

Final First Semester Exam 2022/2023  
 Credit hours- Fourth Level  
 Time allowed: Two hour  
 Exam Date: Saturday (14 -1- 2023)  
 Selected Topics in Mathematics



Damietta University  
 Faculty of Science  
 Department of Mathematics  
 Math Program (J411)  
 Total Mark: 70 marks

**Answer Very Clearly the Following Questions**

**Question One. (20 mark)**

(a) - (10 marks). Assume that the RV,  $X$  can be, 0, 1 or, -1 and the RV  $Y$  can be, 0 or, 1 and the JPMF of,  $(X, Y)$  is given by:  $p_{X,Y}(x, y) = \frac{1}{4(x^2 + y)}$  when,  $x^2 + y > 0$ . What are

$$p_X(x), p_Y(y), F_{X,Y}(x, y), F_X(x), \text{ and } F_Y(y).$$

(b) -(10 marks). If the JPDF of  $(X, Y)$  is,  $f_{X,Y}(x, y) = \begin{cases} 4xy, & \forall 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

Compute the following

1- the marginal probabilities,  $f_X(x)$  and,  $f_Y(y)$ .

2- the conditional probability,  $f_{X|Y}(x | y)$ .

3- the probability that one of the two RVs,  $X$  and,  $Y$  is greater than 0.9.

**Question Two. (20 mark)**

(a) - (10 marks). Show that the expected value of the conditional variance of the RV,  $X$  given,  $Y = y$  is given by:  $E[Var(X|Y)] = E[X^2] - E[(E[X|Y])^2]$ .

(b) - (10 marks). If,  $X$  and,  $Y$  are two independent Poisson RVs with respective parameters,  $\lambda_1$  and,  $\lambda_2$ . Find:  $E[X | X + Y = n]$ , and,  $Var(X | X + Y = n)$ .

**Question Three. (15 mark)**

(a) - (9 marks). If the bivariate RV,  $(X, Y)$  follows the trinomial distribution with parameters,  $(n; p, q)$ . Show that the conditional PMF,  $p_{X|Y}(x | y)$  of,  $X | Y$  follows the

**binomial distribution** with parameters,  $\left(n - y; \frac{p}{1 - q}\right)$ . Find,  $E[X | Y]$  &  $Var(X | Y)$ .

(b) - (6 marks). If the bivariate RV,  $(X, Y)$  follows the bivariate Normal distribution. Complete the following:

i- the JPDF is given by,  $f_{X,Y}(x, y) = Ke^{-\frac{1}{2}Q(x, y)}$  with,  $K = \dots$ , and  $Q(x, y) = \dots$ .

ii- the conditional PDF,  $f_{X|Y}(x | y)$  follows a normal with,  $\mu_{X|Y} = \dots$ , and,  $Var(X | Y) = \dots$ .

**Question Four. (15 mark)**

Let,  $X$  be a continuous RV has PDF,  $f_X(x)$  and, CDF,  $F_X(x)$ . Find the PDF of the RV

(a) - (5 marks).  $Y = aX + b$  where,  $a$  and,  $b$  are constants.

(b) - (5 marks).  $Y = X^2$  in general.

(c) - (5 marks). If,  $X$  has standard normal distribution, what is the distribution of,  $Y = X^2$ .

Good Luck.

Prof. Dr. M. A. El-Shehawey