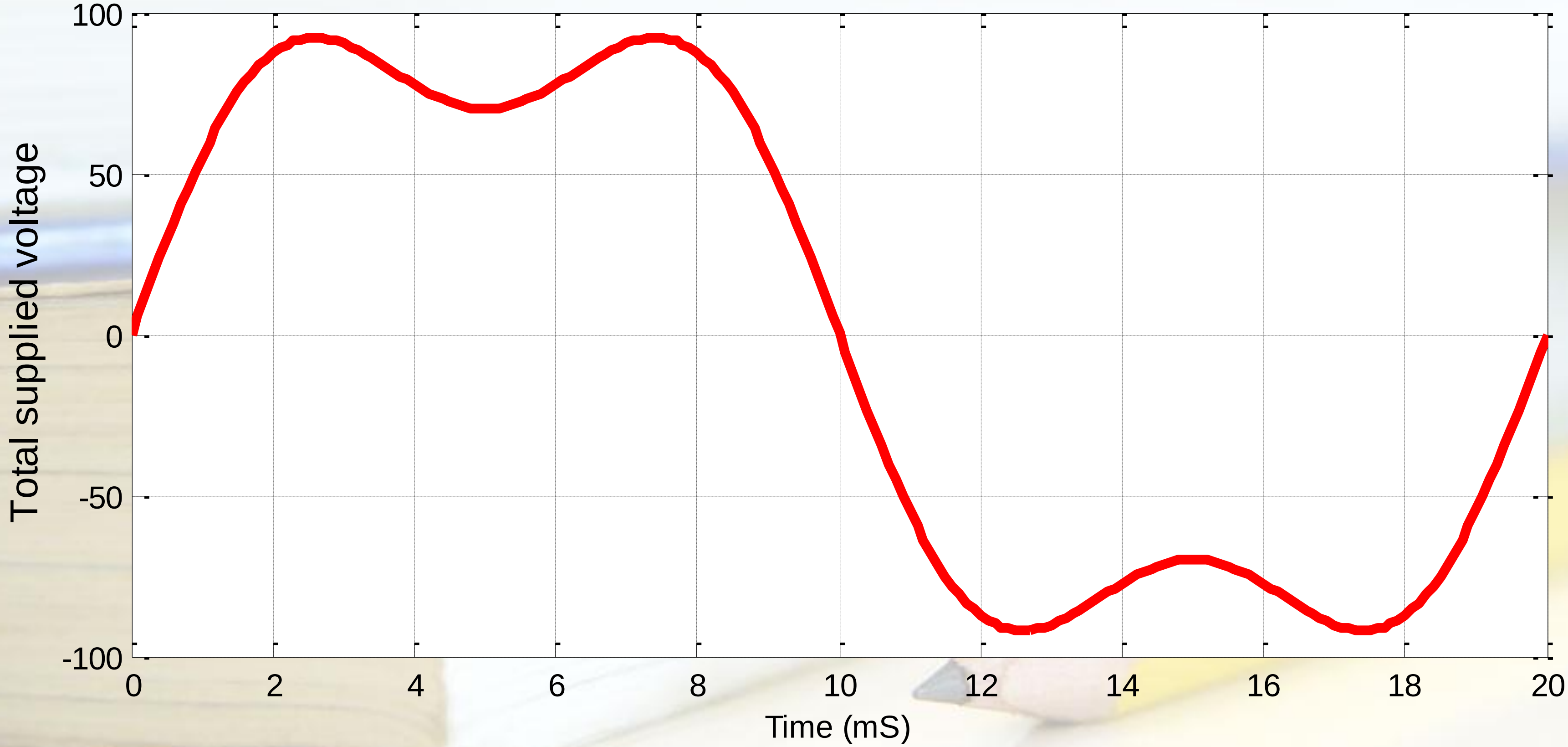
Ideally, an electricity supply should invariably show a perfectly sinusoidal voltage signal at every customer location.



How can a sine wave correctly drawn ???

Introduction

Utilities often find it hard to preserve such desirable conditions.



Basics of Harmonic Theory

Basics of Harmonic Theory

The term “harmonics” was originated in the field of acoustics, where it was related to the vibration of a string or an air column at a frequency that is a multiple of the base frequency.

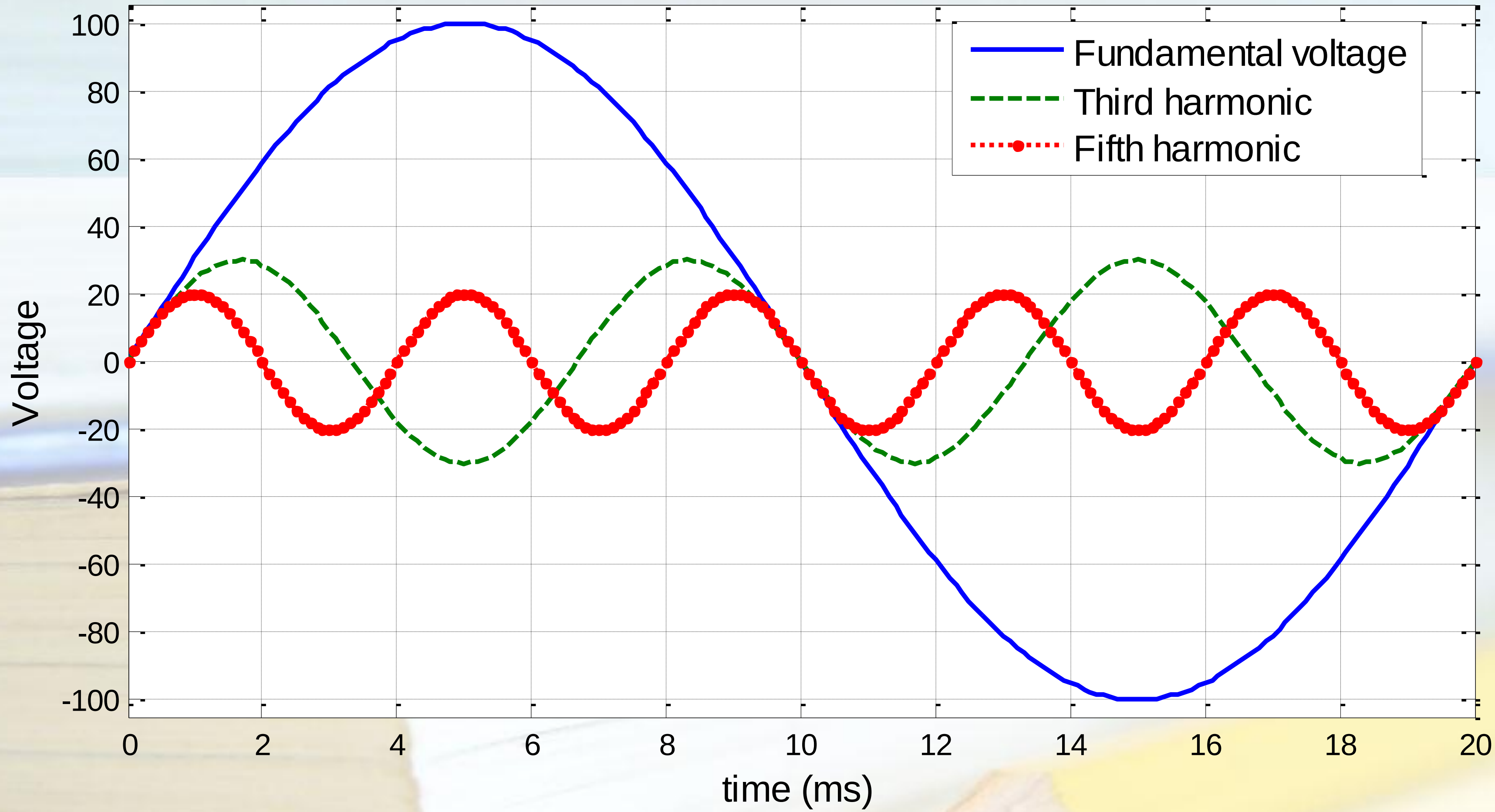
A harmonic component in an AC power system is defined as:-

a sinusoidal component of a periodic waveform that has a frequency equal to an integer multiple of the fundamental frequency of the system.

Harmonics in voltage or current is a sine wave of frequencies multiple of the fundamental frequency:

$$f_h = (h) \text{ x (fundamental frequency)}$$

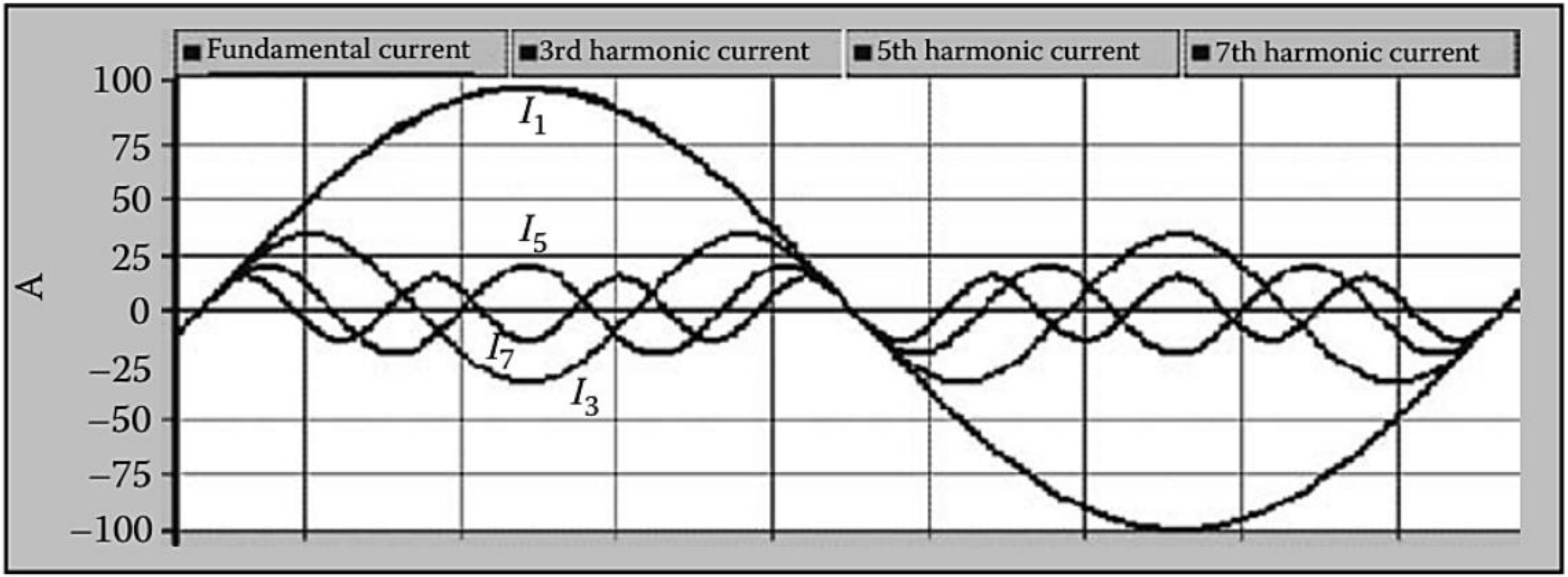
Basics of Harmonic Theory



Harmonics in voltage or current is a sine wave of frequencies multiple of the fundamental frequency:

$$f_h = (h) \times (\text{fundamental frequency})$$

Basics of Harmonic Theory



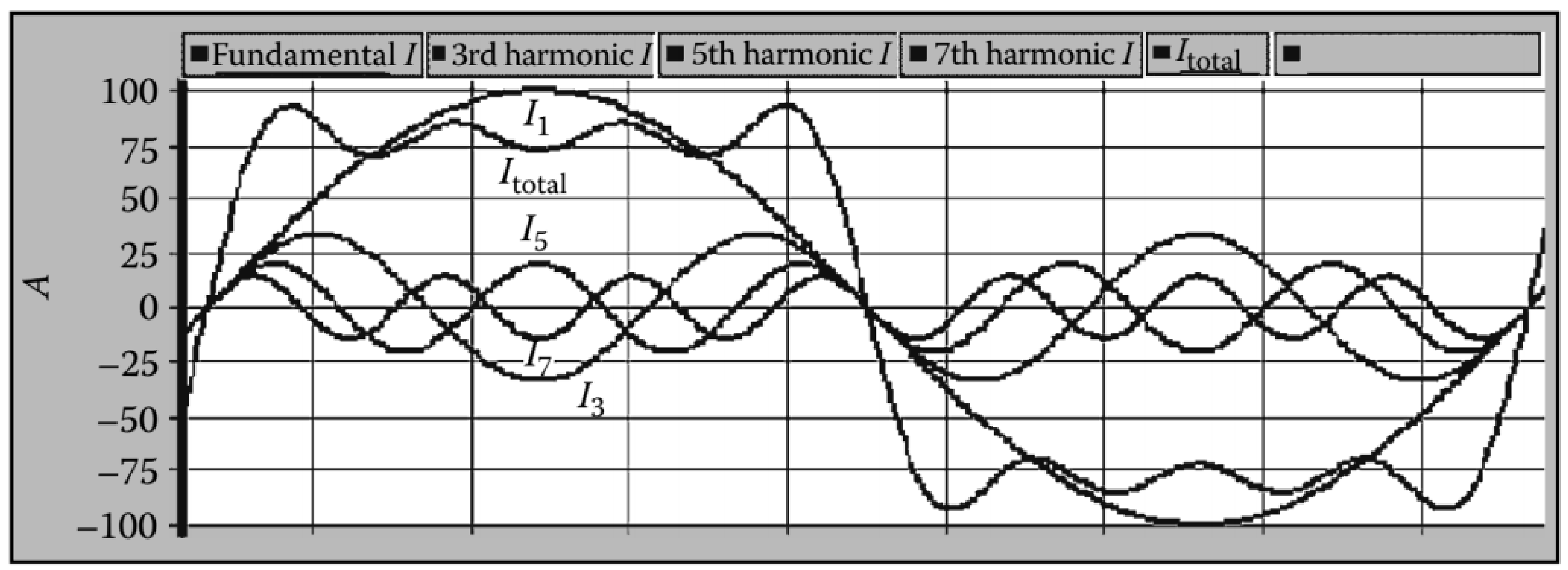
These waveforms can be expressed as:

$$i_1 = I_{m1} \sin(\omega t) \qquad i_3 = I_{m3} \sin(3\omega t - \delta_3)$$

$$i_5 = I_{m5} \sin(5\omega t - \delta_5) \qquad i_7 = I_{m7} \sin(7\omega t - \delta_7)$$

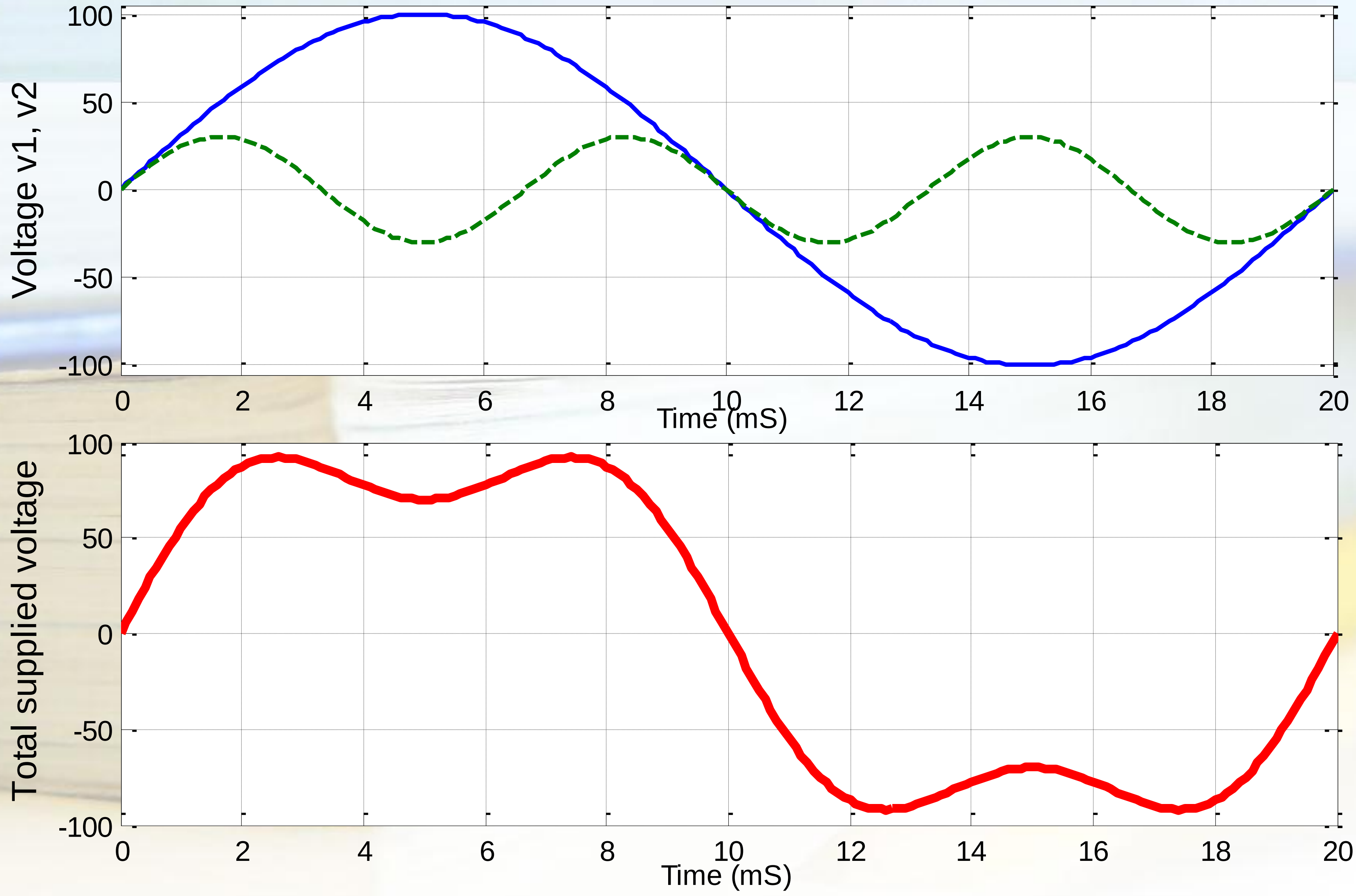
where I_{mh} is the peak RMS value of the harmonic current h.

Basics of Harmonic Theory



This figure shows the same harmonic waveforms as those in previous figure superimposed on the fundamental frequency current yielding I_{total}.

Basics of Harmonic Theory



Linear and Nonlinear Loads

Linear Loads

Linear loads are those in which voltage and current signals follow one another very closely, such as the voltage drop that develops across a constant resistance, which varies as a direct function of the current that passes through it.

This relation is better known as Ohm's law and states that the current through a resistance fed by a varying voltage source is equal to the relation between the voltage and the resistance, as described by:

$$i(t) = \frac{v(t)}{R}$$

Linear Loads

This is why the voltage and current waveforms in electrical circuits with linear loads look alike. Therefore, if the source is a clean open circuit voltage, the current waveform will look identical, showing no distortion.

Resistive elements	Inductive elements	Capacitive elements
• Incandescent lighting	• Induction motors	• Power factor correction capacitor banks
• Electric heaters	• Current limiting reactors	• Underground cables
	• Induction generators (wind mills)	• Insulated cables
	• Damping reactors used to attenuate harmonics	• Capacitors used in harmonic filters
	• Tuning reactors in harmonic filters	

Linear Loads

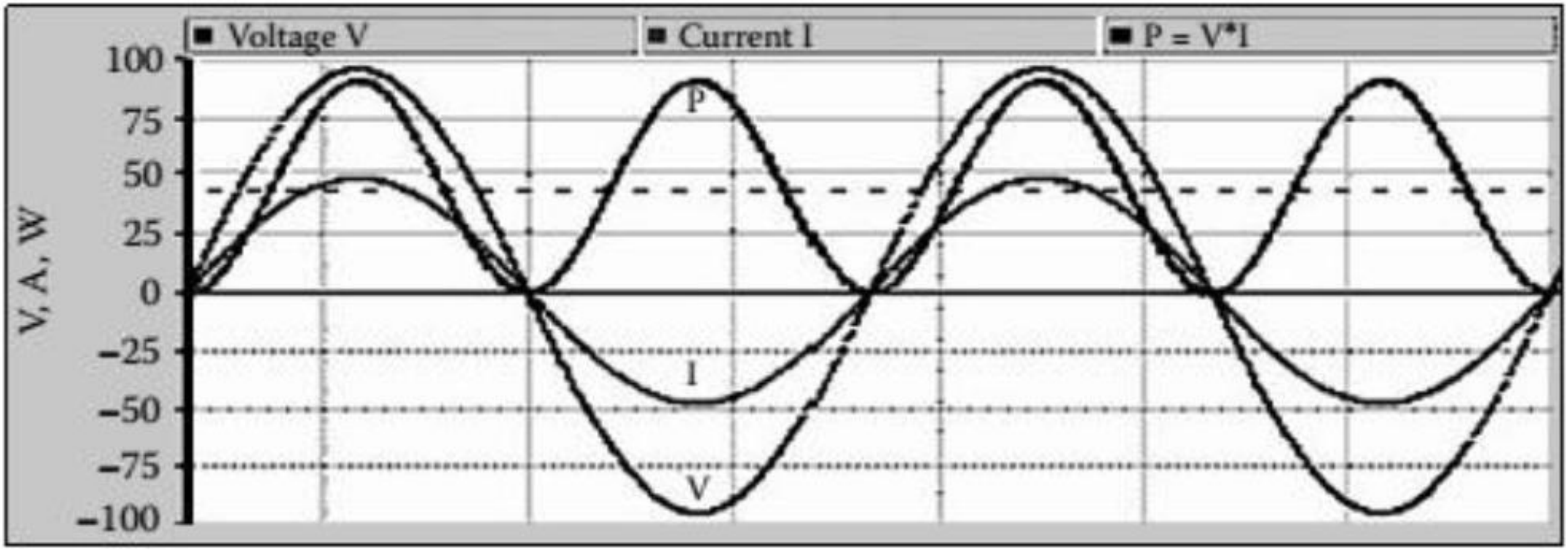


Figure 5.4 Relation among voltage, current, and power in a purely resistive circuit.

Linear Loads

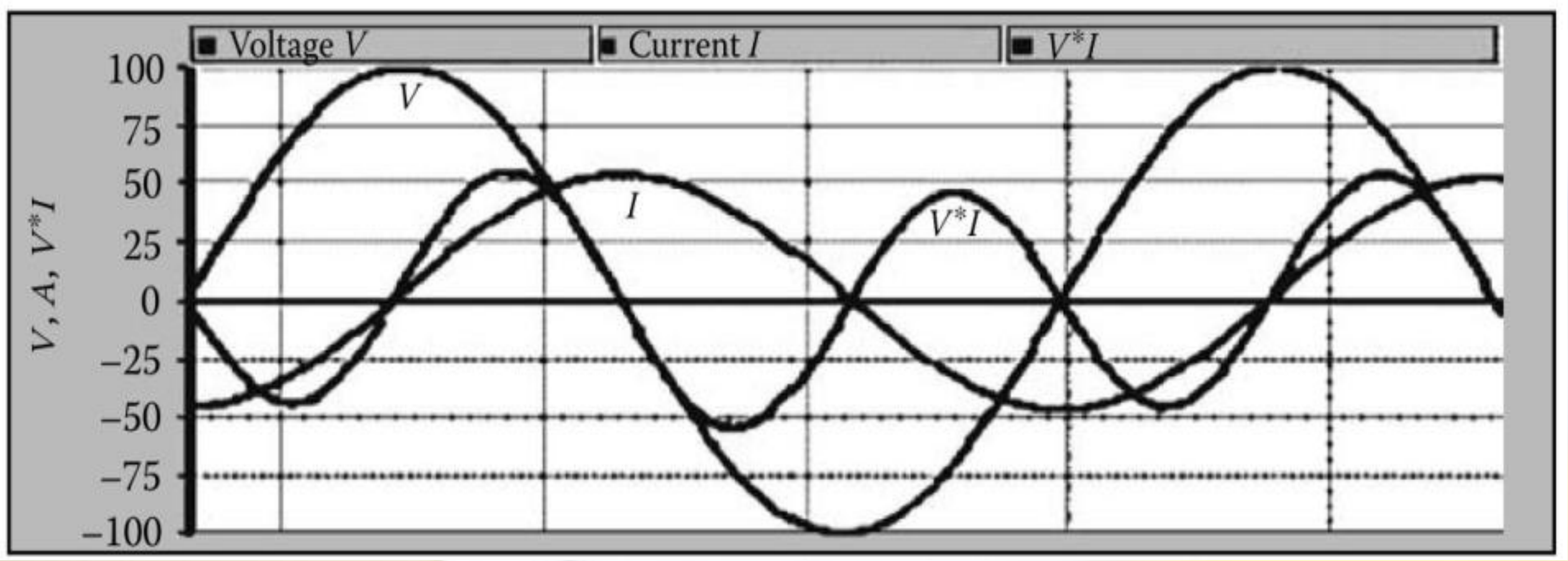


Figure 5.5 Relation among voltage, current, and power in a purely inductive circuit.

Linear Loads

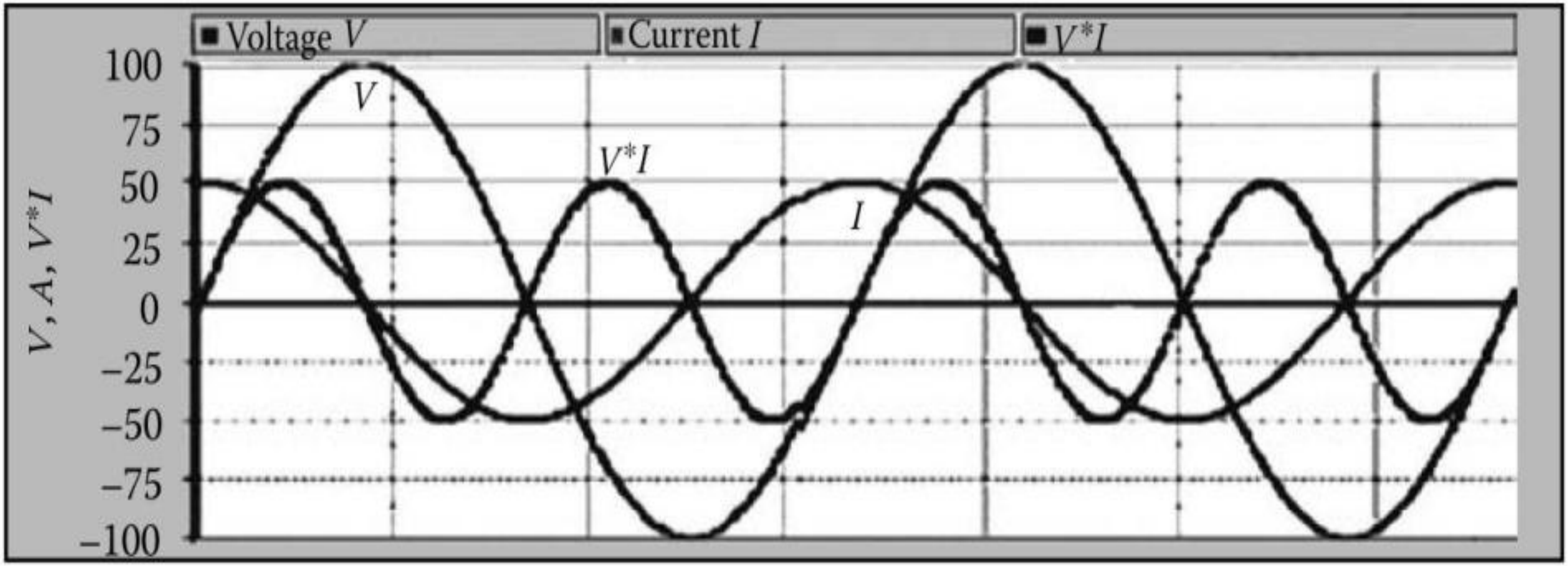


Figure 5.6 Relation among voltage, current, and power in a purely capacitive circuit.

Nonlinear Loads

Nonlinear loads are loads in which the current waveform does not resemble the applied voltage waveform due to a number of reasons, for example, the use of electronic switches that conduct load current only during a fraction of the power frequency period.

We can conceive nonlinear loads as those in which Ohm's law cannot describe the relation between V and I.

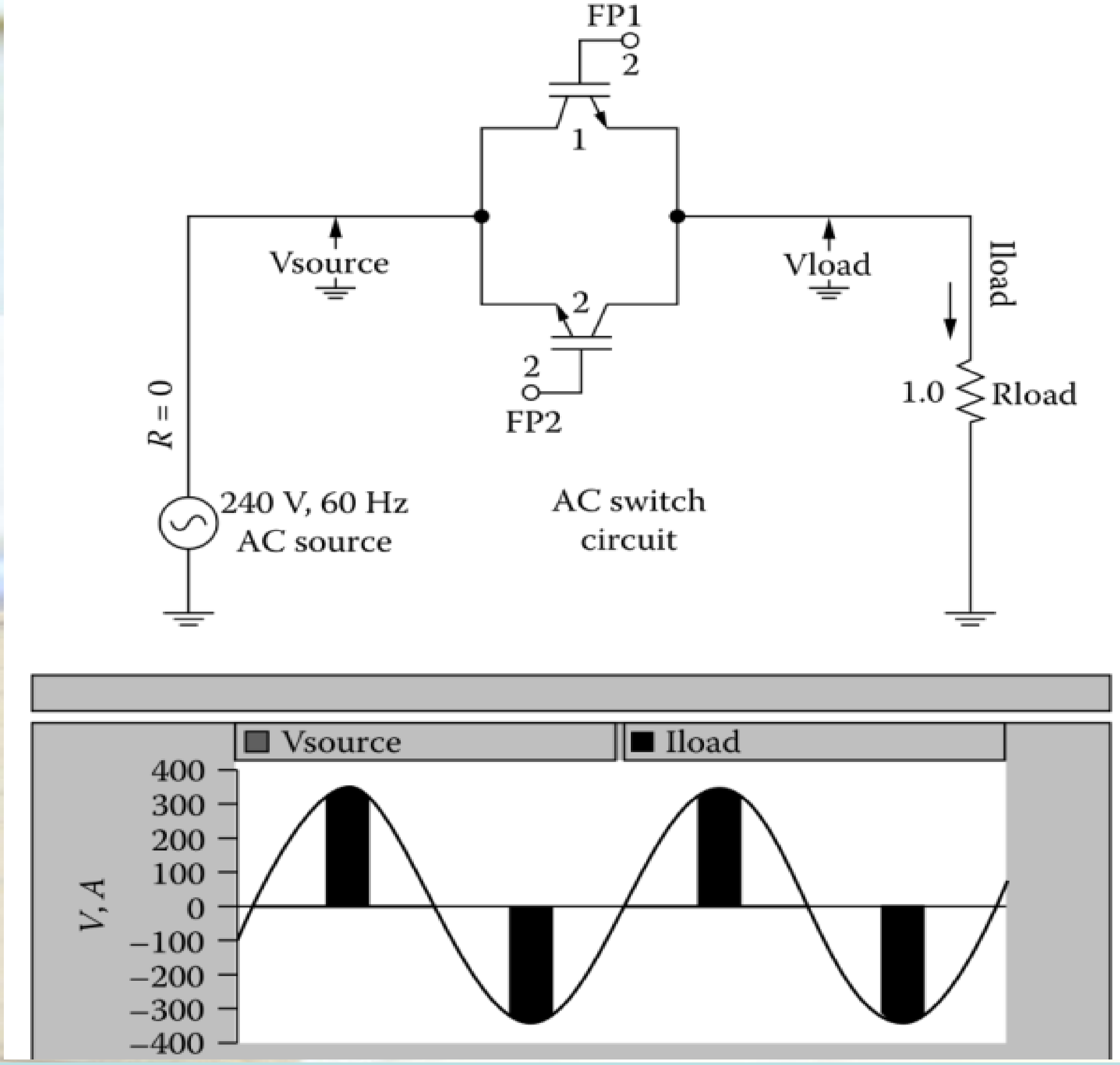
Nonlinear Loads

Power electronics

- Power converters
- Variable frequency drives
- DC motor controllers
- Cycloconverters
- Cranes
- Elevators
- Steel mills
- Power supplies
- UPS
- Battery chargers
- Inverters

ARC devices

- Fluorescent lighting
- ARC furnaces
- Welding machines

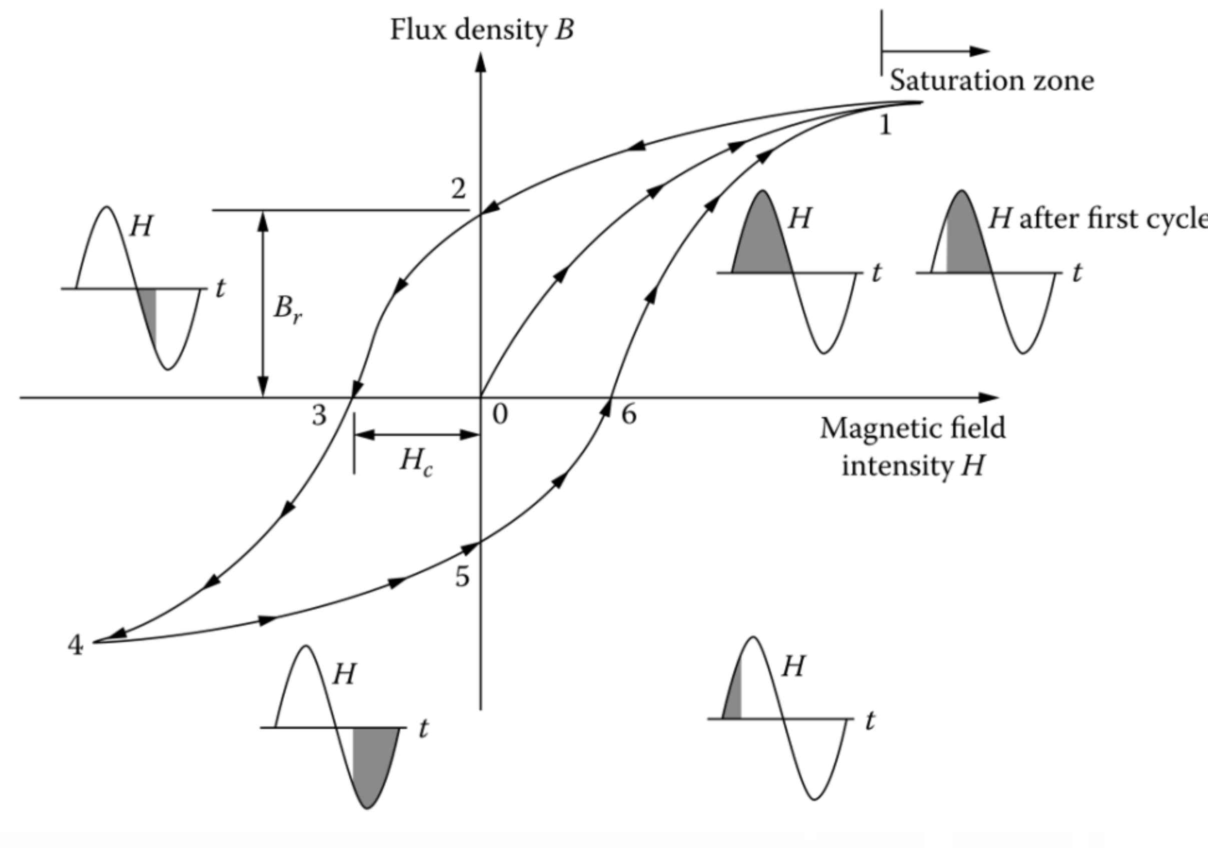


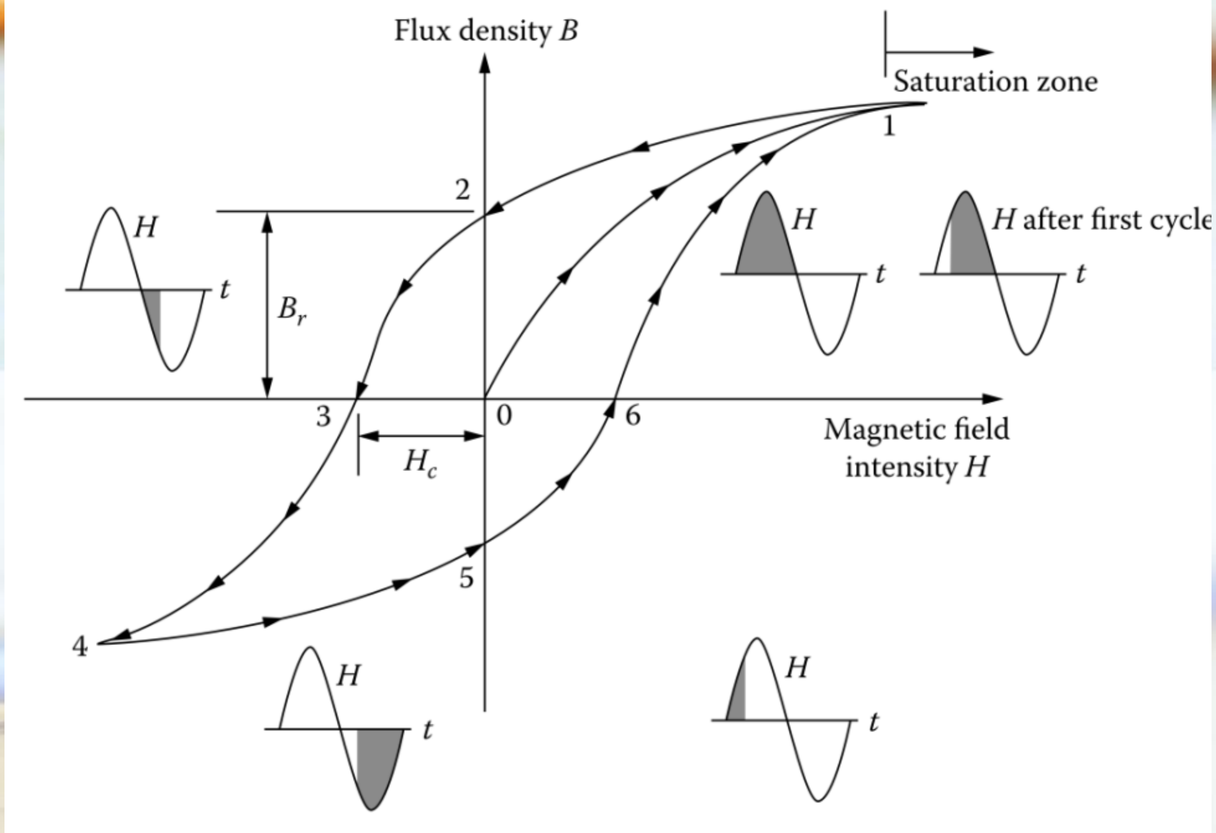
Voltage and current waveforms during the switching action of an insulated gate bipolar transistor (IGBT).

Nonlinear Loads

Is the transformer a linear or non linear element ?????

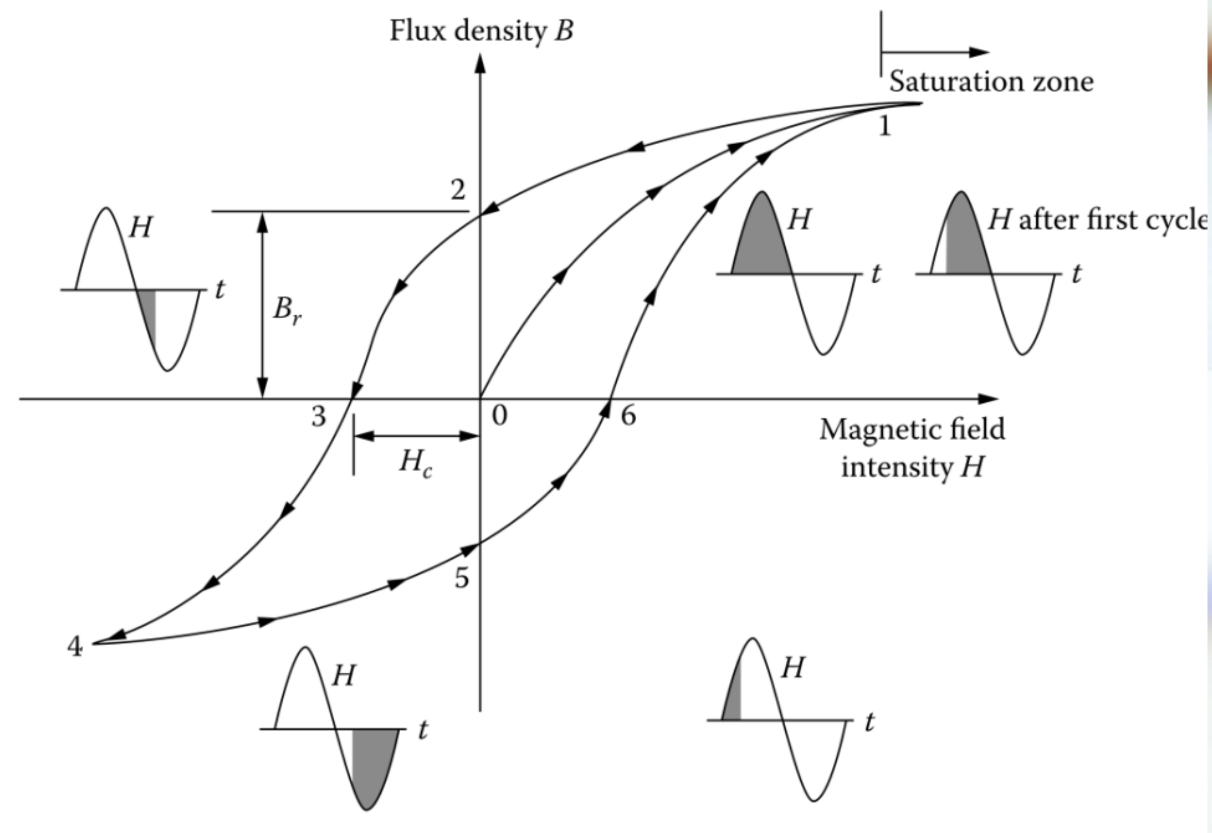
Even linear loads like power transformers can act nonlinear under saturation conditions. What this means is that, in certain instances, the magnetic flux density (B) in the transformer ceases to increase or increases very little as the magnetic flux intensity (H) keeps growing. This occurs beyond the so-called saturation knee of the magnetizing curve of the transformer.





Note that the normal operation of power transformers should be below the saturation region.

However, when the transformer is operated beyond its rated power (during peak demand hours) or above nominal voltage (especially if power factor capacitor banks are left connected to the line under light load conditions), transformers are prone to operate under saturation.



Practically speaking, all transformers reach the saturation region on energization, developing large inrush (magnetizing) currents.

Nevertheless, this is a condition that lasts only a few cycles. Another situation in which the power transformer may operate on the saturation region is under unbalanced load conditions; one of the phases carries a different current than the other phases, or the three phases carry unlike currents.

Effects of Harmonics on Distribution Systems

Effects of Harmonics on Distribution Systems

Is the harmonics useful or not for electric power systems ????

- ✓ **Thermal Effects on Transformers**
- ✓ **Neutral Conductor Overloading**
- ✓ **Effects on Capacitor Banks**
- ✓ **Unexpected Fuse Operation**
- ✓ **Abnormal Operation of Electronic Relays**
- ✓ **Thermal Effects on Rotating Machines**

Each will be discussed later

Harmonic Analyses

Harmonic Analyses

A function f(t) is said to be periodic if f(t+T) = f(t) for all values of t, where T is the interval between two successive repetitions and is called the period of the function f(t).

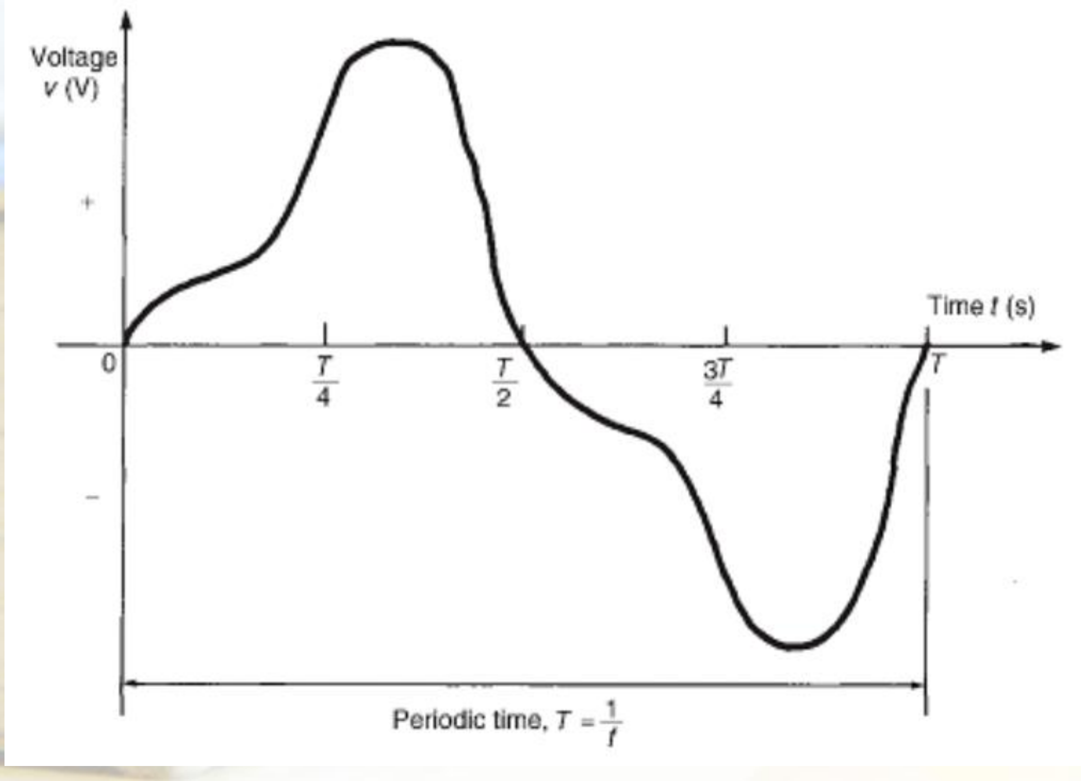
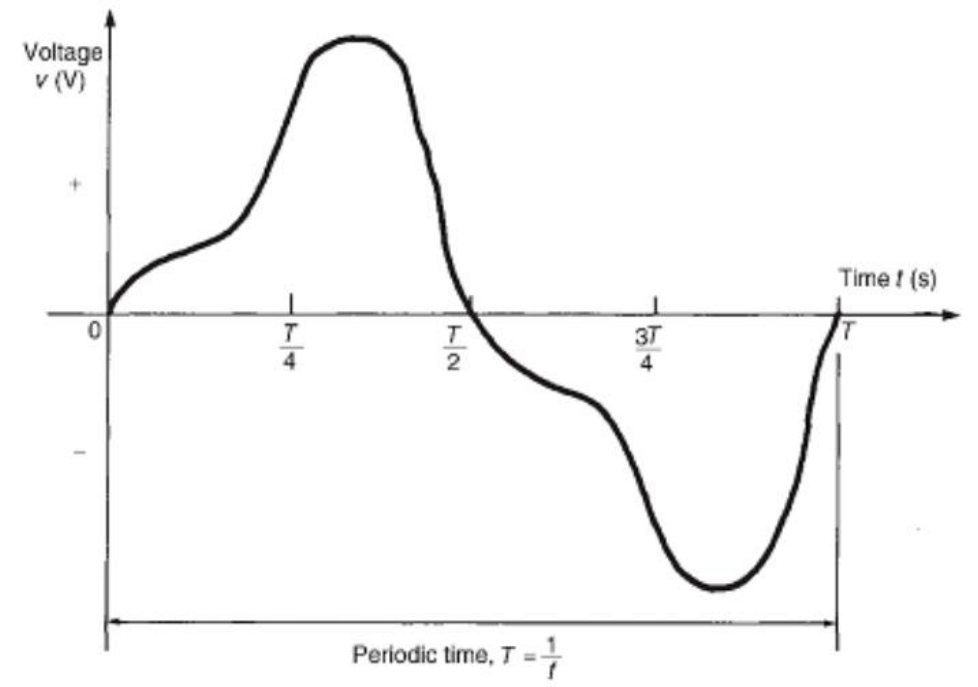


Figure 5.10 Typical complex periodic voltage waveform

The General Equation for a Complex Waveform

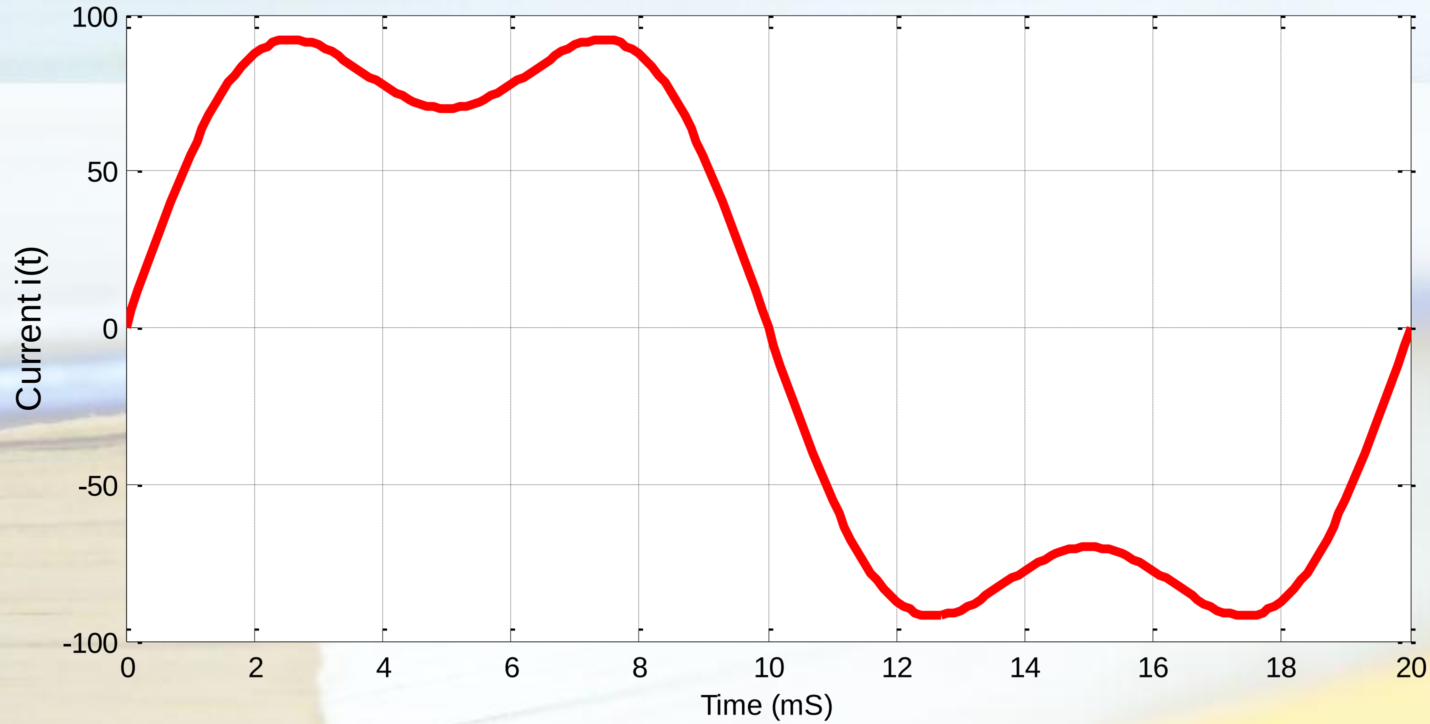


$$v = V_{1m}\sin(\omega t+\psi_1) + V_{2m}\sin(2\omega t+\psi_2) + \ldots\ldots + V_{nm}\sin(n\omega t+\psi_n)$$

$V_{1m}\sin(\omega t+\psi_1)$ represents the fundamental component of which V_{1m} is the maximum or peak value, frequency, $f = \omega/2\pi$ and ψ_1 is the phase angle with respect to time, $t = 0$.

Similarly, $V_{2m}\sin(2\omega t+\psi_2)$ represents the second harmonic component, and $V_{nm}\sin(n\omega t+\psi_n)$ represents the nth harmonic component, of which V_{nm} is the peak value, frequency = $n\omega/2\pi$ (= nf) and ψ_n is the phase angle.

The General Equation for a Complex Waveform



$$i = I_{1m}\sin(\omega t+\theta_1) + I_{2m}\sin(2\omega t+\theta_2) + \ldots\ldots + I_{nm}\sin(n\omega t+\theta_n)$$

Harmonic Synthesis

Example 5–1

Consider the complex voltage expression given by the following complex wave, draw and analyze.

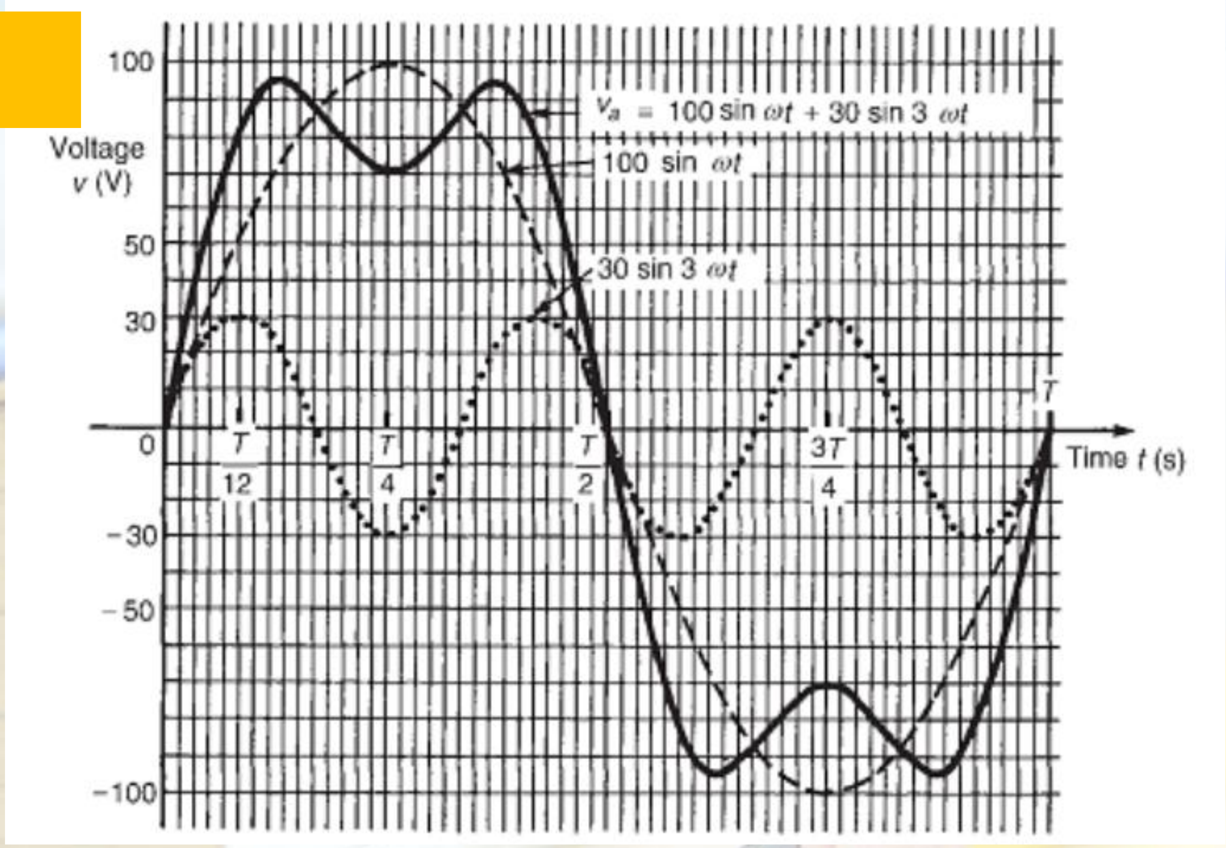
$$v_a = 100 \sin(\omega t) + 30 \sin(3\omega t) \text{ volts}$$

Harmonic Synthesis

Example 5–1

$v_a = 100 \sin(\omega t) + 30 \sin(3\omega t)$ volts

Solution



At time $T/12$ seconds, the fundamental has a value of 50 V and the third harmonic a value of 30 V. Adding gives 80 V for waveform v_{a}, similarly T/8, T/4, T/2,…….

The shapes of the negative and positive half-cycles are identical.

Harmonic Synthesis

Example 5–2

Consider the addition of a fifth harmonic component to the complex waveform of Figure 5.11, giving a resultant waveform expression, draw and analyze.

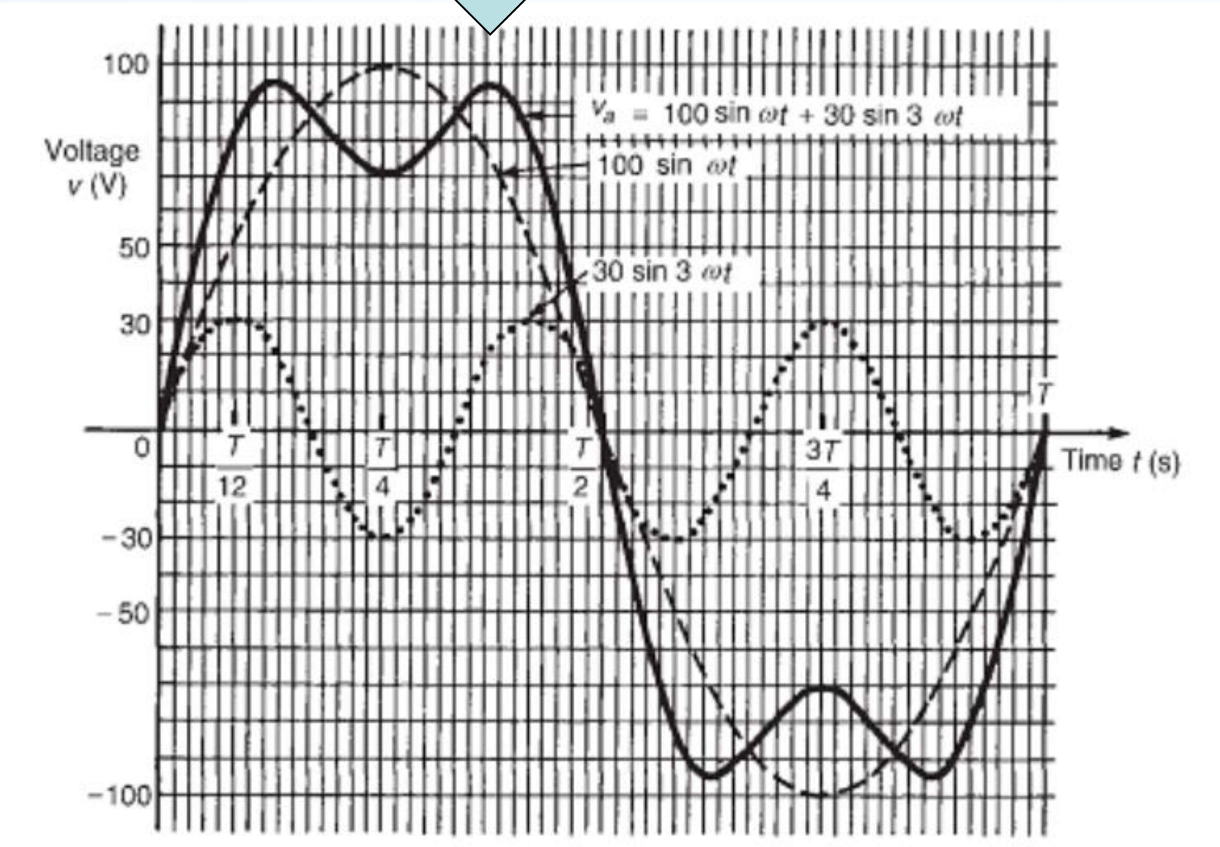
$$v_b = 100 \sin(\omega t) + 30 \sin(3\omega t) + 20 \sin(5\omega t) \quad \text{volts}$$

Harmonic Synthesis

Example 5–2

$$v_b = 100\sin(\omega t) + 30\sin(3\omega t) + 20\sin(5\omega t)$$

Solution

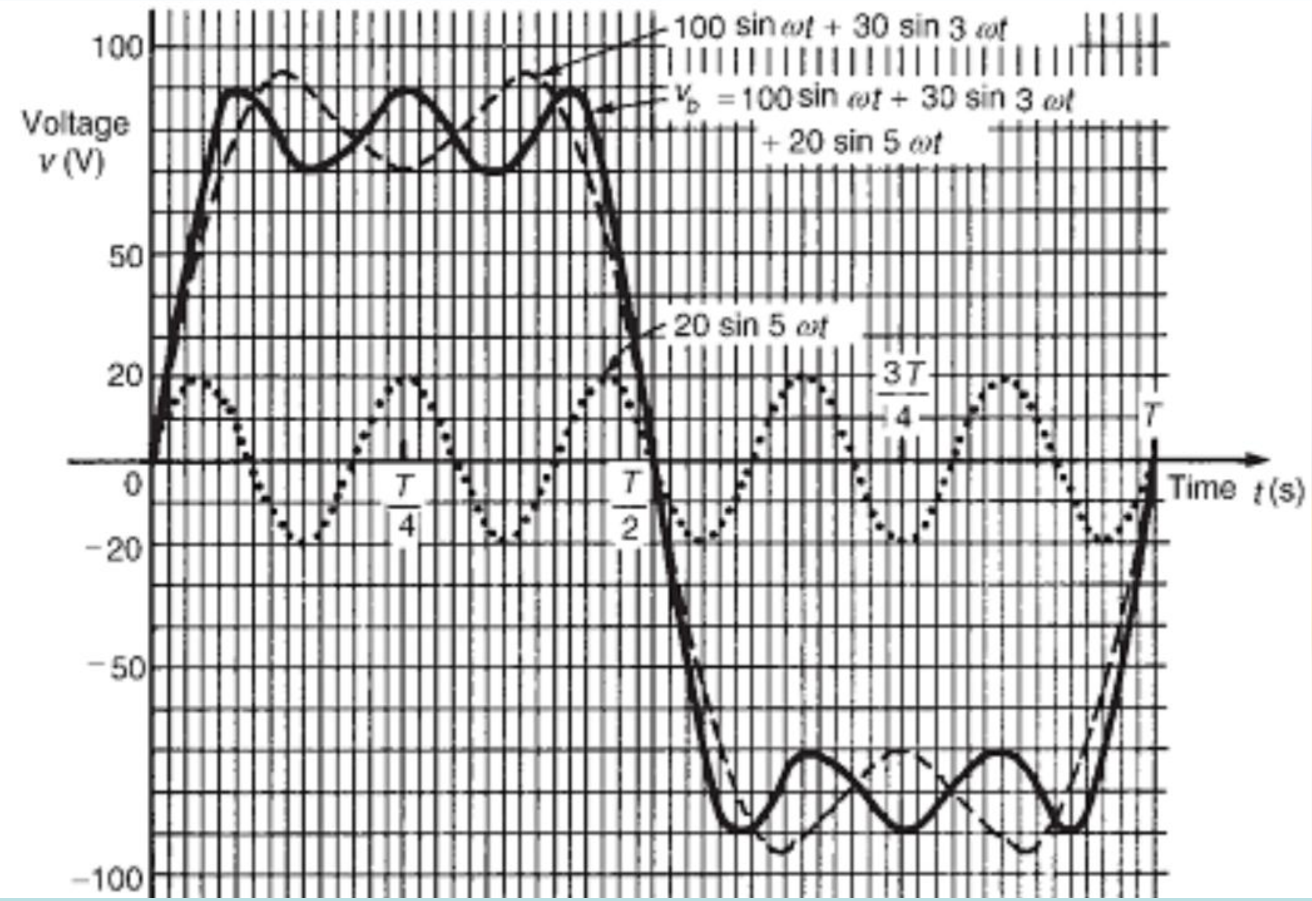


Harmonic Synthesis

Example 5–2

$$v_b = 100\sin(\omega t) + 30\sin(3\omega t) + 20\sin(5\omega t)$$

Solution



The shapes of the negative and positive half-cycles are identical.

Harmonic Synthesis

Example 5–3

Consider the complex voltage expression given by the following complex wave, draw and analyze.

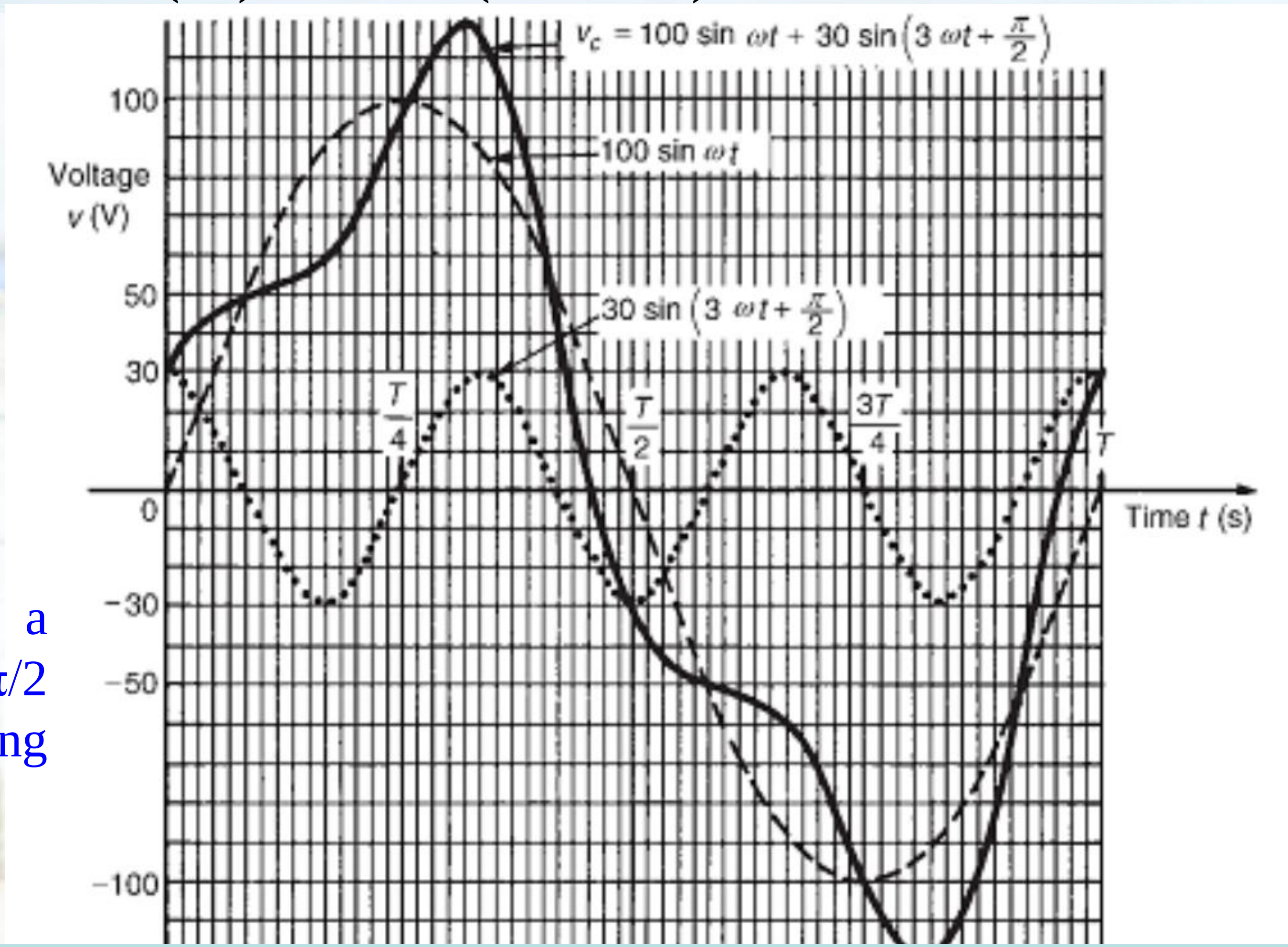
$$v_c = 100 \sin(\omega t) + 30 \sin(3\omega t + \pi/2) \text{ volts}$$

Harmonic Synthesis

Example 5–3

$$v_c = 100\sin(\omega t) + 30\sin(3\omega t + \pi/2) \text{ volts}$$

Solution



The third harmonic has a phase displacement of π/2 radian leading (i.e., leading 30 sin 3ωt by π/2 radian).

The shapes of the negative and positive half-cycles are identical.

Harmonic Synthesis

Example 5–4

Consider the complex voltage expression given by the following complex wave, draw and analyze.

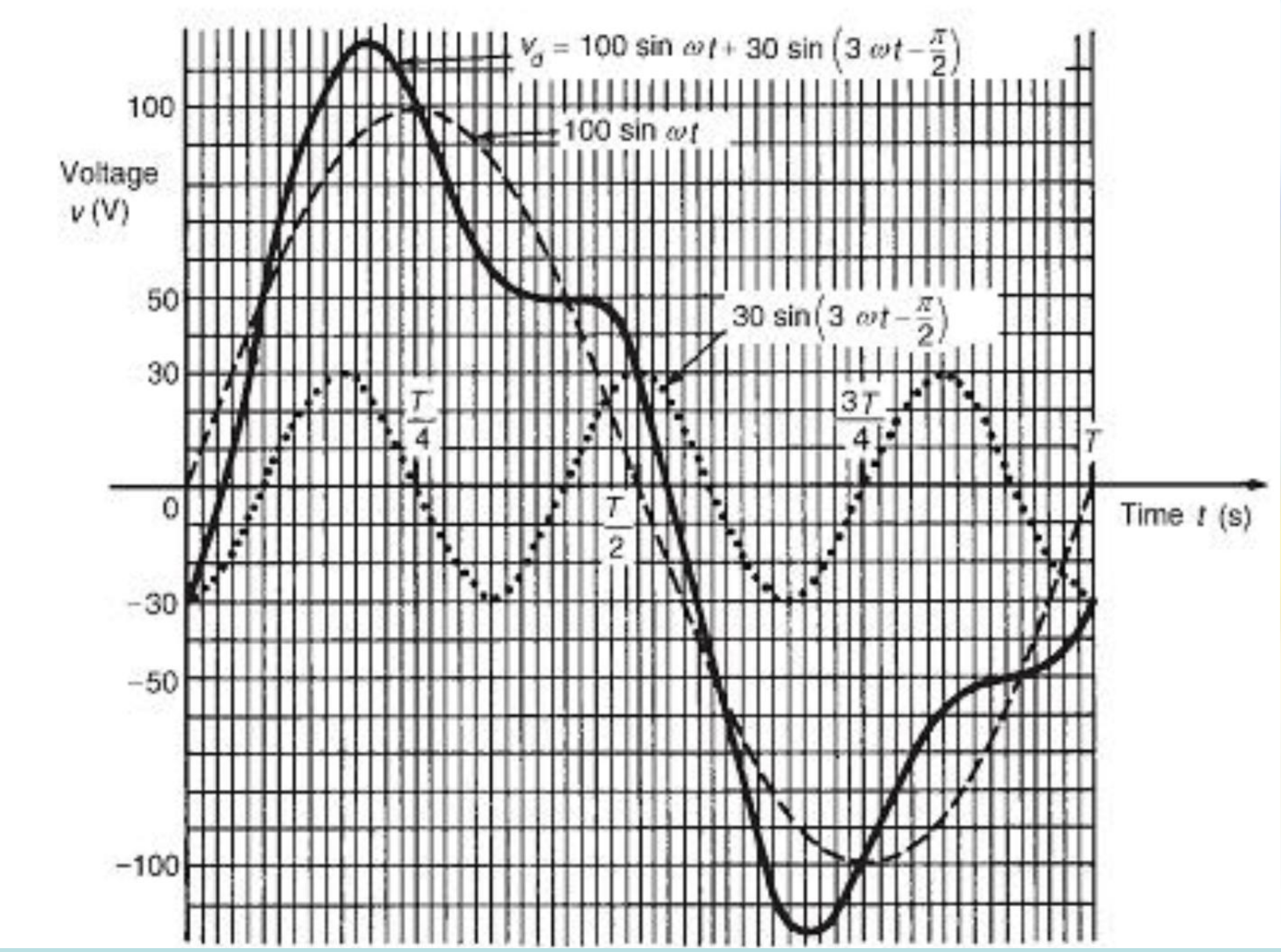
$$v_d = 100\ \sin(\omega t) + 30\ \sin(3\omega t - \pi/2)\ \text{volts}$$

Harmonic Synthesis

Example 5–4

$v_d = 100 \sin(\omega t) + 30 \sin(3\omega t - \pi/2)$ volts

Solution



The shapes of the negative and positive half-cycles are identical.

Harmonic Synthesis

Example 5–5

Consider the complex voltage expression given by the following complex wave, draw and analyze.

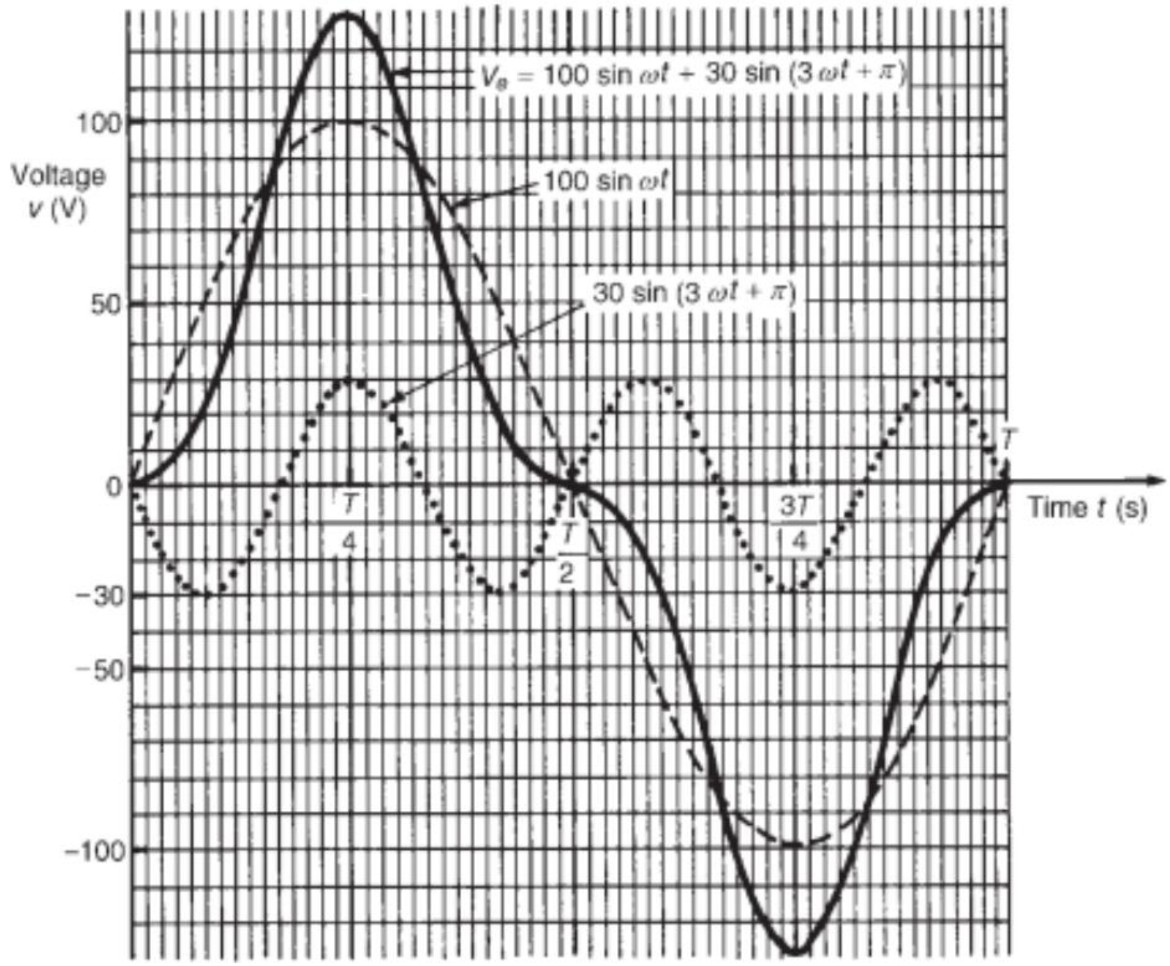
$$v_e = 100\sin(\omega t) + 30\sin(3\omega t + \pi)\text{ volts}$$

Harmonic Synthesis

Example 5–5

$$v_e = 100\ \sin(\omega t) + 30\ \sin(3\omega t + \pi)\ \text{volts}$$

Solution



The shapes of the negative and positive half-cycles are identical.

Harmonic Synthesis

Example 5–6

Consider the complex voltage expression given by the following complex wave, draw and analyze.

$$v_f = 100 \sin(\omega t) - 30 \sin(3\omega t + \pi/2) \text{ volts}$$

Harmonic Synthesis

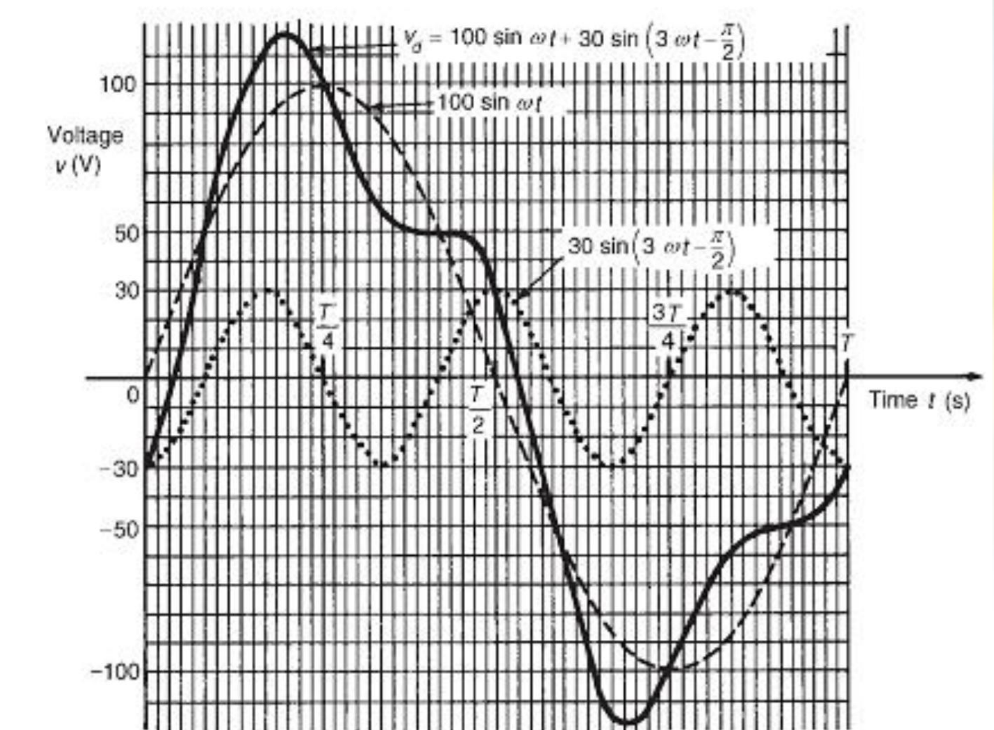
Example 5–6

$$v_f = 100\sin(\omega t) - 30\sin(3\omega t + \pi/2) \text{ volts}$$

Solution

$$v_f = 100\sin(\omega t) - 30\sin(3\omega t + \pi/2)$$
$$= 100\sin(\omega t) + 30\sin(3\omega t - \pi/2)$$

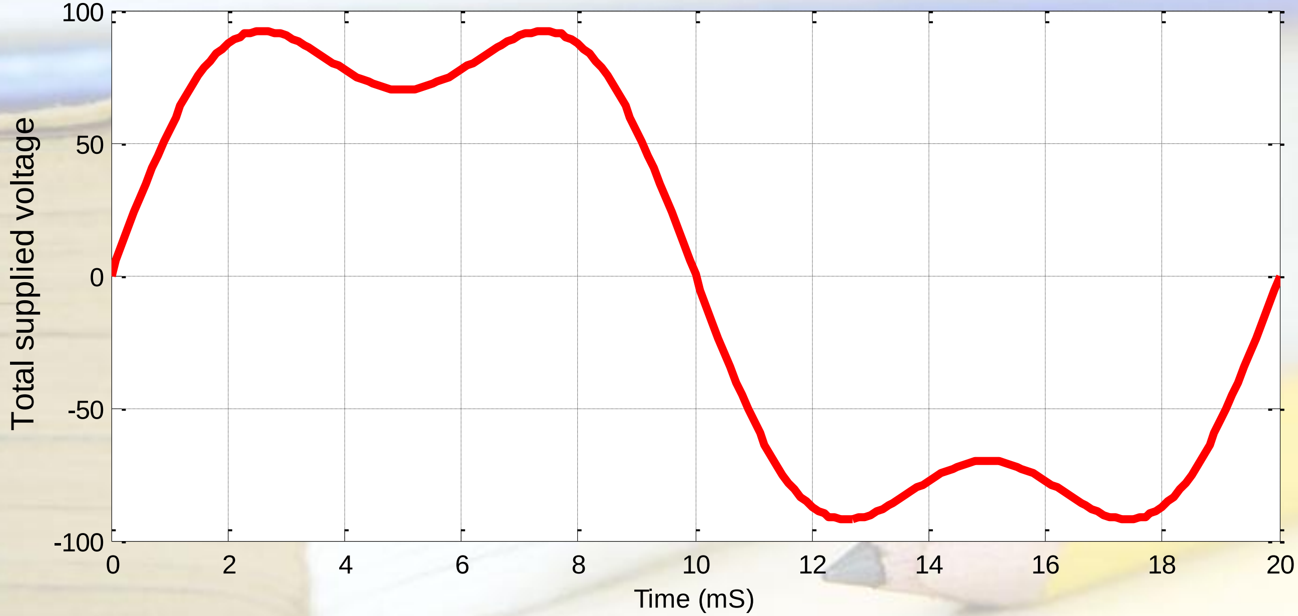
Similar to example 5-4



Harmonic Synthesis

General conclusions on examples 6.1 to 6.6

Whenever odd harmonics are added to a fundamental waveform, whether initially in phase with each other or not, the positive and negative half cycles of the resultant complex wave are identical in shape.

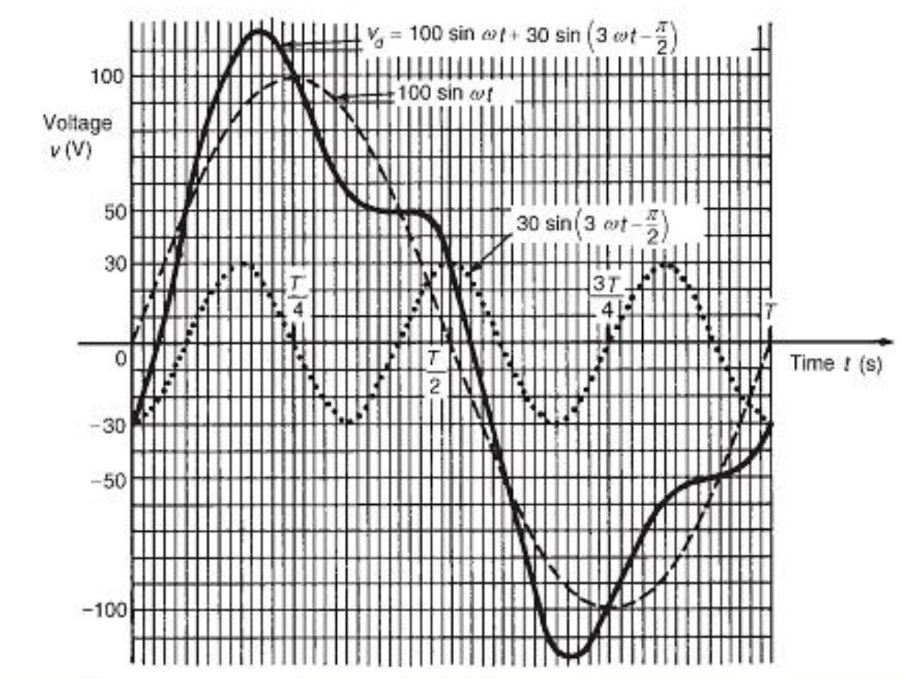


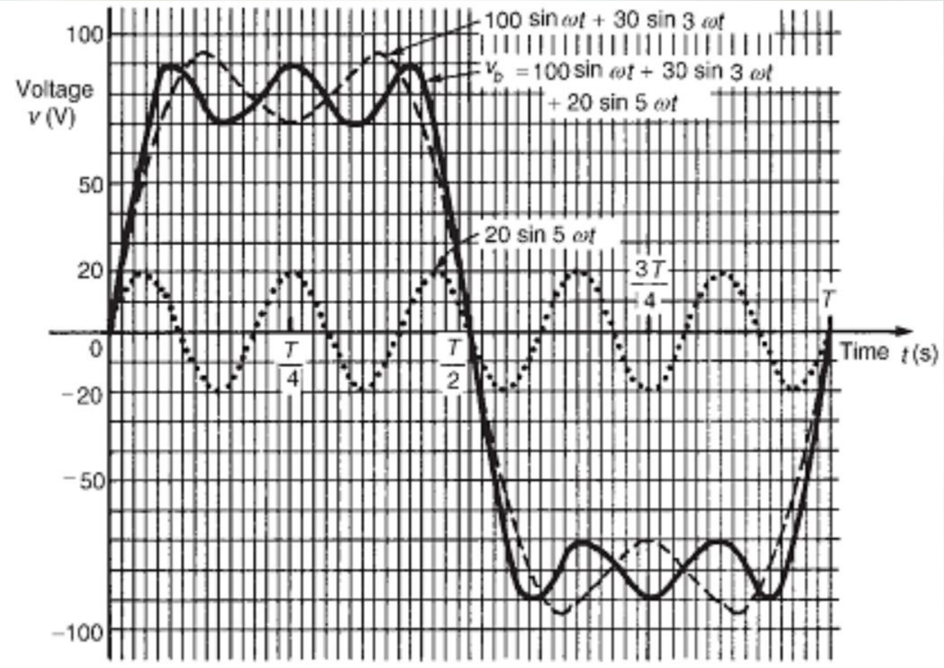
$$f(t) = -f(t + \frac{T}{2})$$

Harmonic Synthesis

General conclusions on examples 6.1 to 6.6

Whenever odd harmonics are added to a fundamental waveform, whether initially in phase with each other or not, the positive and negative half cycles of the resultant complex wave are identical in shape.





Harmonic Synthesis

General conclusions on examples 6.1 to 6.6

When both the positive and negative half cycles of a waveform have identical shapes, the Fourier series contains only odd harmonics.

Harmonic Synthesis

Example 5–7

Consider the complex current expression given by the following complex wave, draw and analyze.

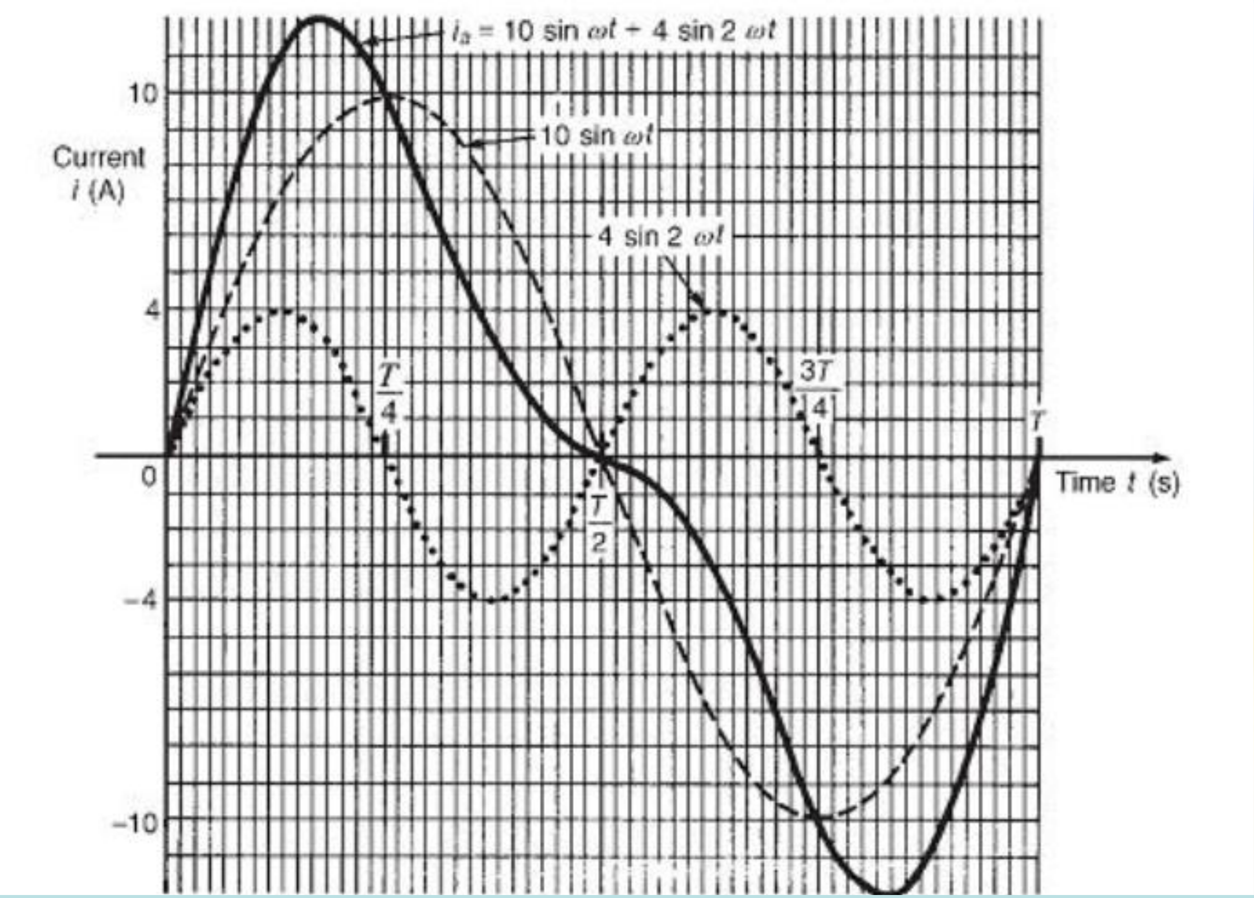
$$i_a = 10 \sin(\omega t) + 4 \sin(2\omega t) \text{ amperes}$$

Harmonic Synthesis

Example 5–7

$i_a = 10 \sin(\omega t) + 4 \sin(2\omega t)$ amperes

Solution



If all the values in the negative half cycle were reversed then this half-cycle would appear as a mirror image of the positive half-cycle about a vertical line drawn at t = $T/2$.

Harmonic Synthesis

Example 5–8

Consider the complex current expression given by the following complex wave, draw and analyze.

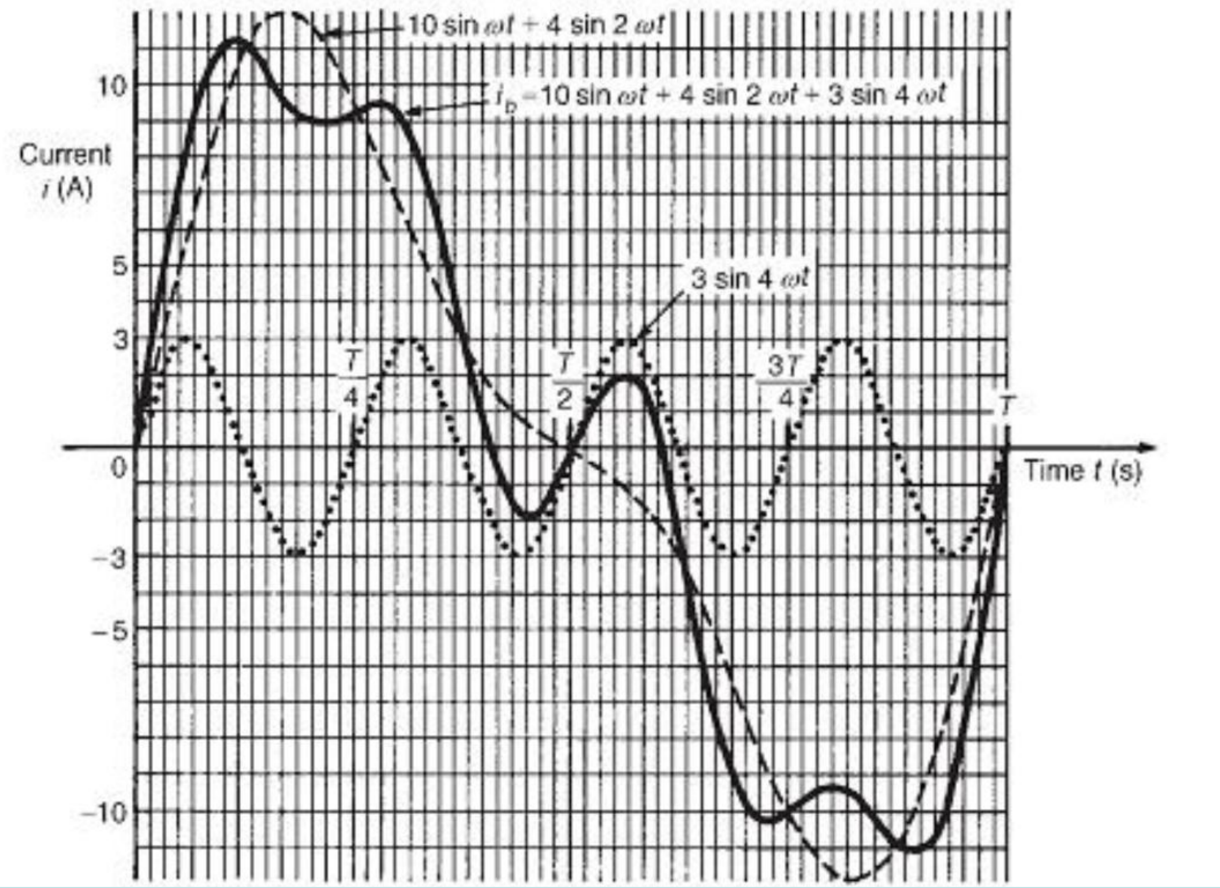
$$i_b = 10\sin(\omega t) + 4\sin(2\omega t) + 3\sin(4\omega t) \quad \text{amperes}$$

Harmonic Synthesis

Example 5–8

$$i_b = 10\sin(\omega t) + 4\sin(2\omega t) + 3\sin(4\omega t) \quad \text{amperes}$$

Solution



The reversed negative half cycle is mirror image of the positive half-cycle about a vertical line drawn at t = *T*/2.

Harmonic Synthesis

Example 5–9

Consider the complex current expressions given by the following complex wave, draw and analyze.

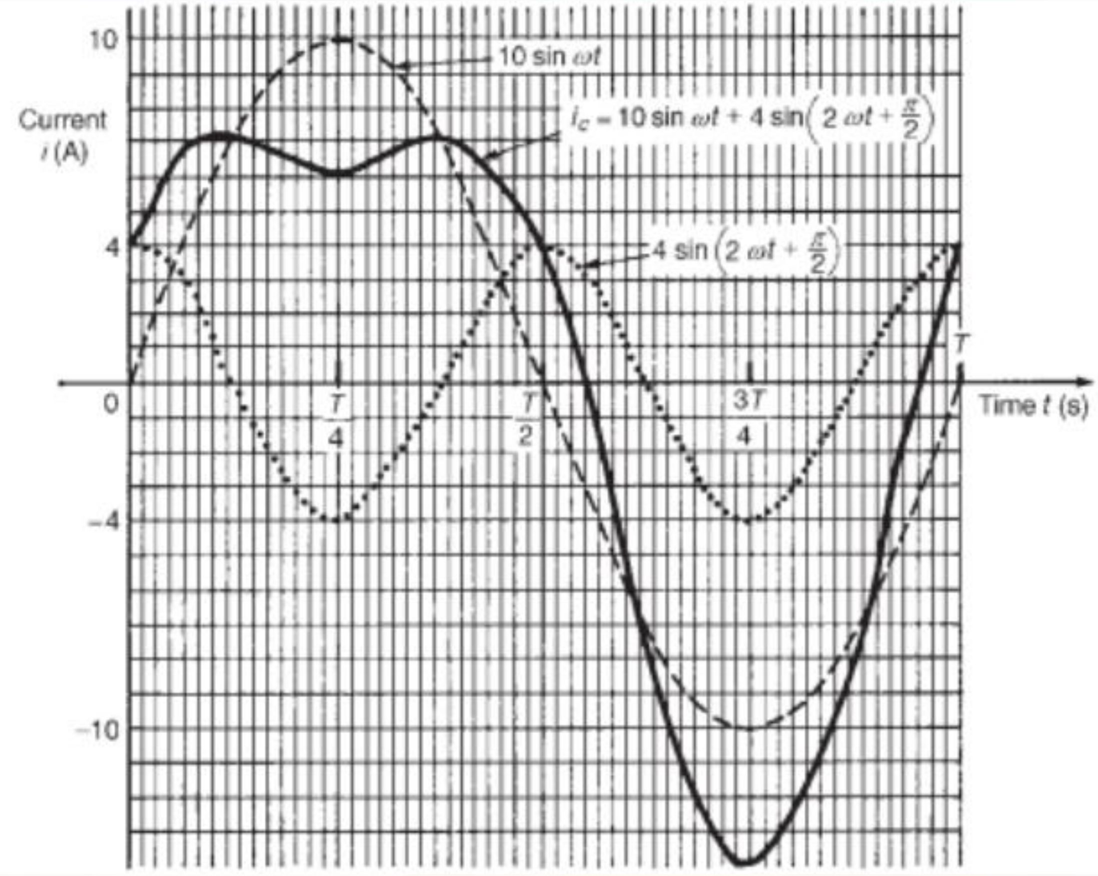
$$i_c = 10 \sin(\omega t) + 4 \sin(2\omega t + \pi/2) \text{ amperes}$$

Harmonic Synthesis

Example 5–9

$$i_c = 10\sin(\omega t) + 4\sin(2\omega t + \pi/2) \text{ amperes}$$

Solution



The positive and negative half-cycles of the resultant waveform ic are seen to be quite dissimilar.

Harmonic Synthesis

Example 5–10

Consider the complex current expression given by the following complex wave, draw and analyze.

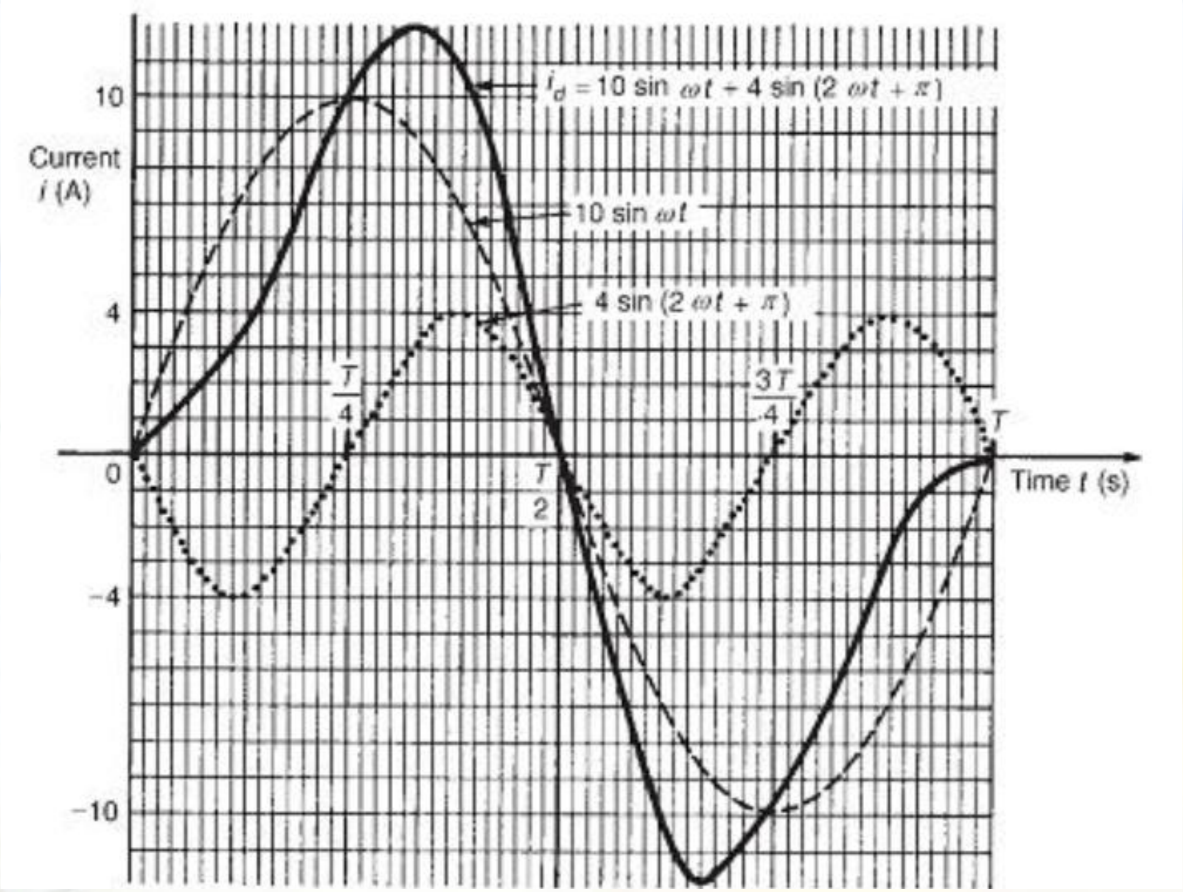
$$i_d = 10 \sin(\omega t) + 4 \sin(2\omega t + \pi) \text{ amperes}$$

Harmonic Synthesis

Example 5–10

$$i_d = 10\sin(\omega t) + 4\sin(2\omega t + \pi) \text{ amperes}$$

Solution



The reversed negative half cycle is mirror image of the positive half-cycle about a vertical line drawn at t = *T*/2.

Harmonic Synthesis

General conclusions on Examples 6.7 to 6.10

(a) If the harmonics are initially in phase or if there is a phase-shift of π rad, the negative half-cycle, when reversed, is a mirror image of the positive half-cycle about a vertical line drawn through time, t = T/2;

$$f(-t) = -f(t)$$

(a) If the harmonics are initially out of phase with each other (i.e., other than π rad), the positive and negative half-cycles are dissimilar.

Harmonic Synthesis

Example 5–11

Consider the complex voltage expression given by the following complex wave, draw and analyze.

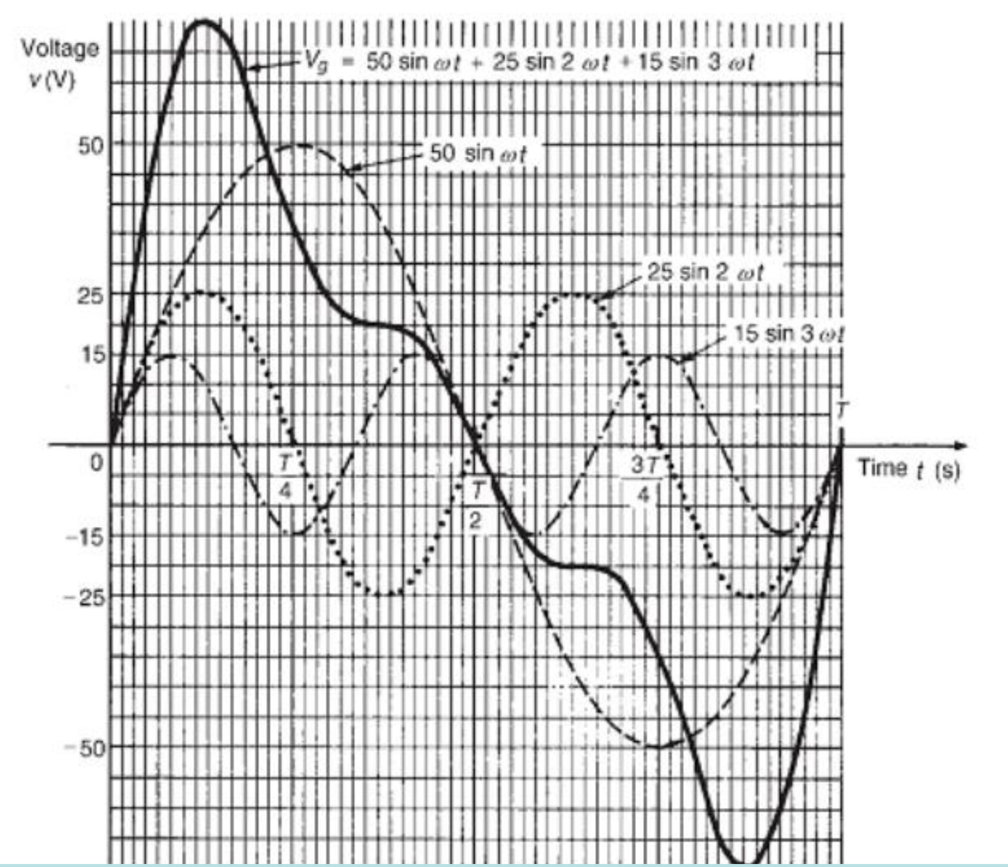
$$v_g = 50\sin(\omega t) + 25\sin(2\omega t) + 15\sin(3\omega t) \text{ volts}$$

Harmonic Synthesis

Example 5–11

$$v_g = 50 \sin(\omega t) + 25 \sin(2\omega t) + 15 \sin(3\omega t) \text{ volts}$$

Solution



The reversed negative half cycle is mirror image of the positive half-cycle about a vertical line drawn at t = $T/2$.

Harmonic Synthesis

Example 5–12

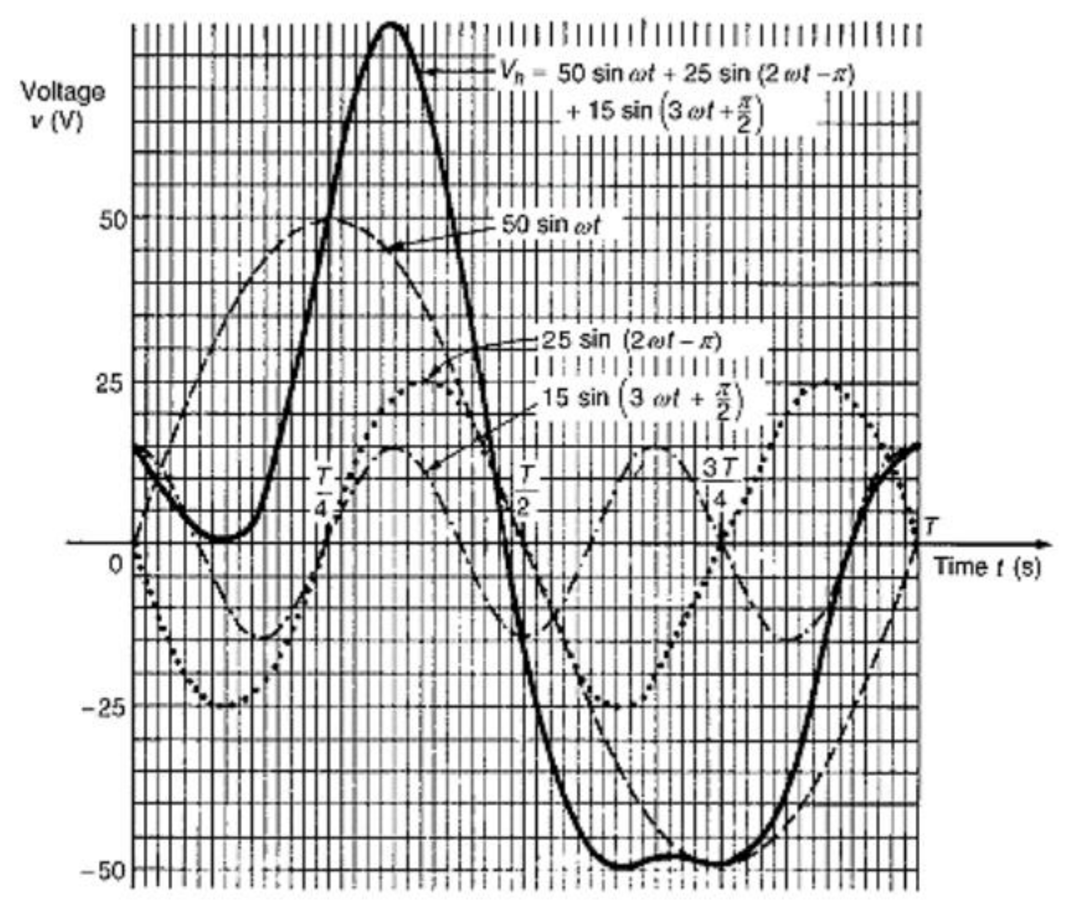
Consider the complex voltage expression given by the following complex wave, draw and analyze.

$$v_h = 50 \sin(\omega t) + 25 \sin(2\omega t - \pi) + 15 \sin(3\omega t + \pi/2) \text{ volts}$$

Harmonic Synthesis

Example 5–12

$$v_h = 50 \sin(\omega t) + 25 \sin(2\omega t - \pi) + 15 \sin(3\omega t + \pi/2) \text{ volts}$$



The positive and negative half-cycles of the resultant waveform ic are seen to be quite dissimilar.

Harmonic Synthesis

General conclusions on examples 11 and 12

(a) If the harmonics are initially in phase with each other, the negative cycle, when reversed, is a mirror image of the positive half-cycle about a vertical line drawn through time, $t = T/2$;

(b) If the harmonics are initially out of phase with each other, the positive and negative half-cycles are dissimilar.

Harmonic Synthesis

Example 5–13

Consider the complex current expression given by the following complex wave, draw and analyze.

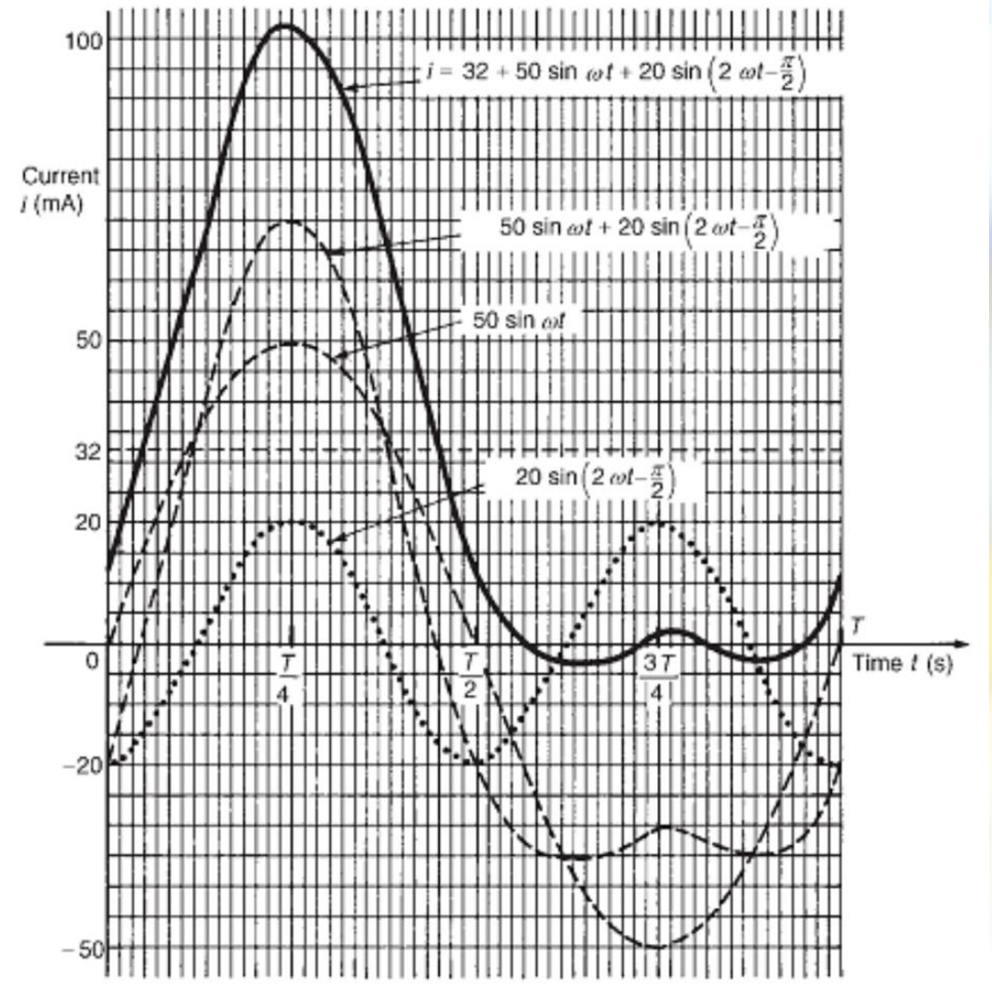
$$i = 32 + 50\sin(\omega t) + 20\sin(2\omega t - \pi/2) \quad \text{mA}$$

Harmonic Synthesis

Example 5–13

$$i = 32 + 50\sin(\omega t) + 20\sin(2\omega t - \pi/2) \quad \text{mA}$$

Solution



Thanks

Week	Required
1st 2nd 3rd	**Chapter (1)** Methods of AC Analysis
4th	**Chapter (2)** Graphical Solution of DC Circuits Contains Nonlinear Elements
5th	**Chapter (3)** **Exam-1** Circle Diagrams
6th 7th	**Chapter (4)** Transient Analysis of Basic Circuits
8th 9th	**Chapter (5)** **Mid Term** Harmonics
10th 11th	**Chapter (6)** Resonance
12th 13th	**Chapter (7)** Passive Filters