جامعة دمياط
Damietta University

كلية العلوم جامعة دمياط
Mathematics Department

Exam: Final

Term : 2 Level: 4

Program: Math

Course (code) : Abstract Algebra2 Rings and Fields (412R)

Date: 15/6/2025 Time: 3 h

Full Marks: 105

Answer the following four questions (The questions in two pages):

Question 1: (18 Marks)

(1) Let Z [$\sqrt{2}$] = $\{a+b\sqrt{2} \in R \mid a,b \in Z\}$. Prove that Z [$\sqrt{2}$] is a ring under the ordinary addition and multiplication of real numbers. **(6 Marks)**

(2) Let R = Z ⊕ Z ⊕ Z and $S = \{(a,b,c) \in R \mid a+b=c\}$. Prove or disprove that S is a subring of R. **(6 Marks)**

(3) Show that 2Z ∪ 3Z is not a subring of Z. **(6 Marks)**

Question 2: (30 Marks)

(1) Define the following: (a) an integral domain (b) a field. **(4 Marks)**

(2) Let R= {0, 2, 4, 6, 8} under addition and multiplication modulo 10. Prove that R is a field. **(6 Marks)**

(3) Give an example of (explain your answer):

(a) A non-commutative ring without unity. **(5 Marks)**

(b) A ring with zero divisors. **(5 Marks)**

(c) A commutative ring without zero-divisors that is not an integral domain. **(5 Marks)**

(d) A field of characteristic 3. **(5 Marks)**

Question 3: (25 Marks)

(1) Define the following: (a) A prime ideal (b) A maximal ideal. **(4 Marks)**

(2) Let S = {a + bi| a, b∈Z, b is even}. Show that S is a subring of Z[i] but not an ideal of Z[i]. **(5 Marks)**

(3) Show that R[x]/<x^2+1> is a field. **(8 Marks)**

(4) Give an example of an ideal of a commutative ring with unity that is prime but maximal. **(8 Marks)**

Question 4: (32 Marks)

(1) Define a ring homomorphism. **(2 Marks)**

(2) State and prove the first isomorphism theorem for rings. **(10 Marks)**

(3) Let R = $\left\{\begin{pmatrix} a & b \\ b & a \end{pmatrix} \mid a \,.\, b \in Z\right\}$, and let ∅ be the mapping that takes $\begin{pmatrix} a & b \\ b & a \end{pmatrix}$ to a-b.

(i) Show that ∅ is a homomorphism. **(4 Marks)**

(ii) Determine the kernel of ∅. **(4 Marks)**

(iii) Show that R/Ker∅ is isomorphic to Z. **(4 Marks)**

(iv) Is Ker∅ a prime ideal? **(4 Marks)**

(v) Is Ker∅ a maximal ideal? **(4 Marks)**

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My Best Wishes

Prof. Hader Elgendy