

Answer the following questions:

Question One: [20marks] prove that the Maxwell's equations in free space have the following form:

(1)
$$div\vec{E} = 4\pi\rho$$
 (2) $div\vec{H} = 0$ (3) $Curl\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}$ (4) $Curl\vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$

Question Two: [23 marks] (a) [13 marks] Show that the equations of the potentials \vec{A} and $\vec{\phi}$ in free space are:

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \vec{A} = -\frac{4\pi}{c} \vec{j} \qquad and \left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \vec{\phi} = -4\pi\rho$$

(b) [10 marks] By using the Maxwell's equations in free space Show that the Law of conservation of charge is: $div\vec{J} + \frac{\partial \rho}{\partial t} = 0$ where \vec{J} is the current and ρ is the density.

Question Three: [20 marks] prove that the Maxwell's equations in the medium have the following form:

(1)
$$div\vec{D} = 4\pi\rho$$
 (2) $div\vec{B} = 0$ (3) $Curl\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$ (4) $Curl\vec{H} = \frac{4\pi}{c}J + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$

where $\vec{D} = \vec{E} + 4\pi\rho$ and $\vec{B} = \vec{H} + 4\pi M$

Question Four: [20 marks] show that the wave equation in the vacuum is:

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) A = 0 \qquad and \left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \phi = 0$$

And show that \vec{E} and \vec{H} satisfy the same relation in the vacuum.

Question five: [22 marks] (a) [11 marks] prove the Law of conservation of the energy of the electromagnetic field in a medium.

(b) [11 marks] Solve the plane waves equations

$$\frac{\partial^2 A}{\partial t^2} - c^2 \frac{\partial^2 A}{\partial x^2} = 0 \qquad and \qquad \frac{\partial^2 \phi}{\partial t^2} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0$$

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(3)
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 (4) $Curl\vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$

Question Two: [23 marks]

(a) [13 marks] Show that the equation of the potentials \vec{A} and $\vec{\phi}$ in free space have the following form:

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \vec{A} = -\frac{4\pi}{c} \vec{j} \qquad and \left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \vec{\phi} = -4\pi\rho$$

(b) [10 marks] by using the Maxwell's equations in free space Show that the Law of conservation of charge is: $div\vec{J} + \frac{\partial \rho}{\partial t} = 0$ where \vec{J} is the current and ρ is the density.

Question Three: [20 marks]

prove that the Maxwell's equations in the medium have the following form:

(1)
$$div\vec{D} = 4\pi\rho$$

(2)
$$div\vec{B} = 0$$

(3)
$$Curl\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$
 (4) $Curl\vec{H} = \frac{4\pi}{c}J + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$

where
$$\vec{D} = \vec{E} + 4\pi\rho$$
 and $\vec{B} = \vec{H} + 4\pi M$

Question Four: [20 marks]

show that the wave equation in the vacuum is:

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) A = 0 \qquad and \left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \phi = 0$$

And show that \vec{E} and \vec{H} satisfy the same relation in the vacuum.

Question five: [22 marks]

(a) [11 marks] prove the Law of conservation of the energy of the electromagnetic field in a medium.

(b) [11 marks] Solve the plane waves equations

$$\frac{\partial^2 A}{\partial t^2} - c^2 \frac{\partial^2 A}{\partial x^2} = 0 \qquad and \frac{\partial^2 \phi}{\partial t^2} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0$$

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