



المستوى: الرابع (ساعات معتمدة)
البرنامج: الرياضيات
المادة: نظرية القياس
الكود: (416 ر)
التاريخ: 2023/ 6 / 22
الزمن: ساعتان
الدرجة الكلية: 70 درجة

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الفصل الدراسي الثاني
2023/2022



Answer only four questions:

Q1:

(14 Marks)

- a- Give the meaning of following: σ -algebra, Measure on a set, Outer measure on a set.
b- Let $X = \mathbb{N}$ and $\Sigma = P(\mathbb{N})$. Define $\mu: \Sigma \rightarrow [0, \infty)$ by

$$\mu(A) = \begin{cases} 0 & \text{if } A \text{ is finite} \\ \infty & \text{otherwise} \end{cases}$$

Is μ a measure space on $P(\mathbb{N})$?

- c-Let (X, Σ, μ) be a measure space, and (A_n) is a sequence in Σ such that $A_n \subseteq A_{n+1}, n \in \mathbb{N}$. Prove that

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n \rightarrow \infty} \mu(A_n)$$

Q2:

(14 Marks)

- a- Let X be a set, μ^* an outer measure on X and M be the collection of all μ^* -measurable subsets of X . Prove that M is σ -algebra.
b- Let $A \subset \mathbb{R}$ and $m^*: P(\mathbb{R}) \rightarrow [0, \infty)$ by

$$m^*(A) = \inf \left\{ \sum_n l(I_n) : I_n \in I, A \subset \bigcup_n I_n \right\}$$

Show that m^* is an outer measure.

Q3:

(14 Marks)

- a- Prove that the outer measure of an interval I is its length.
b- Show that the Lebesgue outer measure is translation-invariant.
c- Show that every countable subset of \mathbb{R} has outer measure zero.

Q4:

(14 Marks)

- a- Show that every interval in \mathbb{R} is measurable.
b- Assume that f and g are measurable real-valued function defined on a domain $E \in \Sigma$. Show that $f + g, f \cdot g$ and $|f|$ are measurable functions.

Q5:

(14 Marks)

- a- Let (X, Σ, μ) be a measure space, φ and ψ non-negative simple functions and c -non-negative real number. Prove all the following statements:
1. $\int_{A \cup B} \varphi d\mu = \int_A \varphi d\mu + \int_B \varphi d\mu, \forall A, B \subseteq X$.
2. $\int_X (\varphi + \psi) d\mu = \int_X \varphi d\mu + \int_X \psi d\mu$.
b- Let (X, Σ, μ) be a measure space. For any $A \in \Sigma$ and the function $\nu: \Sigma \rightarrow [0, \infty]$ defined by $\nu(A) = \int_A \varphi d\mu$, Show that the function $\nu(A)$ is a measure on X .

انتهت الأسئلة

مع أطيب التمنيات بالتوفيق

رئيس قسم الرياضيات: أ.د/أحمد محمد كامل طرايبه

دكتور المادة: د / وفاء قوطه