جامعة دمياط
Damietta University

EXAM: Final TERM: 2
COURSE TITLE: Theory of Differential equations
PROGRAM: Mathematics
DATE: 21-6-2024

LEVEL: 3
CODE: 311 Math
MARKS: 105
DURATION: 3 hrs

DEPARTMENT OF MATHEMATICS

Marks

Answer each of the following questions:

Q1: (27 marks)

4 1- Define the following: Regular singular point – Stable critical point.

5 2- Show that the IVP $y'' = f(x,y), y(x_0) = y_0, y'(x_0) = y_1$ where $f(x,y)$ is continuous in a domain D containing the point (x_0, y_0), is equivalent to the integral

6 equation $y(x) = y_0 + (x - x_0)y_1 + \int_{x_0}^{x}(x-t)f(t, y(t))dt$.

3- Speake about the importance of Lipschitz condition for the existence of a unique solution of the IVP $y' = f(x,y),\ y(x_0) = y_0$.

6 4- Show that the IVP $y' = e^{\sin(xy)} + y\, , y(0) = 1$ has a unique solution in the domain $D: |x| \le 1,\ |y-1| \le 1$.

6 5- State and prove Peano's existence Theorem.

Q2: (24 marks)

1- Check the linearity and homogeneity of each of the following systems:

2 (i)
$$x'(t) = x(t) + y(t)$$
$$y'(t) = x^2(t) - y(t)$$

2 (ii)
$$x'(t) = y(t) + t^2$$
$$y'(t) = z(t) + e^t$$
$$z'(t) = x(t)$$

2- Solve each of the following linear systems:

5 (i) $$u' = \begin{pmatrix} 2 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix} u$$

5 (ii) $$X' = \begin{pmatrix} 4 & -1 & 1 \\ 2 & 5 & -1 \\ 0 & 0 & -3 \end{pmatrix} X$$

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5 (iii) $\mathbf{x}' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} t \\ 0 \end{pmatrix}$

5 3- If A is a continuous square matrix on R, prove that the system $u' = A(t)u$ has a fundamental matrix on R.

Q3: (28 marks)

7 1- Determine the type and stability nature of the critical point $(0,0)$ for the system

$$\frac{dx}{dt} = x + 2y, \qquad \frac{dy}{dt} = 3x + y$$

7 2- Investigate the stability of every critical point in each of the following cases:

(i) $\frac{dx}{dt} = x + x^2 + y^2, \qquad \frac{dy}{dt} = y - xy$

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7 (ii) $\frac{dx}{dt} = y^3 - x^3, \qquad \frac{dy}{dt} = -2xy^2 - 2y^3.$

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7 (iii) $u' = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ -2 & 0 & 0 \end{pmatrix} u$

Q4: (26 marks)

4 1- Determine the regular singular points of the equation:

$$x(x-1)^2 y'' + (x-1)y' - y = 0$$

6 2- Solve the following equation in series form about $x = 0$:

$$2xy'' - y' + 2y = 0.$$

6 4- State and prove Sturm–Picone's oscillation Theorem.

5+5 5- Test the oscillation of each of the following equations:

(i) $y'' + \frac{2+\sin x}{x^2} y = 0$ (ii) $(xy')' + (1 + \cos 2x)y = 0$

With my best wishes

Professor Dr. Hassan El-Morshedy