Answer the following questions:

Question One: [30 marks]

(a) [15 marks] Show that the following wave function ψ satisfies the Heisenberg uncertainty principle

$$\psi(x) = \begin{cases} 2\alpha^{\frac{3}{2}} xe^{-\alpha x} & if \ x \succ 0 \\ 0 & if \ x \prec 0 \end{cases}$$

(b) [15 marks] By using the Schrodinger equation show that the expectation value satisfies the following relation:

$$\frac{d}{dt} \prec p_y \succ = - \prec \frac{\partial V}{\partial y} \succ$$

Question Two: [25 marks]

Solve the Schrodinger equation of the two -dimensional free particle

Where the potential energy is defined by $v(x,y) = \begin{cases} 0 & if \ 0 \le x \le a \ , 0 \le y \le b \\ \infty & otherwise \end{cases}$

(Find the wave function ψ and the energy E).

Question Three: [25 marks]

(a) [05 marks] Writ the origins of quantum mechanics.

(b) [20 marks] show that the following operators $p_x^2, \hat{p_z}$ are linear and Hermitian.

Question Four: [25 marks]

(a) [15 marks] Show that the Hermitian operator $\hat{H}$ satisfies the following relation:

$$\left[\hat{H}, \hat{p_z} \right] = ih \frac{\partial V}{\partial z}$$

(b) [10 marks] Show that the angular momentum operator satisfies the following relation:

$$\left[L^2, \hat{L_x} \right] = o$$

With Best Wishes

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