Rate of Return Analysis

The most commonly quoted measure of economic worth for a project or alternative is its rate of return (ROR). The ROR is known by other names such as the *internal rate of return* (IROR), which is the technically correct term, and *return on investment* (ROI).

Example 1

To get started in a new telecommuting position with AB Hammond Engineers, Jane took out a $1000 loan at i = 10% per year for 4 years to buy home office equipment. From the lender's perspective, the investment in this young engineer is expected to produce an equivalent net cash flow of $315.47 for each of 4 years.

$$A = \$1000(A/P, 10\%, 4) = \$315.47$$

This represents a 10% per year rate of return on the unrecovered balance. Compute the amount of the unrecovered investment for each of the 4 years using (a) the rate of return on the unrecovered balance (the correct basis) and (b) the return on the initial $1000 investment. (c) Explain why all of the initial $1000 amount is not recovered by the final payment in part (b).

Solution

(*a*) Table 1 shows the unrecovered balance at the end of each year in column 6 using the 10% rate on the *unrecovered balance at the beginning of the year*. After 4 years the total $1000 is recovered, and the balance in column 6 is exactly zero.

TABLE 1 Unrecovered Balances Using a Rate of Return of 10% on the Unrecovered Balance

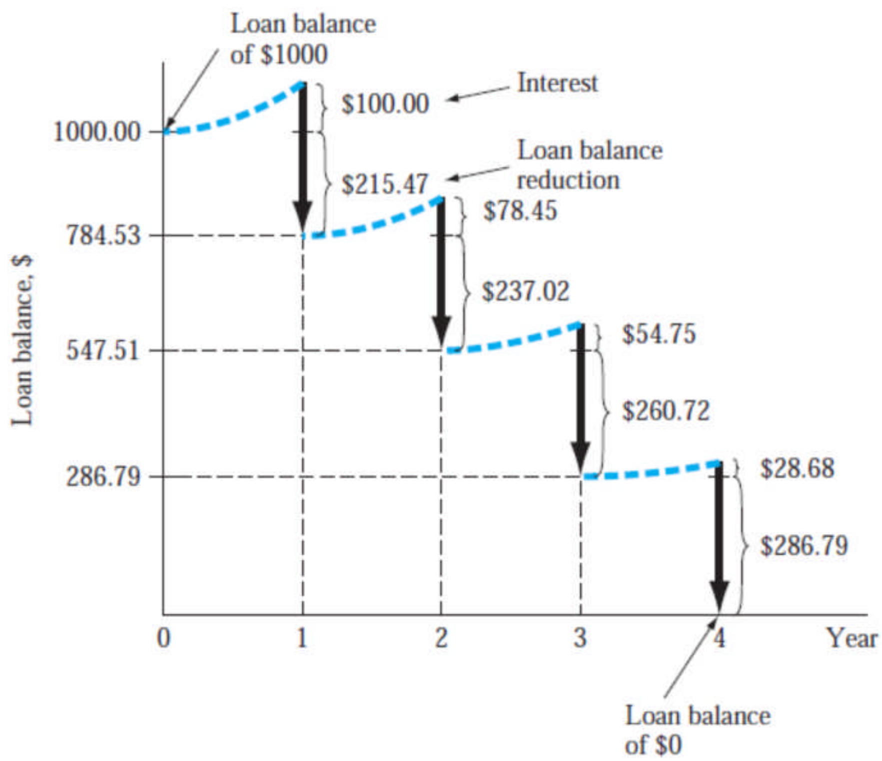
(1)	(2)	(3) = 0.10 × (2)	(4)	(5) = (4) − (3)	(6) = (2) + (5)
Year	Beginning Unrecovered Balance	Interest on Unrecovered Balance	Cash Flow	Recovered Amount	Ending Unrecovered Balance
0	—	—	$−1000.00	—	$−1000.00
1	$−1000.00	$100.00	+315.47	$215.47	−784.53
2	−784.53	78.45	+315.47	237.02	−547.51
3	−547.51	54.75	+315.47	260.72	−286.79
4	−286.79	28.68	+315.47	286.79	0
		$261.88		$1000.00	

TABLE 2 Unrecovered Balances Using a 10% Return on the Initial Amount

(1)	(2)	(3) = 0.10 × (2)	(4)	(5) = (4) − (3)	(6) = (2) + (5)
Year	Beginning Unrecovered Balance	Interest on Initial Amount	Cash Flow	Recovered Amount	Ending Unrecovered Balance
0	—	—	$−1000.00	—	$−1000.00
1	$−1000.00	$100	+315.47	$215.47	−784.53
2	−784.53	100	+315.47	215.47	−569.06
3	−569.06	100	+315.47	215.47	−353.59
4	−353.59	100	+315.47	215.47	−138.12
		$400		$861.88	

(*b*) Table 2 shows the unrecovered balance if the 10% return is always figured on the *initial $1000.* Column 6 in year 4 shows a remaining unrecovered amount of $138.12, because only $861.88 is recovered in the 4 years (column 5).

(*c*) As shown in column 3, a total of $400 in interest must be earned if the 10% return each year is based on the initial amount of $1000. However, only $261.88 in interest must be earned if a 10% return on the unrecovered balance is used. There is more of the annual cash flow available to reduce the remaining loan when the rate is applied to the unrecovered balance as in part (a*)* and Table 1.



Plot of unrecovered balances and 10% per year rate of return on a $1000 amount, Table 1.

Rate of Return Calculation Using a PW or AW Relation

To determine the rate of return, develop the ROR equation using either a PW or AW relation, set it equal to 0, and solve for the interest rate. Alternatively, the present worth of cash outflows (costs and disbursements) PW_O may be equated to the present worth of cash inflows (revenues and savings) PW_I. That is, solve for *i* using either of the relations

0 = PW (1)

Or $\mathbf{PW_O = PW_I}$

The annual worth approach utilizes the AW values in the same fashion to solve for *i*.

0 = AW (2)

Or $\mathbf{AW_O = AW_I}$

The guideline is as follows:

If $i^* \geq$ MARR, accept the project as economically viable.

If $i^* <$ MARR, the project is not economically viable.

Example 2

Applications of green, lean manufacturing techniques coupled with value stream mapping can make large financial differences over future years while placing greater emphasis on environmental factors. Engineers with Monarch Paints have recommended to management an investment of $200,000 now in novel methods that will reduce the amount of wastewater, packaging materials, and other solid waste in their consumer paint manufacturing facility. Estimated savings are $15,000 per year for each of the next 10 years and an additional savings of $300,000 at the end of 10 years in facility and equipment upgrade costs. Determine the rate of return

Solution

Use the trial-and-error procedure based on a PW equation.

1. Figure 5–3 shows the cash flow diagram.

2. Use Equation (1) format for the ROR equation.

$0 = -200{,}000 + 15{,}000(P/A, i^*, 10) + 300{,}000(P/F, i^*, 10)$

3. Use the estimation procedure to determine *i* for the first trial. All income will be regarded as a single *F* in year 10 so that the *P*/*F* factor can be used. The *P*/*F* factor is selected because most of the cash flow ($300,000) already fits this factor and errors created by neglecting the time value of the remaining money will be minimized. Only for the first estimate of *i*, define *P* = $200,000, *n* = 10, and *F* = 10(15,000) + 300,000 = $450,000. Now we can state that

200,000 = 450,000(P/*F, i, 10*)

(P/*F, i, 10*) = 0.444

The roughly estimated *i* is between 8% and 9%. Use 9% as the first trial because this approximate rate for the *P*/*F* factor will be lower than the true value when the time value of money is considered.

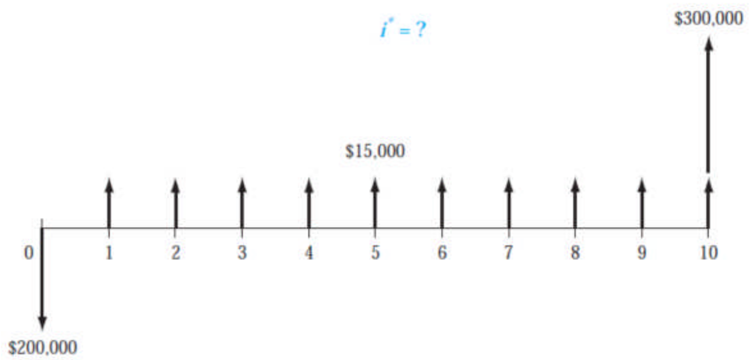


Figure 5–3 Cash flow diagram

0 = -200,000 + 15,000(P/*A, 9*%, 10) + 300,000(P/*F, 9*%, 10)

0 < $22,986

The result is positive, indicating that the return is more than 9%. Try *i* = 11%.

0 = -200,000 + 15,000(P/*A, 11*%, 10) + 300,000(P/*F, 11*%, 10)

0 > $-6002

Since the interest rate of 11% is too high, linearly interpolate between 9% and 11%.

$$i^* = 9.00 + \frac{22{,}986 - 0}{22{,}986 - (-6002)}(2.0)$$

= 9.00 + 1.58 = 10.58%

Just as *i* * can be found using a PW equation, it may equivalently be determined using an AW relation. This method is preferred when uniform annual cash flows are involved. Solution by hand is the same as the procedure for a PW-based relation, except Equation (2) is used. In the case of Example 5.2, i^* = 10.55% is determined using the AW-based relation.

$0 = -200{,}000(A/P, i^*, 10) + 15{,}000 + 300{,}000(A/F, i^*, 10)$