الفرقة: الرابعة علوم
الشعبة: إحصاء وعلوم الحاسب
المادة: تحليل تتابعي
الكود: (٤٣٢ ر)

امتحان نهائي دور مايو ٢٠٢٣

جامعة دمياط
كلية العلوم
قسم الرياضيات

التاريخ: الإثنين ١٢ / ٦ /٢٠٢٣ الزمن: ٣ ساعات الدرجة الكلية: 90 درجة

Answer the following questions (18 Marks for each)

Question no. 1 (18 marks)

1) Let $k > 0$, be a constant and W be a critical region of size α such that

$W = \left\{x \in S\colon \frac{f(x,\theta_1)}{f(x,\theta_0)} > k\right\}$, or $W = \left\{x \in S\colon \frac{L_1}{L_0} > k\right\}$

and $\bar{W} = \left\{x \in S\colon \frac{L_1}{L_0} \le k\right\}$ where L_0 and L_1 are the likelihood functions of the sample observations X: $\{x_1, x_2, \ldots, x_n\}$ under $H_0{:}\theta = \theta_0$ and $H_1{:}\theta = \theta_1$, respectively. Prove that W is the most powerful critical region of the given MP test.

2) Show that for the normal distribution with zero mean and variance σ^2 , the best critical region for testing $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma = \sigma_1$is of the form $\sum_{i=1}^{n} {x^2}_i \le a_\alpha$ for $\sigma_0 > \sigma_1$ and $\sum_{i=1}^{n} {x^2}_i \ge b_\alpha$ for $\sigma_0 < \sigma_1$. Show that the power of the best critical region when $\sigma_0 > \sigma_1$is:

$$F\left(\frac{{\sigma_0}^2}{{\sigma_1}^2}{\chi_{\alpha,n}}^2\right) = p(\chi^2(n) \le \frac{{\sigma_0}^2}{{\sigma_1}^2}{\chi_{\alpha,n}}^2)$$

Where${\chi_{\alpha,n}}^2$is lower 100 $\alpha - per\ cent\ point$ and $F(.)$ is the distribution function of the $\chi^2(n)$-distribution with n degrees of freedom. (9 marks)

Question no. 2 (18 marks)

1) Define A sufficient statistics

2) Use the Neyman-Pearson Lemma to obtain the best critical region for testing $H_0\colon \theta = \theta_0$ (simple hypothesis) Against $H_1\colon \theta = \theta_1 > \theta_0$and $\theta = \theta_1 < \theta_0$. In the case of a normal $N(\theta, \sigma^2)$, where σ^2 is known. Find the power of the test.

Question no. 3 (18 marks)

1. Explain what we mean by the *Parameter Space*
2. Let X have a PDF of the form $f(x,\theta) = \frac{1}{\theta}Exp(\frac{-x}{\theta})$, $0 < x < \infty, \theta > 0$

➡ Please see the 2nd paper of the exam.

Test $H_0 : \theta = 2$ against $H_1 : \theta = 1$. Use a random sample with size two X_1, X_2 and define a critical region $= \{(x_1, x_2) : 9.5 \le x_1 + x_2\}$. Find Significance level of the test.

3. Examine whether a best critical region exists for testing the null hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 > \theta_0$, for the parameter θ of the distribution

$$f(x, \theta) = \frac{1+\theta}{[x+\theta]^2} \quad , \qquad 1 < x < \infty.$$

<u>Question no. 4</u> (18 marks)

1. Define a likelihood ratio test for testing a null hypothesis H_0 .
2. Given two independent random samples X_{1i}, i = 1,2,3,..m, and $X_{2j,}$, $j = 1,2,3,\ldots,n$ form $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively, where the means μ_1 and μ_2 and the variances σ_1^2 and σ_2^2 are unspecified . Develop LR test for testing $H_0 : \sigma_1^2 = \sigma_2^2 = \sigma^2$, ($\boldsymbol{unspecified}$) against

   $H_1 : \sigma_1^2 \ne \sigma_2^2 : \mu_1$ and μ_2 ($\boldsymbol{unspecified}$).

<u>Question no. 5</u> (18 marks)

1. Construct SPRT for testing the hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ based on a random sample drawn from the following distribution with density function $f(x;\theta) = \begin{cases} \theta\, e^{-\theta x}, & x \ge 0, \theta > 0 \\ 0 \quad , & elsewhere \end{cases}$

   Also, obtain the operating characteristic (OC) function and average sample number (ASN) function in sequential analysis.
2. What a sequential method is characterized by?

مع أطيب التمنيات بالتوفيق

Prof. Dr. Ahmed M.K. Tarabia
Head of Mathematics Department.