Marks

Answer each of the following questions:

Q1: (28 marks)

7 **1-** Define the following: Oscillatory equation – Stable critical point – Singular point.

7 **2-** State and prove Peano's uniqueness theorem for the IVP $y' = f(x,y),\ y(x_0) = y_0$.

7 **3-** Test if Lipschitz condition holds for the function $f(x,y) = \sin(xy)$ on the domain $D: |x| \le 1,\ |y| < \infty$.

7 **4-** Prove the existence of a unique solution of the IVP $y' = x\cos y + y^2,\quad y(0) = 0$ on the domain $D: |x| \le 1,\ |y| \le 1$.

Q2: (24 marks)

1- Check the linearity and homogeneity of each of the following systems:

2 **(i)**
$$\begin{aligned} x'(t) &= 2t + x(t) + y(t) \\ y'(t) &= 2x(t) - y(t) \end{aligned}$$

2 **(ii)**
$$\begin{aligned} x'(t) &= x^2(t) - y(t) + t \\ y'(t) &= x(t) + y(t) + e^{z(t)} + t \\ z'(t) &= 2x(t) + 3y(t) + z(t) \end{aligned}$$

2- Solve each of the following linear systems:

5 **(i)** $u' = \begin{pmatrix} 2 & 2 & 2 \\ -3 & -3 & -3 \\ 1 & 1 & 1 \end{pmatrix} u$

5 **(ii)** $u' = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} u$

5 **(iii)** $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^t \\ 0 \end{pmatrix}$

5 **3-** Prove that the fundamental matrix Φ of the system $\mathrm{x}' = A\mathrm{x}$, ($A$ *is* $n \times n$ constant matrix) on R, with $\Phi(0) = \mathrm{I}$, has the exponential representation $\Phi(t) = e^{At}$ on R.

Please, see the other page

Q3: (25 marks)

9 **1-** Determine the type and stability nature of the critical point $(0, 0)$ for the system

$$\frac{dx}{dt} = 2x - y, \qquad \frac{dy}{dt} = x + 2y$$

2- Investigate the stability of the critical point $(0, 0)$ in each of the following cases:

8 **(i)** $\frac{dx}{dt} = x + y + x^4y^4, \qquad \frac{dy}{dt} = 4x + y - x^3y^3$

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8 **(ii)** $\frac{dx}{dt} = -x^3 + xy^3, \qquad \frac{dy}{dt} = -x^2y - x^2y^2.$

Q4: (28 marks)

7 **1-** Determine the ordinary, singular, and regular singular points of the equation:

$$x(x-1)^2y'' + xy' - y = 0$$

7 **2-** Solve the equation $3xy'' + y' - y = 0$.

7 **3-** State and prove Sturm's comparison theorem for second order differential equations.

7 **4-** Study the oscillation of the equation $y'' + \frac{1+x}{4x^2}y = 0$ on $(0, \infty)$.

With my best wishes

Professor Dr. Hassan El-Morshedy