

Marks | **Answer each of the following questions:**

Q1: (12 Marks) Classify each of the following equations according to *degree, order, homogeneity, and linearity*:

- 4 1. $\frac{y'''}{y'} + y = 0$
- 4 2. $y'y'' - \sin y = 0$
- 4 3. $(y'')^{1/3} + (y')^{1/2} = 0$

Q2: (10 Marks)

- 5 1. Form a differential equation by eliminating the arbitrary constants A, B from $y(x) = Ax^2 + Bx$.
- 5 2. Find all possible values of the constant α so that the function $y(x) = e^{\alpha x}$ is a solution of the equation $yy'' - (y')^2 = 0$.

Q3:(40 Marks) Find the general solution of each of the following equations:

- 8 1. $y' - \frac{2}{x}y = x$
- 8 2. $y' = (x + y)^2$
- 8 3. $y' = -\frac{1+x+y^2}{2xy+y}$
- 8 4. $p^6 + p^4 + 2p - 4 = 0$ ($p = y'$)
- 8 5. $\cos y y' + x \sin y = x$

Q4:(43 Marks) Solve each of the following equations:

- 8 1. $y'' - 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = 0$
- 8 2. $x^2y'' + y = 0$
- 9 3. $y'' - 2y' - 3y = e^{3x}$
- 9 4. $\frac{dx}{dt} - 2y = 0, \quad \frac{dy}{dt} + x = t$ (System)
- 9 5. $y'' + y = t, \quad y(0) = y'(0) = 0$ (Use Laplace transformation)

Hint: $\mathcal{L}\{t\} = \frac{1}{s^2}, \quad \mathcal{L}\{\sin bt\} = \frac{b}{s^2+b^2}, \quad s > 0.$