

الشعبة: إحصاء وعلوم الحاسب الشعبة: إحصاء وعلوم الحاسب المتحان نهائي دور مايو ٢٠٢٣ المادة: تحليل تتابعي الكود: (٤٣٢ ر)



التاريخ: الإثنين ١٢ / ٦ /٢٠٢٣ الزمن: ٣ ساعات الدرجة الكلية: 90 درجة

Answer the following questions (18 Marks for each)

Question no. 1 (18 marks)

1) Let k > 0, be a constant and W be a critical region of size α such that

$$W = \left\{ x \in S : \frac{f(x, \theta_1)}{f(x, \theta_0)} > k \right\}, \text{ or } W = \left\{ x \in S : \frac{L_1}{L_0} > k \right\}$$

and $\overline{W} = \left\{ x \in S : \frac{L_1}{L_2} \le k \right\}$ where L_0 and L_1 are the likelihood functions of the sample observations X: $\{x_1, x_2, ..., x_n\}$ under $H_0: \theta = \theta_0$ and $H_1: \theta = \theta_1$, respectively. Prove that W is the most powerful critical region of the given MP test.

2) Show that for the normal distribution with zero mean and variance σ^2 , the best critical region for testing H_0 : $\sigma = \sigma_0$ against H_1 : $\sigma = \sigma_1$ is of the form $\sum_{i=1}^n x_i^2 \le a_\alpha$ for $\sigma_0 > \sigma_1$ and $\sum_{i=1}^n x_i^2 \ge b_\alpha$ for $\sigma_0 < \sigma_1$. Show that the power of the best critical region when $\sigma_0 > \sigma_1$ is:

$$F\left(\frac{{\sigma_0}^2}{{\sigma_1}^2}\chi_{\alpha,n}^2\right) = p(\chi^2(n) \le \frac{{\sigma_0}^2}{{\sigma_1}^2}\chi_{\alpha,n}^2)$$

Where $\chi_{\alpha,n}^2$ is lower 100 α – per cent point and F(.) is the distribution function of the $\chi^2(n)$ -distribution with n degrees of freedom. (9 marks)

Question no. 2 (18 marks)

- 1) Define A sufficient statistics
- 2) Use the Neyman-Pearson Lemma to obtain the best critical region for testing H_0 : $\theta = \theta_0$ (simple hypothesis) Against H_1 : $\theta = \theta_1 > \theta_0$ and $\theta = \theta_1 < \theta_0$. In the case of a normal $N(\theta, \sigma^2)$, where σ^2 is known. Find the power of the test.

Question no. 3 (18 marks)

- 1. Explain what we mean by the *Parameter Space*
- 2. Let X have a PDF of the form $f(x,\theta) = \frac{1}{\theta} Exp(\frac{-x}{\theta}), \quad 0 < x < \infty, \theta > 0$

 \longrightarrow Please see the 2^{nd} paper of the exam.

Test $H_0: \theta=2$ against $H_1: \theta=1$. Use a random sample with size two X_1, X_2 and define a critical region = $\{(x_1, x_2): 9.5 \le x_1 + x_2\}$. Find Significance level of the test.

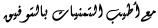
3. Examine whether a best critical region exists for testing the null hypothesis H_0 : $\theta = \theta_0$ against H_1 : $\theta = \theta_1 > \theta_0$, for the parameter θ of the distribution $f(x,\theta) = \frac{1+\theta}{\lceil x+\theta \rceil^2} \quad , \qquad 1 < x < \infty.$

Question no. 4 (18 marks)

- 1. Define a likelihood ratio test for testing a null hypothesis H_0 .
- 2. Given two independent random samples X_{1i} , i=1,2,3,...m, and X_{2j} , j=1,2,3,...,n form $N(\mu_1,\sigma_1^2)$ and $N(\mu_2,\sigma_2^2)$ respectively, where the means μ_1 and μ_2 and the variances σ_1^2 and σ_2^2 are unspecified. Develop LR test for testing $H_0: \sigma_1^2 = \sigma_2^2 = \sigma^2$, (*unspecified*) against $H_1: \sigma_1^2 \neq \sigma_2^2: \mu_1$ and μ_2 (*unspecified*).

Question no. 5 (18 marks)

- 1. Construct SPRT for testing the hypothesis H_0 : $\theta = \theta_0$ against H_1 : $\theta = \theta_1$ based on a random sample drawn from the following distribution with density function $f(x;\theta) = \begin{cases} \theta e^{-\theta x}, & x \ge 0, \theta > 0 \\ 0, & elsewhere \end{cases}$
 - Also, obtain the operating characteristic (OC) function and average sample number (ASN) function in sequential analysis.
- 2. What a sequential method is characterized by?



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