جامعة دمياط
كلية العلوم
قسم الرياضيات

امتحان نهائي دور مايو ٢٠٢٣

الفرقة: الرابعة علوم
الشعبة: إحصاء وعلوم الحاسب
المادة: حساب متغيرات
الكود: (٤٣٣ر)

التاريخ: الإثنين ٥/ ٦ /٢٠٢٣ **الزمن:** ٣ ساعات **الدرجة الكلية:** 90 درجة

Answer the following questions (18 Marks for each)

Question no. 1 (18 Marks ,6 Marks each)

1. Define ***Strong relative minimum for the functional?***
2. **Consider** $[y(x)] = \int_0^1 (x\,y^2 + y'^3)\,dx$. **Obtain** δI **, and** $\delta^2 I$.
3. Find the equation of the curve between the two points (0,0) , $(2\pi, 0)$ and makes the following functional has extreme points.

$$I = \int_1^2 \sqrt{\frac{1+y'^2}{y}}\,dx$$

Question no. 2 (18 Marks, 9 each)

1. Prove that Euler-Lagrange equation satisfied if and only if its solution is a linear function in y'.
2. Discuss the functional $t[y(x)] = \int_{x_0}^{x_1} \frac{1}{x}\sqrt{1+y'^2}\,dx$, which represents the time taken to move from (x_0, y_0) to (x_1, y_1) on the curve $y(x)$.

Question no. 3 (18 Marks, 9 each)

1. Consider $I = \int_A \int F(x, y, z, z_x, z_y)\,dx\,dy$, where A is the area determined by the curve C .Obtain **Euler-Lagrange equation** for **double Integration** to determine the plane $Z = z(x, y)$ which makes this integration extreme.
2. Obtain the extreme functions for the Integration:

$$I[y, z] = \int_0^{\pi} (y'^2 + 2yz - z'^2 - 2y^2)\,dx$$

Which satisfy the Boundary conditions $y(0) = 0,\ y(\pi) = 1,\ z(0) = 0,$

$z(\pi) = -1$

▬► Please see the 2nd paper of the exam.

<u>Question no. 4</u> (18 Marks, 9 each)

1. What are the available two approaches to solve the conditional Extremum problem?
2. Find the curve $y = y(x)$ which satisfies $y(a) = y(-a) = 0$ and has a length $l > 2a$ and surround the segment $-a \leq x \leq a$ to give the maximum area.

<u>Question no. 5</u> (18 Marks, 9 each)

1. Specify the functional field of the following functional:

$$I[y] = \int_0^2 \left(y'^3 + sin^2 x\right)dx$$

Where $y(0) = 1,\ y(2) = 4$

2. Study if Jacobi's condition satisfies for the extreme curve of the following functional :

$$I[y] = \int_0^a (y'^2 + y^2 + x^2)\, dx$$

and passes by the points $A(0,0)$, $B(a,0)$.

مع أطيب التمنيات بالتوفيق

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