Answer the following questions: (Total Degree: 70 marks)

1- Explain the equivalent between Active gravitational mass and Passive gravitational mass

(10 marks)

2- Prove that the components of the Christoffel symbols $\Gamma^{\alpha}_{\mu\beta}$ is not a tensor. (10 marks)

3- Calculate Christoffel symbols for the field in the form of surface of sphere with unit radius, where the metric of this field is given by

$$ds^2 = d\theta^2 + \sin^2\theta \, d\phi^2$$
(10 marks)

4- Derive the geodesic equations. (10 marks)

5- Explain the Length contraction in a gravitational field. (10 marks)

6- Derive the differential equations the orbit of the planets in a Schwartzschild spacetime, where the Schwartzschild timeline element is given by

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)c^2 dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\phi^2$$

(10 marks)

7- Show how the general theory of relativity explains the phenomenon of bending light.

(10 marks)

With Best Wishes

Dr. Mohammed Elhagary