Final First Semester Exam 2023/2024
Credit hours- Fourth Level
Time allowed: Two hours
Exam Date: 11 January 2024
Selected Topics in Mathematics

جامعة دمياط
Damietta University

Damietta University
Faculty of Science
Department of Mathematics
Math Program (ر411)
Total Mark: 70 marks

Answer Very Clearly the Following Questions

Question One [20 marks]

(a) -(10 marks)- In the following, check whether X and Y are independent or not:

i- Let X and Y be integer-valued RV's with JPMF

$$p_{X,Y}(x,y)=\begin{cases} q^2 p^{x+y-2}, & x,y=1,2,\dots,\ p+q=1 \\ 0, & \text{otherwise} \end{cases}$$

ii- If the JPDF the bivariate RV (X,Y) is given by

$$f_{X,Y}(x,y)=\begin{cases} x+y, & 0<x<1,\ 0<y<1 \\ 0, & \text{otherwise} \end{cases}$$

(b) -(10 marks)- If the conditional PDF $f_{Y|\Lambda}(y|\lambda)$ has exponential distribution with parameter λ, and the RV Λ has a PDF:

$$f_\Lambda(x)=\frac{\beta^\alpha x^{\alpha-1}}{\Gamma(\alpha)}e^{-\beta x},\quad x>0.$$

i- Calculate the PDF of the RV Y .

ii-Find the conditional PDF $f_{\Lambda|Y}(\lambda|y)$ and write your remark for the corresponding distribution.

Question Two [20 mark]

(a) -(4 marks)- Show that the expected value of the conditional variance of the RV X given $Y=y$ is given by: $E[Var(X|Y)]=E[X^2]-E[\mu^2_{X|Y}]$.

(b) If the JPDF of the bivariate RV (X,Y) is given by

$$f_{XY}(x,y)=\begin{cases} 2x, & 0\le x\le k,\ 0\le y\le 1 \\ 0, & \text{otherwise} \end{cases}$$

i- (6 marks)- Compute the value of k , and the values of the JCDFs $F_{XY}(0.7,0.5)$, $F_{XY}(2,0)$ and $F_{XY}(0.2,3)$.

ii- (8 marks)- Find

- the marginal PDFs for the two RV's X and Y .
- the conditional PDF for X given Y and the conditional PDF for Y given X .

iii- (2 marks)- Write your remark, with type the cause, about the relation between the two RVs X and Y.

Question Three [15 mark]

(a)-(5 marks)- Show that the value of the correlation coefficient between two RVs X and Y is $-1 \le \rho_{X,Y} \le 1$.

(b)-(10 marks)- If the bivariate RV, (X,Y) follows the trinomial distribution with parameters, $(n;p,q)$. Show that the conditional PMF, $p_{X|Y}(x|y)$ of, $X|Y$ follows the **binomial distribution** with parameters, $\left(n-y;\frac{p}{1-q}\right)$. Find, $E[X|Y]$ & $Var(X|Y)$.

Question Four [15 mark]

(a)-(3 marks)- If the bivariate RV, (X,Y) follows the bivariate normal distribution. Complete the following:

(i) the JPDF is given by, $f_{X,Y}(x,y) = Ke^{-\frac{1}{2}Q(X,Y)}$ with,

$$K = \cdots, \text{ and } Q(x,y) = \cdots .$$

(ii) the conditional PDF, $f_{X|Y}(x|y)$ follows a normal with,

$$\mu_{X|Y} = \cdots, \text{ and } Var(X|Y) = \cdots.$$

(b)-(12 marks)- Let X be a continuous RV has PDF, $f_X(x)$ and, CDF, $F_X(x)$. Find the CDF and the PDF of the following RVs:

(i) $Y = aX + b$ where, a and, b are constants.

(ii) $Y = aX^2$. where, a is constant.

(iii) $Y = X^2$, X has standard normal distribution.

Good Luck **Prof. Dr. M A. El-Shehawey**