الفرقة: الرابعة الشعبة: الاحصاء وعلوم الحاسب المادة: نظرية القياس الكود: (424 ر) التاريخ: 23 / 1 /2022 الزمن: 3 ساعات الدرجة الكلية: 90 درجة	نموذج امتحان نهائي الفصل الدراسي الاول لعام 2023-2022	جامعة دمياط Damietta University جامعة دمياط كلية العلوم قسم الرياضيات

Answer the following questions:

Q1: **(22 Marks)**

a- Let $I = \{[a,b): a, b \in \mathbb{R}, a \leq b\}, \sigma(I)$ be the smallest σ- algebra containing I and $\beta(\mathbb{R})$ be Borel σ-algebra of $\mathbb{R}$. Show that $\sigma(I) = \beta(\mathbb{R})$. **(8 Marks)**

b- Give the meaning of following: **σ –algebra, an outer measure on X, Lebesgue measurable.** **(6 Marks)**

c- Let X be a nonempty set and $\Sigma = P(X)$. Define $\mu: \Sigma \to [0, \infty)$ by

$$\mu(A) = \begin{cases} n & \text{if } A \text{ has n element} \\ \infty & \text{otherwise.} \end{cases}$$

Is μ a measure space on $P(X)$? **(8 Marks)**

Q2: **(22 Marks)**

a- Let (X, Σ, μ)be a measure space, and (A_n) is a sequence in Σ. Prove that **(10 Marks)**

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} \mu(A_n).$$

b- Let X be a set, μ^* an outer measure on X and, E and F are μ^* measurable subsets of X. Show that $E \cup F$ is μ^* measurable on X. **(6 Marks)**

c- Let X be a set, μ^* an outer measure on X and $E_1, E_2 \in M$, $E_1 \cap E_2 = \emptyset$ where M is the collection of all μ^*-measurable subsets of X. Show that

$$\mu^*(A \cap (E_1 \cup E_2)) = \mu^*(A \cap E_1) + \mu^*(A \cap E_2)$$ **(6 Marks)**

Q3: **(22 Marks)**

a- Show that every countable subset of $\mathbb{R}$ has outer measure zero. **(6 Marks)**

b- Let (X, Σ) be a measurable space and $E \in \Sigma$, $f: E \to \mathbb{R}$ and f is measurable. Show that the set $\{x \in E: f(x) \geq \alpha, \alpha \in \mathbb{R}\}$ is measurable. **(6 Marks)**

c- Assume that f and g are measurable real-valued function defined on a domain $E \in \Sigma$. Show that $f - g, f.g$ and $|f|$ are measurable functions. **(10 Marks)**

Q4: **(24 Marks)**

a- Let (X, Σ, μ) be a measure space, φ non-negative simple functions. Show for any $A, B \subseteq X$, that $\int_{A \cup B} \varphi d\mu = \int_A \varphi d\mu + \int_B \varphi d\mu$. **(6 Marks)**

b- Let (X, Σ, μ) be a measure space. For any $A \in \Sigma$ and the function $\nu: \Sigma \to [0, \infty]$ defined by $\nu(A) = \int_A \varphi d\mu$, Show that the function $\nu(A)$ is a measure on X. **(6 Marks)**

c- Let f be non-negative measurable functions and c-non-negative real number. Show that

$$\int_X cf d\mu = c \int_X f d\mu.$$ **(6 Marks)**

d- Let f and g be non-negative measurable functions and $f \leq g$. Show that

$$\int_X f d\mu \leq \int_X g d\mu.$$ **(6 Marks)**

مع أطيب التمنيات بالتوفيق

أستاذ المقرر: د / وفاء قوطه

رئيس قسم الرياضيات: أ.د/أحمد محمد كامل طرابيه