Damietta University
Faculty of science
Mathematical Department

Third Level Mathematics Students
Final Exam of the first semester 2022/2023
Theory of Elasticity
Time: three Hours **Date: 15-01-2023**

Answer the following questions:

1- a- Defined the Kronecker Delta and Permutation tensors, then expand the following expressions and simplify where possible.

i) $\varepsilon_{3jk}a_j a_k$ ii) $\varepsilon_{ijk}\delta_{kj}$ iii) $\varepsilon_{1jk}a_2 T_{kj}$ **(10 marks)**

b- If A_i is a first-order Cartesian tensor, show that its derivative with respect to x_k, namely $A_{i,k}$ is a second-order Cartesian tensor. **(11 marks)**

2- a- Determine the principal values and principal directions of the second-order tensor **T** whose matrix representation is **(9 marks)**

$$[T_{ij}]=\begin{bmatrix}5 & 2 & 0\\ 2 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$$

b- With respect to axes $Ox_1x_2x_3$ the stress state is given in terms of the coordinates by the matrix

$$[\sigma_{ij}]=\begin{bmatrix}x_1x_2 & x_2^2 & 0\\ x_2^2 & x_2x_3 & x_3^2\\ 0 & x_3^2 & x_3x_1\end{bmatrix}$$

Determined

i) The body force components as functions of the coordinates if the equilibrium equations are to be satisfied everywhere. **(6 marks)**

ii) The stress vector at point $P(1,2,3)$ on the plane whose outward unit normal makes equal angles with the positive coordinate axes. **(6 marks)**

3- a- Drive the equilibrium equations of the elastic body **(9 marks)**

b- Given the deformation expressed by

$$x_1=X_1+AX_2^2, \qquad x_2=X_2, \qquad x_3=X_3-AX_2^2$$

where A is a constant, determine the Lagrangian finite strain tensor **E**. **(12 marks)**

4- a- What is the physical meaning of the diagonal elements of the strain tensor ε_{22}? **(9 marks)**

b- Show that for an isotropic linear elastic solid the principal axes of the stress and strain tensors coincide, and develop an expression for the relationship among their principal values. **(12 marks)**

5- a- Prove that the tensor of elastic coefficients C_{ijkl} is a fourth order tensor and show that it satisfy

$C_{ijkl}=C_{jikl}=C_{ijlk}$. **(10 marks)**

b- Use Hooke's law for isotropic media to drive the equations of motion in the terms of displacement components for thermoelastic theory (Navier's Equations). **(11 marks)**

With best wishes

Dr. Mohammed El-Hagary